Computational analysis of parameters affecting economy of one gas and one steam turbine system with scheduled inspection

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Abstract

Introduction: A model considering variation in demand and power production capacity has been developed for a system comprising one gas and one steam turbine. Concept of scheduled inspection at regular intervals of time for maintenance has also been introduced. Initially both the unit i.e. the gas turbine as well as the steam turbine are operating. On failure of the gas turbine, system goes to down state, whereas on failure of the steam turbine, the system may be kept in up state with only gas turbine working or put to down state according as the buyer of the power so generated is ready to pay higher amount or not. Computational work has been done for the cost benefit analysis and interesting numerical results have been obtained regarding various parameters involved which affect the economy of the system. Semi-Markov process and regenerative point technique have been used to analyze the system.

Keyword: turbine system, steep high temperature.

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INTRODUCTION

A large number of researcher in the field of reliability have widely discussed and analysed two unit systems. Contributors for the analysis of reliability models for systems with two similar units include Tuteja et al. (1991), Rizwan et. al. (2010) and Mathew et al. (2011). Systems with two dissimilar units have also been analyzed by numerous researchers including Baohe (1997) and Taneja et. al. (2011). In most of the studies on two dissimilar units, one unit was taken as operative and other as standby. Both the dissimilar units have also been taken as operative simultaneously in some of the studies. Two units for the system discussed by them were totally dissimilar i.e. their nature was different. However, there may be practical situations where the two units are dissimilar but the nature of the work done by them is same; and failure in either of the units affects the working of the other. Such a situation was observed by the author on visiting gas turbine plants and hence a model considering variation in demand and power production capacity has been developed by Singh and Taneja (2013) wherein a power generating system comprising one gas and one steam turbine has been taken into consideration. Initially, both the gas turbine as well as the steam turbine are operative. On failure of the Gas turbine, the
system goes to down state as steam turbine cannot work in this case; whereas on failure of the steam turbine, it may be
kept in upstate with only gas turbine working or put to down state according as the buyer of the power so generated is
ready to make higher payment or not to compensate the heavy losses. The concept of scheduled inspection is also
incorporated as the same was observed while gathering information from gas turbine plant during the visit. The
scheduled inspection is done at regular intervals of time for maintenance and is of three types — Minor, Path and Major
Inspection. Though Singh and Taneja (2013) obtained various measures of system effectiveness, yet no computational
work has been done by them for the analysis. The present paper deals with the same model but with computational work
do draw interesting conclusions regarding various parameters which affect the economy of the system.

NOTATIONS

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>$O_{gt}$</td>
<td>Gas turbine operative</td>
</tr>
<tr>
<td>$O_{gt1}$</td>
<td>Gas turbine operative after 1st scheduled inspection/maintenance</td>
</tr>
<tr>
<td>$O_{gt2}$</td>
<td>Gas turbine operative after 2nd scheduled inspection/maintenance</td>
</tr>
<tr>
<td>$O_{st}$</td>
<td>Steam turbine operative</td>
</tr>
<tr>
<td>$O_{st1}$</td>
<td>Steam turbine operative after 1st scheduled inspection/maintenance</td>
</tr>
<tr>
<td>$O_{st2}$</td>
<td>Steam turbine operative after 2nd scheduled inspection/maintenance</td>
</tr>
<tr>
<td>$U_{rgt}$</td>
<td>Gas turbine under repair</td>
</tr>
<tr>
<td>$U_{rst}$</td>
<td>Steam turbine under repair</td>
</tr>
<tr>
<td>$U_{Rst}$</td>
<td>Repair of the steam turbine is continuing from previous state</td>
</tr>
<tr>
<td>$d_{gt}$</td>
<td>Gas turbine put to down mode</td>
</tr>
<tr>
<td>$d_{st}$</td>
<td>Steam turbine put to down mode</td>
</tr>
<tr>
<td>$W_{rgt}$</td>
<td>Gas turbine waiting for repair</td>
</tr>
<tr>
<td>$W_{rst}$</td>
<td>Steam turbine waiting for repair</td>
</tr>
<tr>
<td>$\text{Insp}_1$</td>
<td>First type of inspection (Minor inspection)</td>
</tr>
<tr>
<td>$\text{Insp}_2$</td>
<td>Second type of inspection (Path inspection)</td>
</tr>
<tr>
<td>$\text{Insp}_3$</td>
<td>Third type of inspection (Major inspection)</td>
</tr>
<tr>
<td>$\lambda$</td>
<td>Failure rate of gas turbine</td>
</tr>
<tr>
<td>$\alpha$</td>
<td>Failure rate of steam turbine</td>
</tr>
<tr>
<td>$p$</td>
<td>Probability that there is dire demand of electricity and the customer is ready to make higher payments.</td>
</tr>
<tr>
<td>$q$</td>
<td>$1-p$ i.e the probability that the customer is not ready to make the payment higher than the normal rates.</td>
</tr>
<tr>
<td>$g_1(t)$, $G_1(t)$</td>
<td>pdf and cdf of repair time of gas turbine</td>
</tr>
<tr>
<td>$g_2(t)$, $G_2(t)$</td>
<td>pdf and cdf of repair time of steam turbine</td>
</tr>
<tr>
<td>$\beta_1$</td>
<td>Rate of requirement of scheduled inspection/ Maintenance</td>
</tr>
<tr>
<td>$\gamma_1$</td>
<td>Rate of doing minor inspection/ maintenance</td>
</tr>
<tr>
<td>$\gamma_2$</td>
<td>Rate of doing path inspection/maintenance</td>
</tr>
<tr>
<td>$\gamma_3$</td>
<td>Rate of doing major inspection/maintenance</td>
</tr>
</tbody>
</table>

The possible states of transition for Model will be as:

<table>
<thead>
<tr>
<th>State Number</th>
<th>Status</th>
<th>State Number</th>
<th>Status</th>
<th>State Number</th>
<th>Status</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>$O_{gt}, O_{st}$</td>
<td>6</td>
<td>$O_{gt1}, O_{st1}$</td>
<td>12</td>
<td>$O_{gt2}, O_{st2}$</td>
</tr>
<tr>
<td>1</td>
<td>$u_{rgt}, d_{st}$</td>
<td>7</td>
<td>$u_{rgt1}, d_{st1}$</td>
<td>13</td>
<td>$u_{rgt2}, d_{st2}$</td>
</tr>
<tr>
<td>2</td>
<td>$O_{gt}, u_{rst}$</td>
<td>8</td>
<td>$O_{gt1}, u_{rst1}$</td>
<td>14</td>
<td>$O_{gt2}, u_{rst2}$</td>
</tr>
<tr>
<td>3</td>
<td>$d_{gt}, u_{rst}$</td>
<td>9</td>
<td>$u_{rst1}, g_{gt}$</td>
<td>15</td>
<td>$u_{rst2}, d_{gt2}$</td>
</tr>
<tr>
<td>4</td>
<td>$w_{rgt}, U_{Rgt}$</td>
<td>10</td>
<td>$w_{rgt1}, U_{Rgt1}$</td>
<td>16</td>
<td>$w_{rgt2}, U_{Rgt2}$</td>
</tr>
<tr>
<td></td>
<td>5</td>
<td>11</td>
<td>$\text{Insp I}$</td>
<td>17</td>
<td>$\text{Insp III}$</td>
</tr>
</tbody>
</table>

The possible transitions are as:
0 to 1, 0 to 2, 0 to 3, o to 5, 1 to 0, 2 to 0, 2 to 4, 2 to 1 via 4, 3 to 0, 5 to 6, 6 to 8, 6 to 9, 6 to 11, 6 to 9, 8 to 6, 8 to 10, 8 to 7 via 10, 9 to 6, 11 to 12, 12 to 14, 12 to 15, 12 to 17, 13 to 12, 14 to 12, 14 to 16, 14 to 13 via 16, 15 to 12, 17 to 0.

The epochs of entry into states 0, 1, 2, 3, 5, 6, 7, 8, 9, 11, 12, 13, 14 and 15 are regeneration points and thus 0, 1, 2, 3, 4, 5, 6, 7, 8, 9, 11, 12, 13, 14 and 15 are called regenerative states. States 4, 10, and 16 are failed states. States 0, 6 and 12 are up states. States 2, 8, and 14 are up states in single cycle. States 5, 11, and 17 are down states due to inspection; and 1, 3, 7, 9, 13, and 15 are also down states as these are non-working states even though the steam turbine/gas turbine is operable.

**TRANSITION PROBABILITIES**

\[
p_{01} = \frac{\lambda}{\lambda + \alpha + \beta_1}, \quad p_{02} = \frac{p\alpha}{\alpha + \lambda + \beta_1}, \quad p_{03} = \frac{q\alpha}{\alpha + \lambda + \beta_1}, \quad p_{05} = \frac{\beta_1}{\alpha + \lambda + \beta_1}, \quad p_{10} = 1
\]

\[
p_{20} = g_2^*(\lambda), \quad p_{24} = 1 - g_2^*(\lambda)
\]

\[
p_{21} = 1 - g_2^*(\lambda), \quad p_{30} = 1, \quad p_{56} = 1
\]

\[
p_{67} = \frac{\lambda}{\lambda + \alpha + \beta_1}
\]

\[
p_{12,13} = \frac{\lambda}{\alpha + \lambda + \beta_1}, \quad p_{12,14} = \frac{p\alpha}{\alpha + \lambda + \beta_1}
\]

\[
p_{12,15} = \frac{q\alpha}{\alpha + \lambda + \beta_1}, \quad p_{12,17} = \frac{\beta_1}{\alpha + \lambda + \beta_1}
\]

\[
p_{13,12} = 1, \quad p_{14,12} = g_2^*(\lambda)
\]

\[
p_{14,16} = 1 - g_2^*(\lambda), \quad p_{14,13} = 1 - g_2^*(\lambda)
\]

\[
p_{15,12} = 1, \quad p_{17,0} = 1
\]

**Mean Sojourn times:**

\[
m_0 = \int_0^\infty e^{-(\lambda + \alpha + \beta_1)t}dt = \frac{1}{\lambda + \alpha + \beta_1}, \quad m_1 = \int_0^\infty t g_1(t)dt
\]

\[
m_2 = \int_0^\infty e^{-\lambda t} G_1(t)dt = \frac{1 - g_2^*(\lambda)}{\lambda}, \quad m_3 = \int_0^\infty t g_2(t)dt
\]

\[
m_5 = \int_0^\infty e^{-\gamma_1 t} dt = \frac{1}{\gamma_1}, \quad m_6 = m_0, \quad m_7 = m_1, \quad m_8 = m_2
\]

\[
m_9 = m_3, \quad m_{11} = \int_0^\infty e^{-\gamma_2 t} dt = \frac{1}{\gamma_2}, \quad m_{12} = m_0, \quad m_{13} = m_1
\]

\[
m_{14} = m_2, \quad m_{15} = m_3, \quad m_{17} = \int_0^\infty e^{-\gamma_3 t} dt = \frac{1}{\gamma_3}
\]

**Availability at Full Capacity** in steady state
\[ A_0 = \lim_{s \to 0} sA_0^*(s) = \frac{N_1}{D_1} \]

**Availability in Single Cycle**

\[ A_0^* = \lim_{s \to 0} sA_0^{**}(s) = \frac{N_2}{D_1} \]

**Expected Down Time Excluding Failed State**

\[ DT_0 = \frac{N_3}{D_1} \]

**Expected Time for Minor Inspection**

\[ MI_0 = \lim_{s \to 0} sMI_0^*(s) = \frac{N_4}{D_1} \]

**Expected Time for Path Inspection**

\[ PI_0 = \lim_{s \to 0} sPI_0^*(s) = \frac{N_5}{D_1} \]

**Expected Time for Major Inspection**

\[ MJ_0 = \lim_{s \to 0} sMJ_0^*(s) = \frac{N_6}{D_1} \]

**Busy period Analysis for doing Repair**

\[ B_0 = \lim_{s \to 0} sB_0^*(s) = \frac{N_7}{D_1} \]

**Expected Number of Visits of the Repairman**

\[ V_0 = \lim_{s \to 0} (sV_0^{**}(s)) = \frac{N_8}{D_1} \]

where

\[ N_1 = \mu_0[(p_{05} + p_{6,11})p_{12,17} + p_{05}p_{6,11}] \]

\[ D_1 = p_{6,11}p_{12,17}[\mu_0 + (p_{01} + p_{02}P_{21}^{(4)})\mu_1 + (p_{02} + p_{03})\mu_2] + p_{05}p_{12,17}[\mu_0 + (p_{67} + p_{68}P_{87}^{(10)})\mu_1 \]

\[ + (p_{68} + p_{69})\mu_2 + p_{5}p_{6,11}[\mu_0 + (p_{12,13} + p_{12,14}P_{14,13})\mu_4 + (p_{12,14} + p_{12,15})\mu_5] \]

\[ + p_{05}p_{6,11}p_{12,17}(\mu_5 + \mu_1 + \mu_7) \]

\[ N_2 = p_{12,17}[p_{02}p_{6,11} + p_{05}p_{6,11}] \mu_2 + p_{05}p_{6,11}p_{12,14} \mu_2 \]

\[ N_3 = p_{12,17}[p_{05}p_{6,11} + p_{03}p_{6,11} + (p_{01} + p_{02}P_{21}^{(4)})\mu_4]p_{6,11} + (p_{69} + p_{69}p_{87}^{(10)})\mu_1p_{05} \]

\[ + [p_{12,17}\mu_1 + p_{12,13} + p_{12,14}P_{14,13})\mu_4p_{05}p_{6,11} \]

\[ N_4 = \mu_5p_{05}p_{6,11}p_{12,17} \]

\[ N_5 = p_{12,17}p_{12,17} \mu_{17} \]

\[ N_6 = p_{05}p_{6,11}p_{12,17} \mu_{17} \]

\[ N_7 = [(p_{05} + p_{02})\mu_5 + (p_{01} + p_{02}P_{21}^{(4)})]p_{6,11}p_{12,17} + [(p_{69} + p_{68})\mu_5 + (p_{67} + p_{68}p_{87}^{(10)})\mu_1]p_{05}p_{12,17} \]

\[ + [(p_{12,15} + p_{12,14})p_{6,11} + (p_{12,13} + p_{12,14}P_{14,13})\mu_4p_{05}p_{6,11} \]

\[ N_8 = (p_{05} + p_{6,11})p_{12,17} + p_{05}p_{6,11} \]

**Cost-Benefit Analysis**

Expected profit incurred to the system is the excess of revenue over cost and in steady state is given by
P ROF3 = C_{30}A_0 + C_{315}A_0' - C_{32}DT_0 - C_{33}MI_0 - C_{34}PI_0 - C_{35}MJ_0 - C_{36}B_0 - C_{37}V_0

C_{30} = \text{Revenue per unit uptime with full capacity.}
C_{315} = \text{Revenue per unit uptime in single cycle}
C_{32} = \text{Loss per unit time for which the system is in down state (other than failed state)}
C_{33} = \text{Cost per unit time for doing minor inspection/maintenance.}
C_{34} = \text{Cost per unit time for doing path inspection/maintenance.}
C_{35} = \text{Cost per unit time for doing major inspection/maintenance.}
C_{36} = \text{Cost per unit time for engaging the repairman for doing repair.}
C_{37} = \text{Cost per visit of the repairman.}

\text{COMPUTATIONAL ANALYSIS}

The following particular case is considered for numerical calculations where all the distributions of times have been taken as exponential. One may, however, take that distribution which will be best fitted to the actual data. The goodness-of-fit tests given in Chapter 1 may be applied to find which one of the distributions can be fitted to the given data.

\begin{align*}
g_1(t) &= \delta_1 e^{-\delta_1 t},
g_2(t) &= \delta_2 e^{-\delta_2 t}
\end{align*}

Using the estimated values on the basis of gathered information and assumed values for other parameters, i.e.,

\begin{align*}
\lambda &= 0.000023, \beta = 0.0001, \gamma_1 = 0.042, \gamma_2 = 0.0019, \gamma_3 = 0.0014, p = 0.5; \\
\text{the values of various measures of system effectiveness are obtained as:}
\end{align*}

Mean time to system failure = 277567300 hrs
Availability at full capacity (A_0) = 0.969129300
Availability in single cycle (A_0') = 0.000242143
Expected down time excluding failed state (DT_0) = 0.016125980
Expected time for minor inspection (MI_0) = 0.004807190
Expected time for path inspection (PI_0) = 0.010626420
Expected time for major inspection (MJ_0) = 0.014421570
Busy period analysis for repair (B_0) = 0.001015411
Expected number of visits of the repairman (V_0) = 0.000102243

Graphical study has been made for the MTSF, availability at full capacity, availability in single cycle and the profit with respect to failure rate of steam turbine (\(\alpha\)), revenue per unit uptime in case of working at full Capacity (C_{30}), revenue per unit uptime in single Cycle (C_{31}) for different values of probability of demand on higher payment (\(p\)) and for different values of loss during down time (C_{32}). \textbf{Fig. 2} shows the behaviour of MTSF w.r.t. failure rate (\(\alpha\)) of steam turbine for different values of probability of demand on higher payment (\(p\)). It is clear from the graph that MTSF gets decreased with the increase in the values of the failure rate (\(\alpha\)) of steam turbine. It has higher values for lower values of probability of demand on higher payment (\(p\)).
**Figure 2:**

**Fig. 3** reveals the behaviour of availability at full Capacity ($A_0$) w. r. t. failure rate ($\alpha$) of steam turbine. It can be seen from the graph that availability decreases with the increase in the values of failure rate ($\alpha$) of steam turbine with negligible change in availability ($A_0$) for different values of probability of demand on higher payment ($p$).

![Graph showing availability versus failure rate](image)

**Figure 3:**

**Fig. 4** depicts the behaviour of availability ($A_S$) in single cycle w.r.t. failure rate ($\alpha$) of steam turbine for different values of probability of demand on higher rates ($p$). It is clear from the graph that the availability ($A_S$) increases with increase in the values of failure rate ($\alpha$) of steam turbine. Also, it has higher values for higher values of the probability of demand on higher payment ($p$). **Fig. 5** shows the behaviour of the availabilities $A_0$, $A_S$ w.r.t. failure rate ($\alpha$) of steam turbine. It can be observed from the graph that the availability ($A_0$) decreases as failure rate ($\alpha$) increases whereas the availability $A_S$ increases with increase in the values of failure rate ($\alpha$). It can also be observed that $A_0$ is greater than or less than $A_S$ according to whether failure rate ($\alpha$) is lesser or greater than 0.080046053. **Fig. 6** shows the behaviour of profit w.r.t. to failure rate.
(α) of steam turbine for different values of probability (p). At the initial stages when the values of probability (p) are small, the profit decreases as failure rate (α) of steam turbine increases. But after certain probability (p) level, there comes a stage where profit starts increasing with respect to failure rate (α) of steam turbine. In any case, it has higher values for higher values of probability (p).

**Fig. 5:**

**PROFIT VERSUS FAILURE RATE OF STEAM TURBINE (α) FOR DIFFERENT VALUES OF PROBABILITY OF DEMAND ON HIGHER RATES (p)**

**Fig. 6:**

Fig. 7 depicts the behaviour of the profit w.r.t. revenue per unit uptime when the system works at full Capacity (C₃₀) for different values of loss during down time (C₃₂). It can be interpreted that the profit increases with increase in the values of C₃₀. Following can also be concluded from the graph:
1. For $C_{32} = 400000$, profit is positive or zero or negative according as $C_{30} >$ or $= or < 12333.00$ i.e. the price per unit of the electricity should be fixed in such a way so as to give $C_{30}$ not less than 12333.00 to get positive profit.
2. For $C_{32} = 450000$, profit is positive or zero or negative according as $C_{30} >$ or $= or < 13909.03$ the price per unit of the electricity should be fixed in such a way so as to give $C_{30}$ not less than 13909.03 to get positive profit.
3. For $C_{32} = 500000$, profit is positive or zero or negative according as $C_{30} >$ or $= or < 15485.06$ i.e. the price per unit of the electricity should be fixed in such a way so as to give $C_{30}$ not less than 15485.06 to get positive profit.

**Figure 7:**

**Figure 8** depicts the behaviour of profit w.r.t. revenue per unit uptime in single cycle ($C_{31S}$) for different values of loss during down time($C_{32}$). It can be noticed that the profit increases with increase in the values of $C_{31S}$. Following observations can also be made from the graph:
1. For $C_{32} = 633340$, profit is positive or zero or negative according as $C_{31S} >$ or $= or < 1362.86$ i.e. the price per unit of the electricity produced in single cycle should be fixed in such a way so as to give $C_{31S}$ not less than 1362.86 to get positive profit.
2. For $C_{32} = 633390$, profit is positive or zero or negative according as $C_{31S} >$ or $= or < 7669.31$ i.e. the price per unit of the electricity produced in single cycle should be fixed in such a way so as to give $C_{31S}$ not less than 7669.31 to get positive profit.
3. For $C_{32} = 633440$, profit is positive or zero or negative according as $C_{31S} >$ or $= or < 13975.76$ i.e. the price per unit of the electricity produced in single cycle should be fixed in such a way so as to give $C_{31S}$ not less than 13975.76 to get positive profit.
REFERENCES


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