Longitudinal wave Propagation at an Imperfect Boundary of Micropolar Viscoelastic Solid and Fluid Saturated Incompressible Porous Solid

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Abstract: The aim of this study is to discuss the reflection and transmission of longitudinal wave through the imperfect boundary of micropolar viscoelastic solid half space and a fluid saturated incompressible half space. A longitudinal wave (P-wave) incident obliquely at the interface of half spaces. The amplitude ratios for reflected and transmitted waves are obtained. Then these amplitude ratios have been computed numerically for a particular model and results thus obtained are depicted graphically with angle of incidence of the incident wave. It is found that these amplitude ratios depend on angle of incidence of the incident wave as well as on the properties of media. The amplitude ratios are affected by the stiffness also. From the present study, a special case when fluid saturated porous half space reduces to empty porous solid is also deduced and discussed graphically.

Keywords: Longitudinal wave, amplitude ratios, micropolar viscoelastic solid, porous, reflection, transmission, stiffness.

Introduction

Most of natural and man-made materials, including engineering, geological and biological media, possess a microstructure. The ordinary classical theory of elasticity fails to describe the microstructure of the material. To overcome this problem, Suhubi and Eringen (1964), Eringen and Suhubi (1964) developed a theory in which they considered the microstructure of the material. Eringen (1967) developed the linear theory of micropolar viscoelasticity. Many researchers like Kumar et.al. (1990), Singh (2000), Singh (2002), discussed the problems of propagation of waves in micropolar viscoelastic medium. Based on the work of Fillunger model (1913), Bowen (1980) and de Boer and Ehlers (1990a, 1990b) developed an interesting theory for porous medium having all constituents to be incompressible. Based on this theory, many researchers like de Boer and Liu (1994, 1995), de Boer and Liu (1996), Liu (1999), Yan et.al. (1999), de Boer and Didwania (2004), Tajuddin and Hussaini (2006), Kumar and Hundal (2007), Kumar et. al. (2011) etc. studied some problems of wave propagation in fluid saturated incompressible porous media. Elastic waves propagation in fluid saturated porous media has its importance in various fields such as soil dynamics, hydrology, seismology, earthquake engineering and geophysics. Imperfect interface considered in this problem means that the stress components are continuous and small displacement field is not. The values of the interface parameters depend upon the material properties of the medium. Recently, using the imperfect conditions at an interface, Chen et.al. (2004), Kumar and Rupender (2009) and Kumar and Chawala (2010) etc. studied the various types of wave problems. Using the theory of de Boer and Ehlers given in 1990 for fluid saturated porous medium and for micropolar viscoelastic solid, the theory given by Eringen in 1967, the reflection and transmission phenomenon of longitudinal wave at an imperfect interface between micropolar viscoelastic solid half space and fluid saturated porous half space is studied. A special case when fluid saturated porous half space reduces to empty porous solid has been deduced and discussed. Amplitudes ratios for various reflected and transmitted waves are computed for a particular model and depicted with help of graphs and discussed accordingly. The model which is considered here is assumed to exist in the oceanic crust part of the earth and the propagation of wave through such a model will be of great use in the fields which are related to earth sciences.

Basic equations and Constitutive Relations

For medium $\mathbf{M_1}$ (micropolar viscoelastic solid)

Following Eringen (1967), the constitutive and field equations for a micropolar viscoelastic solid in the absence of body forces and body couples, are given below

$$t_{kl} = \lambda u_{r,r}\delta_{kl} + \mu(u_{k,l} + u_{l,k}) + \kappa(u_{l,k} - \epsilon_{klr}\phi_r), (1)$$

$$m_{kl} = \alpha\phi_{r,r}\delta_{kl} + \beta\phi_{k,l} + \gamma\phi_{l,k}, (2)$$

$$(c_1^2 + c_3^2)\nabla(\nabla.\mathbf{u}) - (c_2^2 + c_3^2)\nabla\times(\nabla\times\mathbf{u}) + c_3^2\nabla\times\boldsymbol{\phi} = \ddot{\mathbf{u}}, (3)$$

$$(c_4^2 + c_5^2)\nabla(\nabla.\boldsymbol{\phi}) - c_4^2\nabla\times(\nabla\times\boldsymbol{\phi}) + \omega_0^2\nabla\times\mathbf{u} - 2\omega_0^2\boldsymbol{\phi} = \ddot{\boldsymbol{\phi}}, (4)$$

where

$$c_1{}^2 = \frac{(\lambda + 2\mu)}{\rho},\ c_2{}^2 = \frac{\mu}{\rho}, c_3{}^2 = \frac{\kappa}{\rho},$$
$$c_4{}^2 = \frac{\gamma}{\rho j},\ c_5{}^2 = \frac{(\alpha + \beta)}{\rho j},\ \omega_0{}^2 = \frac{\kappa}{\rho j},$$
$$\lambda = \lambda' + \lambda_\upsilon{}'\left(\frac{\partial}{\partial t}\right), \mu = \mu' + \mu_\upsilon{}'\left(\frac{\partial}{\partial t}\right),$$
$$\kappa = \kappa' + \kappa_\upsilon{}'\left(\frac{\partial}{\partial t}\right), \alpha = \alpha' + \alpha_\upsilon{}'\left(\frac{\partial}{\partial t}\right),$$
$$\beta = \beta' + \beta_\upsilon{}'\left(\frac{\partial}{\partial t}\right), \gamma = \gamma' + \gamma_\upsilon{}'\left(\frac{\partial}{\partial t}\right),$$
$$\nabla = i\left(\frac{\partial}{\partial x}\right) + k\left(\frac{\partial}{\partial z}\right). (5)$$

$\lambda', \mu', \kappa', \alpha', \beta', \gamma', \lambda_\upsilon{}', \mu_\upsilon{}', \kappa_\upsilon{}', \alpha_\upsilon{}', \beta_\upsilon{}'$and $\gamma_\upsilon{}'$ are material constants, ρ is the density and j the rotational inertia. $\mathbf{u}$ and $\boldsymbol{\phi}$ are displacement and microrotation vectors respectively. Superposed dots on right hand side of equations (3) and (4) represent the second order partial derivative with respect to time. Take $\mathbf{u} = (u_1, 0, u_3)$ and $\boldsymbol{\phi} = (0, \phi_2, 0)$ and taking the potentials $\phi(x, z, t)$ and $\psi(x, z, t)$ which are related to displacement components are given below

$$u_1 = \frac{\partial\phi}{\partial x} + \frac{\partial\psi}{\partial z}, u_3 = \frac{\partial\phi}{\partial z} - \frac{\partial\psi}{\partial x}. (6)$$

Using the displacement components given by equation (6) in equations (3) and (4), we obtain

$$\left(\nabla^2 - \frac{1}{(c_1{}^2 + c_3{}^2)}\frac{\partial^2}{\partial t^2}\right)\phi = 0, (7)$$
$$\left(\nabla^2 - \frac{1}{(c_2{}^2 + c_3{}^2)}\frac{\partial^2}{\partial t^2}\right)\psi - p\phi_2 = 0, (8)$$
$$\left(\nabla^2 - 2q - \frac{1}{c_4{}^2}\frac{\partial^2}{\partial t^2}\right)\phi_2 + q\nabla^2\psi = 0, (9)$$

where

$$p = \frac{\mu}{\mu + \kappa}, q = \frac{\kappa}{\gamma}. (10)$$

The time variation can be assumed as

$$\phi(x, z, t) = \overline{\phi}(x, z) \exp(i\omega t),$$
$$\psi(x, z, t) = \overline{\psi}(x, z) \exp(i\omega t),$$
$$\phi_2(x, z, t) = \overline{\phi}_2(x, z) \exp(i\omega t). (11)$$

Using equation (11) in equations (7) - (9), we get

$$\left(\nabla^2 + \left(\omega^2/V_1{}^2\right)\right)\overline{\phi} = 0, (12)$$
$$(\nabla^4 + \omega^2 B\nabla^2 + \omega^4 C)(\overline{\psi}, \overline{\phi}_2) = 0, (13)$$

where

$$B = \frac{q(p-2)}{\omega^2} + \frac{1}{(c_2{}^2 + c_3{}^2)} + \frac{1}{c_4{}^2},$$
$$C = \frac{1}{(c_2{}^2 + c_3{}^2)}\left(\frac{1}{c_4{}^2} - \frac{2q}{\omega^2}\right), (14)$$

and V_1 is given by the relation

$$V_1{}^2 = c_1{}^2 + c_3{}^2. (15)$$

For unbounded medium, the solution of equation (12) represents the modified longitudinal displacement wave (LD wave) propagating with velocity V_1.
The solution of equation (13) can be written as

$$\overline{\psi} = \overline{\psi}_1 + \overline{\psi}_2, (16)$$

where
$\overline{\psi}_1$ and $\overline{\psi}_2$ satisfy

$$\left(\nabla^2 + \delta_1{}^2\right)\overline{\psi}_1 = 0, (17)$$
$$\left(\nabla^2 + \delta_2{}^2\right)\overline{\psi}_2 = 0, (18)$$

and

$$\delta_1{}^2 = \lambda_1{}^2\omega^2, \delta_2{}^2 = \lambda_2{}^2\omega^2, (19)$$

where λ_1 and λ_2 are give as

$$\lambda_1{}^2 = \frac{1}{2}\left[B + \sqrt{B^2 - 4C}\right],$$
$$\lambda_2{}^2 = \frac{1}{2}\left[B - \sqrt{B^2 - 4C}\right]. (20)$$

From equation (8) we get

$$\overline{\phi}_2 = E\overline{\psi}_1 + F\overline{\psi}_2,$$

where

$$E = \frac{\left(\frac{\omega^2}{c_2{}^2 + c_3{}^2} - \delta_1{}^2\right)}{p}, F = \frac{\left(\frac{\omega^2}{c_2{}^2 + c_3{}^2} - \delta_2{}^2\right)}{p}. (21)$$

Thus there are two waves propagating with velocities $\lambda_1{}^{-1}$ and $\lambda_2{}^{-1}$, each consisting of transverse displacement ψ and transverse microrotation ϕ_2.According to Parfitt and Eringen in 1969, these waves are modified coupled transverse displacement wave and transverse microrotational waves (CD I and CD II waves) respectively.

For medium M_2 (Fluid saturated incompressible porous medium)

Following de Boer and Ehlers (1990b), the governing equations in a fluid-saturated incompressible porous medium are

$$\mathrm{div}(\eta^S \dot{\mathbf{x}}_S + \eta^F \dot{\mathbf{x}}_F) = 0. (22)$$
$$\mathrm{div}\mathbf{T}_\mathbf{E}^\mathbf{S} - \eta^S \,\mathrm{grad}\, p + \rho^S(\mathbf{b} - \ddot{\mathbf{x}}_s) - \mathbf{P}_\mathbf{E}^\mathbf{F} = 0, (23)$$
$$\mathrm{div}\mathbf{T}_\mathbf{E}^\mathbf{F} - \eta^F \,\mathrm{grad}\, p + \rho^F(\mathbf{b} - \ddot{\mathbf{x}}_F) + \mathbf{P}_\mathbf{E}^\mathbf{F} = 0, (24)$$

where $\dot{\mathbf{x}}_i$ and $\ddot{\mathbf{x}}_i$ $(i = S, F)$ denote the velocities and accelerations, respectively of solid (S) and fluid (F) phases of the porous aggregate and p is the effective pore pressure of the incompressible pore fluid. ρ^S and $\rho^F$are the densities of the solid and fluid phases respectively and $\mathbf{b}$ is the body force per unit volume. $\mathbf{T}_\mathbf{E}^\mathbf{S}$ and $\mathbf{T}_\mathbf{E}^\mathbf{F}$ are the effective stress in the solid and fluid phases respectively, $\mathbf{P}_\mathbf{E}^\mathbf{F}$ is the effective quantity of momentum supply and η^S and η^F are the volume fractions satisfying

$$\eta^S + \eta^F = 1. (27)$$

If $\mathbf{u}_S$ and $\mathbf{u}_F$ are the displacement vectors for solid and fluid phases, then

$$\dot{x}_S = \dot{\mathbf{u}}_S, \ddot{\mathbf{x}}_s = \ddot{\mathbf{u}}_s, \dot{\mathbf{x}}_F = \dot{\mathbf{u}}_F, \ddot{\mathbf{x}}_F = \ddot{\mathbf{u}}_F. (28)$$

The constitutive equations for linear isotropic, elastic incompressible porous medium are given by de Boer, Ehlers and Liu (1993) as

$$\mathbf{T}_{\mathbf{E}}^{\mathbf{S}} = 2\mu^S \mathbf{E}_S + \lambda^S (E_S.\mathbf{I})\mathbf{I}, (29)$$

$$\mathbf{T}_{\mathbf{E}}^{\mathbf{F}} = 0, (30)$$

$$\mathbf{P}_{\mathbf{E}}^{\mathbf{F}} = -\mathbf{S}_v(\dot{\mathbf{u}}_F - \dot{\mathbf{u}}_S), (31)$$

where λ^S and μ^S are the macroscopic Lame's parameters of the porous solid and $\mathbf{E}_S$ is the linearized Langrangian strain tensor defined as

$$\mathbf{E}_S = \frac{1}{2}(\text{grad}\,\mathbf{u}_S + \text{grad}^T\mathbf{u}_S), (32)$$

In the case of isotropic permeability, the tensor $\mathbf{S}_v$ describing the coupled interaction between the solid and fluid is given by de Boer and Ehlers (1990b) as

$$\mathbf{S}_v = \frac{(\eta^F)^2\gamma^{FR}}{K^F}\mathbf{I}, (33)$$

where γ^{FR} is the specific weight of the fluid and K^F is the Darcy's permeability coefficient of the porous medium.

Making the use of (28) in equations (22)-(24), and with the help of (29)-(32), we obtain

$$\text{div}(\eta^S\dot{\mathbf{u}}_S + \eta^F\dot{\mathbf{u}}_F) = 0, (34)$$

$$(\lambda^S + \mu^S)\text{grad div}\,\mathbf{u}_S + \mu^S\text{div grad}\,\mathbf{u}_S - \eta^S\text{grad p} + \rho^S(\mathbf{b} - \ddot{\mathbf{u}}_s) + S_v(\dot{\mathbf{u}}_F - \dot{\mathbf{u}}_S) = 0, (35)$$

$$-\eta^F\text{grad p} + \rho^F(\mathbf{b} - \ddot{\mathbf{u}}_F) - S_v(\dot{\mathbf{u}}_F - \dot{\mathbf{u}}_S) = 0. (36)$$

For the two dimensional problem, we assume the displacement vector $\mathbf{u}_i$ $(i = F, S)$ as

$$\mathbf{u}_i = (u^i, 0, w^i) \text{ where } i = F, S. (37)$$

Equations (34) - (36) with the help of eq. (37) in the absence of body forces take the form

$$\eta^S\left[\frac{\partial^2 u^S}{\partial x\,\partial t} + \frac{\partial^2 w^S}{\partial z\,\partial t}\right] + \eta^F\left[\frac{\partial^2 u^F}{\partial x\,\partial t} + \frac{\partial^2 w^F}{\partial z\,\partial t}\right] = 0, (38)$$

$$\eta^F\frac{\partial p}{\partial x} + \rho^F\frac{\partial^2 u^F}{\partial t^2} + S_v\left[\frac{\partial u^F}{\partial t} - \frac{\partial u^S}{\partial t}\right] = 0, (39)$$

$$\eta^F\frac{\partial p}{\partial z} + \rho^F\frac{\partial^2 w^F}{\partial t^2} + S_v\left[\frac{\partial w^F}{\partial t} - \frac{\partial w^S}{\partial t}\right] = 0, (40)$$

$$(\lambda^S + \mu^S)\frac{\partial \theta^S}{\partial x} + \mu^S\nabla^2 u^S - \eta^S\frac{\partial p}{\partial x} - \rho^S\frac{\partial^2 u^S}{\partial t^2} + S_v\left[\frac{\partial u^F}{\partial t} - \frac{\partial u^S}{\partial t}\right] = 0, (41)$$

$$(\lambda^S + \mu^S)\frac{\partial \theta^S}{\partial z} + \mu^S\nabla^2 w^S - \eta^S\frac{\partial p}{\partial z} - \rho^S\frac{\partial^2 w^S}{\partial t^2} + S_v\left[\frac{\partial w^F}{\partial t} - \frac{\partial w^S}{\partial t}\right] = 0, (42)$$

where

$$\theta^S = \frac{\partial(u^S)}{\partial x} + \frac{\partial(w^S)}{\partial z}, (43)$$

and

$$\nabla^2 = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial z^2}. (44)$$

Also, the normal and tangential stresses ${t_{zz}}^S$ and ${t_{zx}}^S$ respectively in the solid phase are as

$${t_{zz}}^S = \lambda^S\left(\frac{\partial u^S}{\partial x} + \frac{\partial w^S}{\partial z}\right) + 2\mu^S\frac{\partial w^S}{\partial z}, (45)$$

$${t_{zx}}^S = \mu^S\left(\frac{\partial u^S}{\partial z} + \frac{\partial w^S}{\partial x}\right). (46)$$

The displacement components u^j and w^j in terms of the potential ϕ^j and ψ^j can be written as

$$u^j = \frac{\partial\phi^j}{\partial x} + \frac{\partial\psi^j}{\partial z}, w^j = \frac{\partial\phi^j}{\partial z} - \frac{\partial\psi^j}{\partial x}, j = S, F. (47)$$

Using equation (47) in equations (38)-(42), we get the following equations in $\phi^S, \phi^F, \psi^S, \psi^F$ and p as

$$\nabla^2\phi^S - \frac{1}{C^2}\frac{\partial^2\phi^S}{\partial t^2} - \frac{S_v}{(\lambda^S + 2\mu^S)(\eta^F)^2}\frac{\partial\phi^S}{\partial t} = 0, (48)$$

$$\phi^F = -\frac{\eta^S}{\eta^F}\phi^S, (49)$$

$$\mu^S\nabla^2\psi^S - \rho^S\frac{\partial^2\psi^S}{\partial t^2} + S_v\left[\frac{\partial\psi^F}{\partial t} - \frac{\partial\psi^S}{\partial t}\right] = 0, (50)$$

$$\rho^F\frac{\partial^2\psi^F}{\partial t^2} + S_v\left[\frac{\partial\psi^F}{\partial t} - \frac{\partial\psi^S}{\partial t}\right] = 0, (51)$$

$$(\eta^F)^2 p - \eta^S\rho^F\frac{\partial^2\phi^S}{\partial t^2} - S_v\frac{\partial\phi^S}{\partial t} = 0, (52)$$

where

$$C = \sqrt{\frac{(\eta^F)^2(\lambda^S + 2\mu^S)}{(\eta^F)^2\rho^S + (\eta^S)^2\rho^F}}. (53)$$

The solution of the system of equations (48) - (52) can be assumed in the form given below

$$(\phi^S, \phi^F, \psi^S, \psi^F, p) = ({\phi_1}^S, {\phi_1}^F, {\psi_1}^S, {\psi_1}^F, p_1)\exp(i\omega t), (54)$$

where ω denotes the complex circular frequency.

With the help of equations (54) in equations (48)-(52), we get

$$\left[\nabla^2 + \frac{\omega^2}{{C_1}^2} - \frac{i\omega S_v}{(\lambda^S + 2\mu^S)(\eta^F)^2}\right]{\phi_1}^S = 0, (55)$$

$$[\mu^S\nabla^2 + \rho^S\omega^2 - i\omega S_v]{\psi_1}^S = -i\omega S_v{\psi_1}^F, (56)$$

$$[-\omega^2\rho^F + i\omega S_v]{\psi_1}^F - i\omega S_v{\psi_1}^S = 0, (57)$$

$$(\eta^F)^2 p_1 + \eta^S\rho^F\omega^2{\phi_1}^S - i\omega S_v{\phi_1}^S = 0, (58)$$

$${\phi_1}^F = -\frac{\eta^S}{\eta^F}{\phi_1}^S. (59)$$

Equation (55) obtained above corresponds to the longitudinal wave propagating with velocity $\overline{V}_1$, and

$${\overline{V}_1}^2 = \frac{1}{G_1}, (60)$$

where

$$G_1 = \left[\frac{1}{{C_1}^2} - \frac{iS_v}{\omega(\lambda^S + 2\mu^S)(\eta^F)^2}\right]. (61)$$

From equations (56) and (57), we obtain

$$\left[\nabla^2+\frac{\omega^2}{\overline{V}_2{}^2}\right]\psi_1{}^S=0, (62)$$

Equation (62) corresponds to the b transverse wave propagating with velocity $\overline{V}_2$, given by $\overline{V}_2{}^2=1/G_2$
where

$$G_2=\left\{\frac{\rho^S}{\mu^S}-\frac{iS_v}{\mu^S\omega}-\frac{S_v{}^2}{\mu^S(-\rho^S\omega^2+i\omega S_v)}\right\}, (63)$$

Formulation of the problem

Consider a two dimensional problem by taking the z-axis pointing into the lower half-space and imperfect interface at z=0 separating the uniform micropolar viscoelastic solid half space medium M_1 (z>0) and fluid saturated porous half space medium M_2 (z<0). A longitudinal wave propagating through the medium M_1, incident at the plane z=0 and makes an angle θ_0 with normal to the surface. Corresponding to the incident longitudinal displacement wave, we get three reflected waves in the medium M_1 and two transmitted waves in medium M_2 as shown in figure 1.

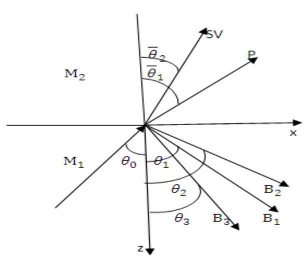

Figure 1: Geometry of the problem

For medium M_1

$$\phi=B_0\exp\{ik_0(x\sin\theta_0-z\cos\theta_0)+i\omega_1 t\}$$
$$+B_1\exp\{ik_0(x\sin\theta_1+z\cos\theta_1)+i\omega_1 t\}, (64)$$
$$\psi=B_2\exp\{i\delta_1(x\sin\theta_2+z\cos\theta_2)+i\omega_2 t\}$$
$$+B_3\exp\{i\delta_2(x\sin\theta_3+z\cos\theta_3)+i\omega_3 t\}, (65)$$
$$\Phi_2=EB_2\exp\{i\delta_1(x\sin\theta_2+z\cos\theta_2)+i\omega_2 t\}$$
$$+FB_3\exp\{i\delta_2(x\sin\theta_3+z\cos\theta_3)+i\omega_3 t\}, (66)$$

In medium M_2

$$\{\phi^S,\phi^F,p\}=\{1,m_1,m_2\}\left[A_1\exp\{i\bar{k}_1(x\sin\overline{\theta}_1-z\cos\overline{\theta}_1)+i\overline{\omega}_1 t\}\right], (67)$$

$$\{\psi^S,\psi^F\}=\{1,m_3\}\left[A_2\exp\{i\bar{k}_2(x\sin\overline{\theta}_2-z\cos\overline{\theta}_2)+i\overline{\omega}_2 t\}\right], (68)$$

where

$$m_1=-\frac{\eta^S}{\eta^F}, m_2=-\left[\frac{\eta^S\omega_1{}^2\rho^F-i\omega_1 S_v}{(\eta^F)^2}\right],$$
$$m_3=\frac{i\omega_2 S_v}{i\omega_2 S_v-\omega_2{}^2\rho^F}, (69)$$

and B_0, B_1, B_2, B_3 are amplitudes of incident P-wave, reflected P-wave, reflected CDI and reflected CDII waves respectively, A_1 and A_2 are amplitudes of transmitted P-wave and transmitted SV-wave, respectively

Boundary conditions

Boundary conditions appropriate here are the continuity of displacement, micro rotation and stresses at the interface separating medium M_1 and M_2. These boundary conditions at z=0 can be written in mathematical form as

$$t_{zz}=t_{zz}{}^S-p, t_{zx}=t_{zx}{}^S, m_{zy}=0,$$
$$t_{zz}{}^S-p=K_n(u_3-w^S),$$
$$t_{zx}{}^S=K_t(u_1-u^S), (70)$$

In order to satisfy the boundary conditions, the extension of the Snell's law is

$$\frac{\sin\theta_0}{V_0}=\frac{\sin\theta_1}{V_1}=\frac{\sin\theta_2}{\lambda_1^{-1}}=\frac{\sin\theta_3}{\lambda_2^{-1}}=\frac{\sin\overline{\theta}_1}{\overline{V}_1}=\frac{\sin\overline{\theta}_2}{\overline{V}_2}, (71)$$

For longitudinal wave,

$$V_0=V_1, \theta_0=\theta_1. (72)$$

Also

$$k_0V_1=\delta_1\lambda_1^{-1}=\delta_2\lambda_2^{-1}=\bar{k}_1\overline{V}_1=\bar{k}_2\overline{V}_2=\omega, (73)$$

With the help of potentials given by equations (64)-(68) in equations (1)-(2),(6), (45)-(47) and (67) and then using the boundary conditions given by equation (70) and using (71)-(73), we obtain a system of five non homogeneous which can be written as

$$\sum_{j=0}^{5}a_{ij}Z_j=Y_i, (i=1,2,3,4,5) (74)$$

where

$$Z_1=\frac{B_1}{B_0}, Z_2=\frac{B_2}{B_0}, Z_3=\frac{B_3}{B_0}, Z_4=\frac{A_1}{B_0}, Z_5=\frac{A_2}{B_0}, (75)$$

i.e. Z_1 to Z_5 be the amplitude ratios of reflected modified longitudinal displacement wave, reflected CD I wave at an angle θ_2, reflected CD II wave at an angle θ_3, transmitted P-wave and transmitted SV-wave, respectively and a_{ij} in non-dimensional form are given as

$$a_{11}=\left[\frac{\lambda}{\mu}+\left(2+\frac{\kappa}{\mu}\right)\cos^2\theta_1\right],$$
$$a_{12}=-\left(2+\frac{\kappa}{\mu}\right)\frac{\delta_1{}^2}{k_0^2}\sin\theta_2\cos\theta_2,$$
$$a_{13}=-\left(2+\frac{\kappa}{\mu}\right)\sin\theta_3\cos\theta_3\frac{\delta_2{}^2}{k_0^2},$$
$$a_{14}=\frac{-\bar{k}_1^2(\lambda^S+2\mu^S\cos^2\overline{\theta}_1)-m_2}{\mu k_0^2},$$
$$a_{15}=\frac{-2\mu^S\bar{k}_2^2\sin\overline{\theta}_2\cos\overline{\theta}_2}{\mu k_0^2}, Y_1=-a_{11},$$
$$a_{21}=\left(2+\frac{\kappa}{\mu}\right)\sin\theta_1\cos\theta_1,$$
$$a_{22}=\frac{\delta_1{}^2}{k_0^2}\left[\left(\left(1+\frac{\kappa}{\mu}\right)\cos^2\theta_2-\sin^2\theta_2\right)+\frac{\kappa}{\mu}\frac{E}{k_0^2}\right],$$

$$a_{23} = \frac{{\delta_2}^2}{k_0^2}\left[\left(\left(1+\frac{\kappa}{\mu}\right)\cos^2\theta_3 - \sin^2\theta_3\right) + \frac{\kappa}{\mu}\frac{F}{k_0^2}\right],$$

$$a_{24} = \frac{\mu^S \bar{k}_1^2 \sin 2\bar{\theta}_1}{\mu k_0^2},\ a_{25} = \frac{\mu^S \bar{k}_2^2\left(\sin^2\bar{\theta}_2 - \cos^2\bar{\theta}_2\right)}{\mu k_0^2},$$

$$Y_2 = a_{21},\ a_{31} = \cos\theta_1, a_{32} = -\frac{\delta_1}{k_0}\sin\theta_2,$$

$$a_{33} = -\frac{\delta_2}{k_0}\sin\theta_3,$$

$$a_{34} = \frac{\bar{k}_1}{k_0}\cos\bar{\theta}_1 + \frac{\bar{k}_1^2\left(\lambda^S + 2\mu^S\cos^2\bar{\theta}_1\right) + m_2}{K_n\, k_0},$$

$$a_{35} = \frac{\bar{k}_2}{k_0}\sin\bar{\theta}_2 + \frac{2\mu^S \bar{k}_2^2 \sin\bar{\theta}_2\cos\bar{\theta}_2}{K_n\, k_0}, Y_3 = a_{31},$$

$$a_{41} = \sin\theta_1, a_{42} = \frac{\delta_1}{k_0}\cos\theta_2, a_{43} = \frac{\delta_2}{k_0}\cos\theta_3,$$

$$a_{44} = -\frac{\bar{k}_1}{k_0}\sin\bar{\theta}_1 - \frac{\mu^S \bar{k}_1^2 \sin 2\bar{\theta}_1}{K_t\, k_0},$$

$$a_{45} = \frac{\bar{k}_2}{k_0}\cos\bar{\theta}_2 - \frac{\mu^S \bar{k}_2^2\left(\sin^2\bar{\theta}_2 - \cos^2\bar{\theta}_2\right)}{K_t\, k_0},$$

$$Y_4 = -a_{41},\ a_{51} = 0, a_{52} = \cos\theta_2,$$

$$a_{53} = \frac{F\delta_2}{E\delta_1}\cos\theta_3,\ a_{54} = 0, a_{55} = 0,$$

$$Y_5 = 0. (76)$$

Particular Cases

Case I Normal force stiffness ($K_n \neq 0, K_t \to \infty$)

In this case, we get a system of four non homogeneous equations as in given by equation (74) with some a_{ij} changed as

$$a_{34} = \frac{\bar{k}_1}{k_0}\cos\bar{\theta}_1,\ a_{35} = \frac{\bar{k}_2}{k_0}\sin\bar{\theta}_2\ (77)$$

Case II Transverse force stiffness ($K_t \neq 0, K_n \to \infty$)

In this case, a system of four non homogeneous equations as those given by equation (74) is obtained but some a_{ij} changed as

$$a_{44} = -\frac{\bar{k}_1}{k_0}\sin\bar{\theta}_1\ a_{45} = \frac{\bar{k}_2}{k_0}\cos\bar{\theta}_2\ (78)$$

Case III: Welded contact ($K_n \to \infty, K_t \to \infty$)

Again in this case, a system of four non homogeneous equations is obtained as in equation (74) with some a_{ij} changed as

$$a_{34} = \frac{\bar{k}_1}{k_0}\cos\bar{\theta}_1,\ a_{35} = \frac{\bar{k}_2}{k_0}\sin\bar{\theta}_2,$$

$$a_{44} = -\frac{\bar{k}_1}{k_0}\sin\bar{\theta}_1\ a_{45} = \frac{\bar{k}_2}{k_0}\cos\bar{\theta}_2, (79)$$

Special case

If pores are absent or gas is filled in the pores then ρ^F is very small as compared to ρ^S and can be neglected, so the relation (53) gives us

$$C = \sqrt{\frac{\lambda^S + 2\mu^S}{\rho^S}}. (80)$$

and the coefficients a_{14}, and a_{34} in (76) changes to

$$a_{14} = \frac{-\bar{k}_1^2\left(\lambda^S + 2\mu^S\cos^2\bar{\theta}_1\right)}{\mu\, k_0^2},$$

$$a_{34} = \frac{\bar{k}_1}{k_0}\cos\bar{\theta}_1 + \frac{\bar{k}_1^2\left(\lambda^S + 2\mu^S\cos^2\bar{\theta}_1\right)}{K_n\, k_0}$$

and the remaining coefficients in (76) remain same. In this situation the problem reduces to the problem of empty porous solid half space lying over micropolar viscoelastic solid half space.

Numerical Results and Discussion

The theoretical results obtained above indicate that the amplitude ratios Z_i ($i = 1,2,3,4,5$) depend on the angle of incidence of incident wave and material properties of half spaces. In order to study in more detail the behaviour of various amplitude ratios, we have computed them numerically for a particular model for which the values of various physical parameters are as under

In medium M_1, the physical parameters for micropolar viscoelastic solid are taken from Gauthier (1982) as

$\lambda' = 7.59 \times 10^{11}$ dyne/cm^2,
$\mu' = 1.89 \times 10^{11}$ dyne/cm^2,
$\kappa' = 0.0149 \times 10^{11}$dyne/cm^2,
$\rho = 2.19$gm/cm^3,

$$\lambda = \lambda'\left(1+\frac{i}{Q_1}\right), \mu = \mu'\left(1+\frac{i}{Q_2}\right),$$

$$\kappa = \kappa'\left(1+\frac{i}{Q_3}\right), \gamma = \gamma'\left(1+\frac{i}{Q_4}\right), (80)$$

where the quality factors $Q_i (i = 1,2,3,4)$ are taken arbitrarily as

$Q_1 = 5, Q_2 = 10, Q_3 = 15, Q_4 = 13.$

In mediumM_2, the physical constants for fluid saturated incompressible porous medium are taken from de Boer, Ehlers and Liu (1993) as

$\eta^s = 0.67, \eta^F = 0.33, \rho^s = 1.34$ Mg/m^3,
$\rho^F = 0.33$ Mg/m^3, $\lambda^s = 5.5833$ MN/m^2,

$$K^F = \frac{0.01\text{m}}{\text{s}}, \gamma^{FR} = \frac{10.00\text{KN}}{\text{m}^3},$$

$$\mu^s = \frac{8.3750\text{N}}{\text{m}^2}. (81)$$

A computer programme in MATLAB has been developed to calculate the modulus of amplitude ratios of various reflected and transmitted waves for the particular model and to depict graphically. In figures (2) - (6) solid lines show the variations of amplitude ratios when medium-I is micropolar viscoelastic solid (MVES) and medium-II is incompressible fluid saturated porous medium (FS) whereas dashed lines show the variations of amplitude ratios when medium-II becomes incompressible empty porous solid (EPS). Figures (2) - (6) indicates the effect of pores fluid.

Longitudinal displacement wave incidence

In all the figures (2)-(16), dashed dotted line signifies the general stiffness case, whereas dashed line depict the case of transverse force stiffness. Also double dashed line represents normal force stiffness case and solid line shows the welded contact case. Figures (2)-(5) show the variations of the amplitude ratios of reflected P-wave, reflected CDI-wave, reflected CDII-wave, transmitted P-wave and transmitted SV-wave with angle of incidence of incident P-wave. The behaviour of all these distribution curves is similar i.e. increasing from normal incidence to maximum value and then decreasing from maximum value to grazing incidence except the figure (5). The values of all amplitude ratios corresponding to reflected waves in figures (2)-(4) in all the stiffness cases are same. But in case of amplitude ratios corresponding to transmitted waves in figures (5)-(6), transverse force stiffness values are small than all other stiffness cases. Figures (7)-(11) depict the variations of the amplitude ratios $|Z_i|$ with angle of incidence of the incident P wave in case when medium M_1 becomes elastic solid half space. The behaviour of curves in figures (7)-(9) is same as in figures(2)-(4) and behaviour of curves in figures (10)-(11) is same as in figures(5)-(6).The effect of viscosity of viscoelastic solid is not significant if we compare the values of corresponding amplitude ratio in figures (2)-(6) and (7)-(11). Figures (12)-(16) show the variations of the amplitude ratios $|Z_i|$ with angle of incidence of the incident P wave when medium M_2 is empty porous solid. The effect of fluid filled in the pores of fluid saturated porous medium is significant by comparing the values of corresponding amplitude ratio in figures (2)-(6) and (12)-(16). In figures (12)-(14) no effect of stiffness is seen in case of amplitude ratios of reflected waves. Effect of stiffness is clear in figures (15)-(16).

Conclusion

In conclusion, a mathematical study of reflection and refraction coefficients at an interface separating micropolar viscoelastic solid half space and fluid saturated incompressible porous half space is made when longitudinal wave is incident. It is observed that the amplitudes ratios of various reflected and transmitted waves depend on the angle of incidence of the incident wave and material properties of half spaces. The effect of fluid filled in the pores of incompressible fluid saturated porous medium is significant on the amplitudes ratios the effect of viscosity of viscoelastic solid is not significant. The model presented in this paper is one of the more realistic forms of the earth models. It may be of some use in engineering, seismology and geophysics etc.

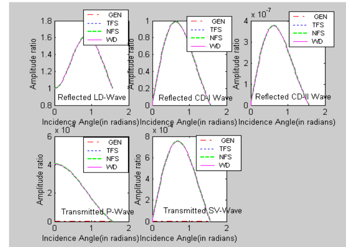

Figure 2-6: Variation of the amplitude ratios of reflected P-wave, reflected CDI-wave, reflected CDII-wave, transmitted P-wave and transmitted SV-wave with angle of incidence of longitudinal displacement wave

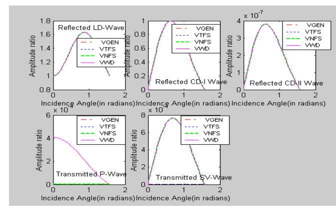

Figure 7-11: Variation of the amplitude ratios of reflected P-wave, reflected CDI-wave, reflected CDII-wave, transmitted P-wave and transmitted SV-wave with angle of incidence of longitudinal displacement wave in case of elastic solid

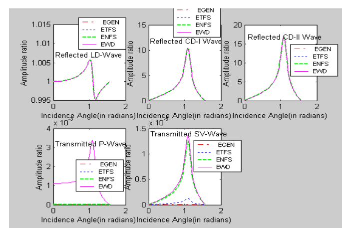

Figure 12-16: Variation of the amplitude ratios of reflected P-wave, reflected CDI-wave, reflected CDII-wave, transmitted P-wave and transmitted SV-wave with angle of incidence of longitudinal displacement wave in case of empty porous solid

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