Cost- benefit analysis of two dissimilar warm standby system subject to failure due to tides and gravitational attractions with switch failure

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**Abstract**

Tidal power, also called tidal energy, is a form of hydropower that converts the energy of tides into useful forms of power, mainly electricity. Although not yet widely used, tidal power has potential for future electricity generation. Tides are more predictable than wind energy and solar power. Among sources of renewable energy, tidal power has traditionally suffered from relatively high cost and limited availability of sites with sufficiently high tidal ranges or flow velocities, thus constraining its total availability. However, many recent technological developments and improvements, both in design (e.g. dynamic tidal power, tidal lagoons) and turbine technology (e.g. new axial turbines, cross flow turbines), indicate that the total availability of tidal power may be much higher than previously assumed, and that economic and environmental costs may be brought down to competitive levels. Historically, tide mills have been used both in Europe and on the Atlantic coast of North America. The incoming water was contained in large storage ponds, and as the tide went out, it turned waterwheels that used the mechanical power it produced to mill grain. The earliest occurrences date from the Middle Ages, or even from Roman times. It was only in the 19th century that the process of using falling water and spinning turbines to create electricity was introduced in the U.S. and Europe. Two-unit standby system subject to environmental conditions such as shocks, change of weather conditions etc. have been discussed in reliability literature by several authors due to significant importance in defences, industry etc. In the present paper we have taken two-non-identical warm standby system with failure time distribution as exponential and repair time distribution as general. The Role of tidal energy generated due to tides and gravitational attractions under which the system operates plays significant role on its working. We are considering system under the influence of (i) tides and (ii) Gravitational attractions causing different types of failure requiring different types of repair facilities. Using semi Markov regenerative point technique we have calculated different reliability characteristics such as MTSF, reliability of the system, availability analysis in steady state, busy period analysis of the system under repair, expected number of visits by the repairman in the long run and gain-function and graphs are drawn.

**Keyword:** warm standby, tides producing tidal energy, gravitational attractions, switches failure.

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INTRODUCTION

Tidal power is taken from the Earth's oceanic tides; tidal forces are periodic variations in gravitational attraction exerted by celestial bodies. These forces create corresponding motions or currents in the world's oceans. Due to the strong attraction to the oceans, a bulge in the water level is created, causing a temporary increase in sea level. When the sea level is raised, water from the middle of the ocean is forced to move toward the shorelines, creating a tide. This occurrence takes place in an unfailing manner, due to the consistent pattern of the moon’s orbit around the earth. The magnitude and character of this motion reflects the changing positions of the Moon and Sun relative to the Earth, the effects of Earth's rotation, and local geography of the sea floor and coastlines. Tidal stream generators (or TSGs) make use of the kinetic energy of moving water to power turbines, in a similar way to wind turbines that use wind to power turbines. Some tidal generators can be built into the structures of existing bridges, involving virtually no aesthetic problems. Land constrictions such as straits or inlets can create high velocities at specific sites, which can be captured with the use of turbines. These turbines can be horizontal, vertical, open, or ducted and are typically placed near the bottom of the water column.

Assumptions

1. The failure time distribution is exponential whereas the repair time distribution is arbitrary of two non-identical units.
2. The repair facility is of four types:
   Type I, II repair facility
   • when failure due to tides producing tidal energy and gravitational attractions of first unit occurs respectively
   Type III, IV repair facility
   • when failure due to tides producing tidal energy and gravitational attractions of the second unit occurs respectively.
3. The repair starts immediately upon failure of units and the repair discipline is FCFS.
4. The repairs are perfect and start immediately as soon as the tides producing tidal energy and gravitational attractions of the system become normal. The tides producing tidal energy and gravitational attractions in both the units do not occur simultaneously.
5. The failure of a unit is detected immediately and perfectly.
6. The switches are perfect and instantaneous.
7. All random variables are mutually independent.

SYMBOLS FOR STATES OF THE SYSTEM

Superscripts: O, WS, SO, FTPTE, FGA, SF
Operative, Warm Standby, Stops the operation, failure due to tides producing tidal energy, failure due to gravitational attractions, Switch failure respectively

Subscripts: npt, tpe, ga, ur, wr, uR
No tides producing tidal energy, tides producing tidal energy, gravitational attractions, under repair, waiting for repair, under repair continued respectively

Up states: 0, 1, 2, 9;
Down states: 3, 4, 5, 6, 7, 8, 10, 11
Regeneration point: 0,1,2,4,7,10

STATES OF THE SYSTEM

\(0(O_{npt}, WS_{npt})\)
One unit is operative and there are no tides producing tidal energy and the other unit is warm standby with no gravitational attractions in both the units.

\(1(SO_{tpe}, O_{npt})\)
The operation of the first unit stops automatically due to tides producing tidal energy and warm standby unit’s starts operating with no tides producing tidal energy.

\(2(FTPTE_{ur}, O_{npt})\)
The first unit fails and undergoes repair after the tides producing tidal energy are over and the other unit continues to be operative with no tides producing tidal energy.
The repair of the first unit is continued from state 2 and the operation of second unit stops automatically due to tides producing tidal energy.

The first unit fails and undergoes repair after the tides producing tidal energy are over and the other unit also stops automatically due to gravitational attractions.

The repair of the first unit is continued from state 4 and the other unit is failed due to gravitational attractions in it and is waiting for repair.

The first unit becomes operative with no tides producing tidal energy and the second unit is failed due to gravitational attractions is under repair.

The operation of the first unit stops automatically due to gravitational attractions and during switchover to the second unit switch fails and undergoes repair and there is no tides producing tidal energy.

The repair of failed switch is continued from state 7 and the first unit is failed after gravitational attractions are waiting for repair.

The first unit is operative with no tides producing tidal energy and the operation of warm standby second unit is stopped automatically due to gravitational force.

The operation of the first unit stops automatically due to gravitational attractions and the second unit switch fails and undergoes repair after the gravitational attractions is over.

The repair of the second unit is continued from state 10 and the first unit is failed due to gravitational attractions is waiting for repair.

Simple probabilistic considerations yield the following expressions:

\[ P_{01} = \frac{\lambda_1}{\lambda_1 + \lambda_2 + \lambda_3}, \quad P_{07} = \frac{\lambda_2}{\lambda_1 + \lambda_2 + \lambda_3} \]
\[ p_{09} = \lambda_2 \left( \frac{\lambda_1}{\lambda_1 + \lambda_2} \right), \quad p_{12} = \lambda_1 \left( \frac{\lambda_1}{\lambda_1 + \lambda_3} \right), \quad p_{14} = \lambda_3 \left( \frac{\lambda_1 + \lambda_3}{\lambda_1 + \lambda_3} \right), \]

\[ P_{20} = P_1 = 1, \quad P_{22}^{(3)} = G_1, \quad P_{72} = G_2, \quad P_{72}^{(8)} = G_3. \]

We can easily verify that

\[ p_{01} + p_{07} + p_{09} = 1, \quad p_{12} + p_{14} = 1, \quad p_{20} + p_{23} = 1, \quad p_{46} = 1, \quad p_{60} = 1, \]

\[ p_{72} + p_{72}^{(3)} = p_{72} + p_{74} = 1, \quad p_{10,1} = 1, \quad p_{10,2} = 1. \]

And mean sojourn time are

\[ \mu_0 = E(T) = \int_0^\infty P[T > t] \, dt \]

**Mean time to system failure**

We can regard the failed state as absorbing

\[ \theta_0(t) = Q_{01}(t) + Q_{07}(t) + Q_{09}(t), \quad \theta_3(t) = Q_{12}(t) + Q_{14}(t) + Q_{20}(t), \quad \theta_4(t) = Q_{9,10}(t). \]

Taking Laplace-Stiltjes transform of eq. (3-5) and solving for

\[ Q_0(s) = N_1(s) / D_1(s) \]

where

\[ N_1(s) = Q_01(s), \quad Q_{12}(s) = Q_{12}^{(3)}(s), \quad Q_{14}(s), \quad Q_09(s), \quad Q_{10,10}(s), \quad Q_{07}(s) \]

\[ D_1(s) = 1 - Q_{01}(s), \quad Q_{12}(s), \quad Q_{20}(s) \]

Making use of relations (1) and (2) it can be shown that \( \theta_0^*(0) = 1 \), which implies that \( \theta_0(t) \) is a proper distribution.

\[ MTSF = E[T] = d/ds \theta_0(s) = (D_1(0) - N_1(0)) / D_1(0) \]

where

\[ \mu_0 = \mu_0 + \mu_0 \mu_9, \quad \mu_1 = \mu_{12} + \mu_{14}, \quad \mu_2 = \mu_{20} + \mu_{22}^{(3)}, \quad \mu_9 = \mu_{9,10} \]

**AVAILABILITY ANALYSIS**

Let \( M_i(t) \) be the probability of the system having started from state I is up at time t without making any other regenerative state belonging to E. By probabilistic arguments, we have The value of \( M_0(t), M_1(t), M_2(t), M_4(t) \) can be found easily.

The point wise availability of \( A_i(t) \) having the following recursive relations

\[ A_0(t) = M_0(t) + q_{01}(t)cA_1(t) + q_{07}(t)cA_2(t) + q_{09}(t)cA_4(t) \]

\[ A_1(t) = M_1(t) + q_{12}(t)cA_2(t) + q_{14}(t)cA_4(t) \]

\[ A_2(t) = M_2(t) + q_{20}(t)cA_0(t) + q_{22}(t)cA_2(t) \]

\[ A_4(t) = M_4(t) + q_{46}(t)cA_4(t) \]

\[ A_5(t) = q_{51}(t)cA_1(t) + q_{54}(t)cA_4(t) \]

\[ A_6(t) = q_{60}(t)cA_0(t) + q_{62}(t)cA_2(t) + q_{64}(t)cA_4(t) \]

\[ A_7(t) = q_{72}(t)cA_2(t) + q_{74}(t)cA_4(t) \]

\[ A_8(t) = q_{80}(t)cA_0(t) + q_{82}(t)cA_2(t) + q_{84}(t)cA_4(t) \]

Taking Laplace Transform of eq. (7-14) and solving for \( \hat{A}_0(s) \)

\[ \hat{A}_0(s) = N_2(s) / D_2(s) \]

where

\[ N_2(s) = \left( 1 - \hat{q}_{22}(3) \right) \{ \hat{M}_0(s) + \hat{q}_{01}(s)\hat{M}_1(s) + \hat{q}_{09}(s)\hat{M}_4(s) \} + \hat{M}_2(s) \]

\[ D_2(s) = \left( 1 - \hat{q}_{22}(3) \right) \{ 1 - \hat{q}_{46}(5) s \hat{q}_{00}(s)\hat{q}_{44}(s) + \hat{q}_{07}(s)\hat{q}_{72}(s) \}

The steady state availability

\[ A_0 = \lim_{t \to \infty} A_0(t) = \lim_{s \to 0} \left[ s N_2(s) / D_2(s) \right] = \lim_{s \to 0} \frac{s N_2(s)}{D_2(s)} \]

Using L’ Hospitals rule, we get

\[ A_0 = \lim_{s \to 0} \frac{s N_2(s) + s N_3(s)}{D_2(s)} = \frac{N_2(0)}{D_2(0)} \]

Where

\[ N_2(0) = p_{20}(\hat{M}_0(0)) + p_{01}(\hat{M}_1(0)) + p_{09}(\hat{M}_4(0)) + \hat{M}_2(0) (p_{01}p_{12} + p_{07}p_{72} \]


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where \( N \)

\[
D_2(0) = \frac{\mu_0 + (p_0 + p_{14} + p_{74}) \mu_4 + p_{07} \mu_7 + p_{09}(\mu_9 + \mu_{10})}{\mu_2 \left(1 - (p_0 + p_{14} + p_{74})\right)}
\]

\[
\mu_4 = \mu_{46}, \mu_7 = \mu_{72} + \mu_{46}, \mu_{10} = \mu_{10.2} + \mu_{72}
\]

The expected up time of the system in \((0, t]\) is

\[
\lambda_u(t) = \int_0^\infty A_0(z) d\zeta \Rightarrow \frac{\lambda_u(s)}{s} = \frac{N_2(s)}{SD_2(s)}
\]

The expected down time of the system in \((0, t]\) is

\[
\lambda_d(t) = \lambda_u(t) \Rightarrow \frac{\lambda_d(s)}{s} = \frac{1}{S^2} - \frac{\lambda_u(s)}{s}
\]

**The expected busy period of the server for repairing the failed unit under gravitational attractions in \((0, t]\)**

\[
R_0(t) = S_0(t) + q_{01}(t)C_0(t) + q_{09}(t)C_0(t) + q_{09}(t)C_1(t)
\]

\[
R_1(t) = S_1(t) + q_{12}(t)C_0(t) + q_{19}(t)C_1(t) + q_{19}(t)C_2(t)
\]

\[
S_2(t) = \frac{q_{20}(t)C_0(t) - q_{22}(t)C_2(t)}{1 - q_{22}(t)C_2(t)}
\]

\[
R_0(t) = q_{46}(t)C_0(t) + q_{46}(t)C_1(t) + q_{46}(t)C_2(t)
\]

\[
R_1(t) = (q_{47}(t) + q_{47}(t))C_2(t) + q_{47}(t)C_2(t)
\]

\[
R_2(t) = (q_{47}(t) + q_{47}(t))C_2(t) + q_{47}(t)C_2(t)
\]

\[
R_3(t) = q_{46}(t)C_0(t) + q_{46}(t)C_1(t) + q_{46}(t)C_2(t)
\]

Taking Laplace Transform of eq. (19-26) and solving for \( \hat{R}_0(s) \)

\[
\hat{R}_0(s) = \frac{N_3(s)}{D_2(s)}
\]

Where

\[
N_2(s) = \left(1 - \hat{q}_{22}(s)\right) \{ \hat{S}_0(s) + \hat{q}_{01}(s)\hat{S}_1(s) + \hat{q}_{09}(s)\hat{S}_2(s) \}
\]

In the long run, \( R_0 = \frac{N_2(0)}{D_2(0)} \)

\[
\lambda_{ru}(t) = \int_0^\infty R_0(z) d\zeta \Rightarrow \frac{\lambda_{ru}(s)}{s} = \frac{N_2(0)}{S^2}
\]

**The expected busy period of the server for repairing the failed units under tides producing tidal energy by the repairman in \((0, t]\)**

\[
B_0(t) = q_{01}(t)C_0(t) + q_{01}(t)C_0(t) + q_{09}(t)C_0(t)
\]

\[
B_1(t) = q_{12}(t)C_0(t) + q_{12}(t)C_0(t) + q_{12}(t)C_0(t)
\]

\[
B_2(t) = q_{46}(t)C_0(t) + q_{46}(t)C_0(t) + q_{46}(t)C_0(t)
\]

\[
B_3(t) = q_{46}(t)C_0(t) + q_{46}(t)C_0(t) + q_{46}(t)C_0(t)
\]

\[
B_4(t) = (q_{47}(t) + q_{47}(t))C_2(t) + q_{47}(t)C_2(t)
\]

\[
B_5(t) = (q_{47}(t) + q_{47}(t))C_2(t) + q_{47}(t)C_2(t)
\]

Taking Laplace Transform of eq. (29-36) and solving for \( \hat{B}_0(s) \)

\[
\hat{B}_0(s) = \frac{N_4(s)}{D_2(s)}
\]

Where

\[
N_4(s) = \left(1 - \hat{q}_{22}(s)\right) \{ \hat{q}_{01}(s)\hat{q}_{12}(s)\hat{T}_4(s) + \hat{q}_{46}(s)\hat{T}_4(s) + \hat{q}_{09}(s)\hat{T}_4(s)\hat{T}_4(s) \}
\]

And \( D_2(s) \) is already defined.

In steady state, \( B_0 = \frac{N_4(0)}{D_2(0)} \)

\[
\lambda_{ru}(t) = \int_0^\infty B_0(z) d\zeta \Rightarrow \frac{\lambda_{ru}(s)}{s} = \frac{B_0(s)}{s}
\]

**The expected busy period of the server for repair of switch in \((0, t]\)**

\[
P_0(t) = q_{01}(t)C_0(t) + q_{01}(t)C_0(t) + q_{09}(t)C_0(t)
\]

\[
P_1(t) = q_{12}(t)C_0(t) + q_{12}(t)C_0(t) + q_{12}(t)C_0(t)
\]

\[
P_2(t) = q_{46}(t)C_0(t) + q_{46}(t)C_0(t) + q_{46}(t)C_0(t)
\]

\[
P_3(t) = q_{46}(t)C_0(t) + q_{46}(t)C_0(t) + q_{46}(t)C_0(t)
\]

\[
P_4(t) = q_{46}(t)C_0(t) + q_{46}(t)C_0(t) + q_{46}(t)C_0(t)
\]

Taking Laplace Transform of eq. (40-47) and solving for \( \hat{P}_0(s) \)

\[
\hat{P}_0(s) = \frac{N_5(s)}{D_2(s)}
\]
where $N_2(s) = \hat{q}_{22}(s) \hat{L_2}(s) (1 - \hat{q}_{22}^{(3)}(s))$ and $D_2(s)$ is defined earlier.

In the long run, $P_0 = \frac{N_5(0)}{D_2(0)}$ (49)

where $N_5(0) = p_{20} p_{07} L_4(0)$ and $D_2(0)$ is already defined.

The expected busy period of the server for repair of the switch in $(0, t]$ is

$$\lambda_{RS}(t) = \int_0^\infty P_0(z) dz$$

So that $\lambda_{RS}(s) = \frac{P_0(0)}{s}$ (50)

The expected number of visits by the repairman for repairing the non-identical units in $(0, t]$

$$H_0(t) = Q_{010}(t)c[cH_2(t) + Q_{00}(t)c[H_2(t) + Q_{00}(t)[cH_4(t)]$$

The expected number of visits by the repairman for repairing the switch in $(0, t]$ $H_0(t)$ is already defined.

The expected number of visits by the repairman for repairing the non-identical units in $(0, t]$ $H_0(t) = Q_{010}(t)c[cH_2(t) + Q_{00}(t)c[H_2(t) + Q_{00}(t)[cH_4(t)]$$

Taking Laplace-Stieltjes transform of eq. (61-68) and solving for $H_0^*(s)$

$$H_0^*(s) = N_0(s) / D_3(s)$$ (59)

Where

$N_0(s) = (1 - Q_{22}^{(3)}(s)) \{ Q_0^{(1)}(s) [Q_0^{(12)}(s) + Q_0^{(14)}(s)] + Q_0^{(90)}(s) Q_0^{(9,10,2)}(s) \}

D_3(s) = (1 - Q_{22}^{(3)}(s)) \{ 1 - (Q_0^{(1)}(s) Q_0^{(14)}(s) + Q_0^{(90)}(s) Q_0^{(9,10,2)}(s) \} Q_{46}^{(5)}(s) Q_0^{(9,10,2)}(s) + Q_0^{(9,10,2)}(s) Q_0^{(9,10,2)}(s) + Q_0^{(9,10,2)}(s) Q_0^{(9,10,2)}(s) \}

In the long run, $H_0 = \frac{N_0(0)}{D_3(0)}$ (60)

where $N_0(0) = p_{20} (p_{01} + p_{09})$ and $D_3(0)$ is already defined.

The expected number of visits by the repairman for repairing the switch in $(0, t]$

$$V_0(t) = Q_{010}(t)c[cV_2(t) + Q_{010}(t)c[V_4(t), V_2(t) = Q_{20}(t)[c]V_0(t) + Q_{22}^{(3)}(t)[c]V_2(t)]

V_4(t) = Q_{22}^{(3)}(t)[c]V_6(t), V_6(t) = Q_{20}(t)[c]V_0(t)

V_7(t) = (Q_{22}^{(3)}(t)[c]V_2(t) + Q_{22}^{(3)}(t)[c]V_4(t) + Q_{22}^{(3)}(t)[c]V_6(t) + Q_{22}^{(3)}(t)[c]V_8(t)]

V_9(t) = Q_{02}(t)[c]V_10(t), V_10(t) = Q_{02}(t)[c]V_2(t) + Q_{02}(t)[c]V_4(t) + Q_{02}(t)[c]V_6(t)

Taking Laplace-Stieltjes transform of eq. (61-68) and solving for $V_0^*(s)$

$$V_0^*(s) = N_4(s) / D_3(s)$$ (69)

where $N_4(s) = Q_0^{(9,7,2)}(s) Q_0^{(9,7,2)}(s) (1 - Q_{22}^{(3)}(s))$ and $D_3(s)$ is the same as $D_3(s)$

In the long run, $V_0 = \frac{N_4(0)}{D_3(0)}$ (70)

where $N_4(0) = p_{20} p_{07} p_{72}$ and $D_3(0)$ is already defined.

**C O S T-B E N E F I T A N A L Y S I S**

The gain-function of the system considering mean up-time, expected busy period of the system under tides producing tidal energy when the units stops automatically, expected busy period of the server for repair of unit due to gravitational attractions and switch, expected number of visits by the repairman for unit failure, expected number of visits by the repairman for switch failure.

The expected total cost-benefit incurred in $(0, t]$ is

$C(t) = $ Expected total revenue in $(0, t]$ - expected total repair cost for switch in $(0, t]$ [1 - expected total repair cost for repairing the units due to tides producing tidal energy in $(0, t]$ when the units automatically stop in $(0, t]$]  
• expected busy period of the system under gravitational attractions  
• expected number of visits by the repairman for repairing the switch in $(0, t]$  
• expected number of visits by the repairman for repairing of the non-identical units in $(0, t]$  

The expected total cost per unit time in steady state is

$$C = \lim_{t \to \infty} C(t) / t = \lim_{s \to 0} (s^2 C(s))$$

$= K_1 A_0 - K_2 P_0 - K_3 B_0 - K_4 R_0 - K_5 V_0 - K_6 H_0$

Where

$K_1$: revenue per unit up-time,  
$K_2$: cost per unit time for which the system is under switch repair,
\(K_3\): cost per unit time for which the system is under repair due to tides producing tidal energy when units automatically stop,

\(K_4\): cost per unit time for which the system is under repair due to gravitational attractions,

\(K_5\): cost per visit by the repairman for which switch repair,

\(K_6\): cost per visit by the repairman for units repair.

**CONCLUSION**

After studying the system, we have analyzed graphically that when the failure rate due to tides producing tidal energy, failure rate due to gravitational attractions increases, the MTSF and steady state availability decreases and the cost function decreased as the failure increases.

**REFERENCES**