Cost- benefit analysis of two dissimilar warm standby system subject to failure due to landslide caused by heavy rainfall, snowmelt and water level change

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<u>Abstract</u>

In the majority of cases the main trigger of landslides is heavy or prolonged rainfall. Generally this takes the form of either an exceptional short lived event, such as the passage of a tropical cyclone or even the rainfall associated with a particularly intense thunderstorm or of a long duration rainfall event with lower intensity, such as the cumulative effect of monsoon rainfall in South Asia. In the former case it is usually necessary to have very high rainfall intensities, whereas in the latter the intensity of rainfall may be only moderate - it is the duration and existing pore water pressure conditions that are important. The importance of rainfall as a trigger for landslides cannot be underestimated. A global survey of landslide occurrence in the 12 months to the end of September 2003 revealed that there were 210 damaging landslide events worldwide. Of these, over 90% were triggered by heavy rainfall. One rainfall event for example in Sri Lanka in May 2003 triggered hundreds of landslides, killing 266 people and rendering over 300,000 people temporarily homeless. In July 2003 an intense rain band associated with the annual Asian monsoon tracked across central Nepal, triggering 14 fatal landslides that killed 85 people. The reinsurance company Swiss Re estimated that rainfall induced landslides associated with the 1997-1998 El Nino event triggered landslides along the west coast of North, Central and South America that resulted in over \$5 billion in losses. Finally, landslides triggered by Hurricane Mitch in 1998 killed an estimated 18,000 people in Honduras, Nicaragua, Guatemala and El Salvador. So why does rainfall trigger so many landslides? Principally this is because the rainfall drives an increase in pore water pressures within the soil. Landslides have always existed on our planet. Generally classified as mass movements of rock, debris, and soil down a slope of land. While landslides are a naturally occurring environmental hazard they have recently increased in frequency in certain areas due to human activity. Two-unit standby system subject to environmental conditions such as shocks, change of weather conditions etc. have been discussed in reliability literature by several authors due to significant importance in defenses, industry etc. In the present paper we have taken two-non-identical warm standby system with failure time distribution as exponential and repair time distribution as general. We are considering system subject to failure due to (i) landslide caused by heavy rainfall and (ii) landslide caused by Snowmelt (iii) landslide caused due to water level change requiring different types of repair facilities. Using semi Markov regenerative point technique we have calculated different reliability characteristics such as MTSF, reliability of the system, availability analysis in steady state, busy period analysis of the system under repair, expected number of visits by the repairman in the long run and gain-function and graphs are drawn.

Keyword: warm standby, failure due to landslide caused by heavy rainfall, failure due to landslide caused by Snowmelt, failure due to landslide caused by water level change.

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SNOWMELT

In many cold mountain areas, snowmelt can be a key mechanism by which landslide initiation can occur. This can be especially significant when sudden increases in temperature lead to rapid melting of the snow pack. This water can then infiltrate into the ground, which may have impermeable layers below the surface due to still-frozen soil or rock, leading to rapid increases in pore water pressure, and resultant landslide activity. This effect can be especially serious when the warmer weather is accompanied by precipitation, which both adds to the groundwater and accelerates the rate of thawing.

Water-Level Change

Rapid changes in the groundwater level along a slope can also trigger landslides. This is often the case where a slope is adjacent to a water body or a river. When the water level adjacent to the slope falls rapidly the groundwater level frequently cannot dissipate quickly enough, leaving an artificially high water table. This subjects the slope to higher than normal shear stresses, leading to potential instability. This is probably the most important mechanism by which river bank materials fail, being significant after a flood as the river level is declining (i.e. on the falling limb of the hydrograph).

Assumptions

- 1. The failure time distribution is exponential whereas the repair time distribution is arbitrary of two non-identical units.
- 2. The repair facility is of four types :
 - Type I, II repair facility

When failure due to landslide caused by heavy rainfall and failure due to landslide caused by Snow melt of first unit occurs respectively

And Type III, IV repair facility

When failure due to landslide caused by heavy rainfall and failure due to landslide caused due to Snow melt of the second unit occurs respectively.

- 3. The repair starts immediately upon failure of units and the repair discipline is FCFS.
- 4. The repairs are perfect and start immediately after failure due to landslide caused by heavy rainfall and failure due to landslide caused by Snowmelt as soon as the of the system become normal. The failure due to landslide cused by heavy rainfall and failure due to landslide caused by Snowmelt in both the units do not occur simultaneously.
- 5. The failure of a unit is detected immediately and perfectly.
- 6. The switches are perfect and instantaneous.
- 7. All random variables are mutually independent.

SYMBOLS FOR STATES OF THE SYSTEM

Superscripts: O, WS, SO, FLSHR, FLSSM, FLSWLC

Operative, Warm Standby, Stops the operation, failure due to landslide caused by heavy rainfall, failure due to landslide caused by Snowmelt, failure due to landslide caused by water level change respectively

Subscripts: nhrf, lshrf, lssm, lswlc, ur, wr, uR

No heavy rainfall, landslide due to heavy rainfall, landslide due to Snowmelt, landslide due to water level change, under repair, waiting for repair, under repair continued respectively

Up states: 0, 1, 2, 9;

Down states: 3,4,5,6,7,8,10,11

Regeneration point: 0,1,2,4,7,10

States of the System

 $0(O_{nhrf}, WS_{nhrf})$

One unit is operative and there is no heavy rainfall and the other unit is warm standby with no heavy rainfall in both the units.

1(SO_{lshrf}, O_{nhrf})

The operation of the first unit stops automatically due to landslide caused by heavy rainfall and warm standby unit's starts operating with no heavy rainfall.

2(FLSHR_{ur}, O_{nhrf})

The first unit fails and undergoes repair after failure due to landslide caused by heavy rainfall is over and the other unit continues to be operative with no heavy rainfall.

3(FLSHR_{uR}, SO lshrf)

The repair of the first unit is continued from state 2 and the operation of second unit stops automatically due to landslide caused by heavy rainfall.

4(FLSHR_{ur}, SO_{lshrf})

The first unit fails and undergoes repair after the landslide caused by heavy rainfall is over and the other unit also stops automatically due to landslide caused by heavy rainfall.

5(FLSHR_{uR}, FLSSM_{sm,wr})

The repair of the first unit is continued from state 4 and the other unit is failed due to landslide caused due to Snowmelt is waiting for repair.

6(O_{nhrf}, FLSSM_{ur})

The first unit becomes operative with no heavy rainfall and the second unit is failed due to landslide caused by Snowmelt is under repair.

7(SO lshrf, FLSWLCnhrf, ur)

The operation of the first unit stops automatically due to landslide caused by heavy rainfall and the second unit fails due to landslide caused by water level change and undergoes repair and there is no heavy rainfall.

8(FLSSM_{lssm, wr}, FLSWLC _{nhrf, uR})

The repair of failed unit due to landslide is continued from state 7 and the first unit is failed due to landslide caused by Snowmelt is waiting for repair.

9(O nhrf, SOlshrf)

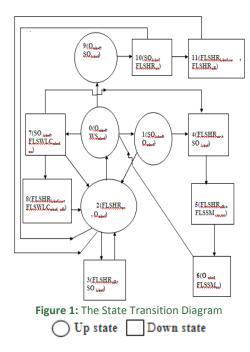
The first unit is operative with no heavy rainfall and the operation of warm standby second unit is stopped automatically due to landslide caused by heavy rainfall.

10(SO_{lshrf}, FLSHR_{ur})

The operation of the first unit stops automatically due to landslide caused by heavy rainfall and the second unit fails due to landslide caused by heavy rainfall undergoes repair after the heavy rainfall is over.

11(FLSHR_{lshrf, wr}, FLSHR_{uR})

The repair of the second unit is continued from state 10 and the first unit is failed due to landslide caused by heavy rainfall is waiting for repair.



TRANSITION PROBABILITIES

Simple probabilistic considerations yield the following expressions :

$$p_{01} = \frac{\lambda 1}{\lambda 1 + \lambda 2 + \lambda 3}, P_{07} = \frac{\lambda 2}{\lambda 1 + \lambda 2 + \lambda 3}$$
$$p_{09} = \frac{\lambda 2}{\lambda 1 + \lambda 2 + \lambda 3}, p_{12} = \frac{\lambda 1}{\lambda 1 + \lambda 3}, p_{14} = \frac{\lambda 3}{\lambda 1 + \lambda 3}$$

$$\begin{array}{ll} P_{20} = G_1^{*}(\lambda_1), P_{22}^{(3)} = G_1^{*}(\lambda_1) = \overline{p_{23}}, P_{72} = G_2^{*}(\lambda_4), \\ P_{72}^{(8)} = G_2^{*}(\lambda_4) = P_{78} \\ \text{We can easily verify that} \\ p_{01} + p_{07} + p_{09} = 1, p_{12} + p_{14} = 1, p_{20} + p_{23} = p_{22}^{(3)} = 1, p_{46}^{(6)} = 1 p_{60} = 1, \\ p_{72} + P_{72}^{(5)} + p_{74} = 1, p_{9,10} = 1, p_{10,2} + p_{10,2}^{(11)} = 1 \\ \text{And mean sojourn time is} \\ \mu_0 = E(T) = \int_0^{\infty} P[T > t] dt \\ \text{(2)} \\ \text{Mean Time to System Failure} \\ \text{We can regard the failed state as absorbing} \\ \theta_0(t) = Q_{01}(t) [s] \theta_1(t) + Q_{09}(t) [s] \theta_9(t) + Q_{07}(t) \\ \theta_1(t) = Q_{12}(t) [s] \theta_2(t) + Q_{14}(t), \theta_2(t) = Q_{20}(t) [s] \theta_0(t) + Q_{22}^{(3)}(t) \\ \theta_4(t) = Q_{9,10}(t) \\ \text{Taking Laplace-Stiltjes transform of eq. (3-5) and solving for} \\ Q_0^{*}(s) = N_{1}(s) / D_{1}(s) \\ \text{where} \\ N_1(s) = Q_{01}^{*}(s) \begin{cases} Q_{12}^{*}(s) Q_{22}^{(3)*}(s) + Q_{14}^{*}(s) \} + Q_{09}^{*}(s) Q_{9,10}^{*}(s) + Q_{07}^{*}(s) \\ D_{1}(s) = 1 - Q_{01}^{*}(s) & Q_{12}^{*}(s) Q_{20}^{*}(s) \\ M_{21}(s) & Q_{12}^{*}(s) Q_{20}^{*}(s) \\ M_{10}(s) = 0 \\ \text{for the transform of eq. (3-5) and solving for} \\ Q_0^{*}(s) = N_1(s) / D_1(s) \\ \text{where} \\ N_1(s) = Q_{01}^{*}(s) & \{Q_{12}^{*}(s) Q_{22}^{*}(s) + Q_{14}^{*}(s)\} + Q_{09}^{*}(s) Q_{9,10}^{*}(s) + Q_{07}^{*}(s) \\ D_{1}(s) = 1 - Q_{01}^{*}(s) & Q_{12}^{*}(s) Q_{20}^{*}(s) \\ \text{Making use of relations (1) and (2) it can be shown that $\theta_0^{*}(0) = 1$, which implies that $\theta_0(t)$ is a proper distribution.
 MTSF = E[T] = d/ds $\theta_0^{*}(s)$$$

s=0 $= (\mu_0 + p_{01} \mu_1 + p_{01} p_{12} \mu_2 + p_{09} \mu_9) / (1 - p_{01} p_{12} p_{20})$ where $\mu_0 = \mu_{01} + \mu_{07} + \mu_{09}, \mu_1 = \mu_{12} + \mu_{14}, \mu_2 = \mu_{20} + \mu_{22}^{(3)}, \mu_9 = \mu_{9.10}$

AVAILABILITY ANALYSIS

Let M_i(t) be the probability of the system having started from state i is up at time t without making any other regenerative state. By probabilistic arguments, we have The value of $M_0(t)$, $M_1(t)$, $M_2(t)$, $M_4(t)$ can be found easily. The point wise availability $A_i(t)$ have the following recursive relations $A_0(t) = M_0(t) + q_{01}(t)[c]A_1(t) + q_{07}(t)[c]A_7(t) + q_{09}(t)[c]A_9(t)$ $A_{1}(t) = M_{1}(t) + q_{12}(t)[c]A_{2}(t) + q_{14}(t)[c]A_{4}(t), A_{2}(t) = M_{2}(t) + q_{20}(t)[c]A_{0}(t) + q_{22}^{(3)}(t)[c]A_{2}(t)$ $\begin{aligned} A_4(t) &= q_{46}^{(3)}(t)[c]A_6(t), A_6(t) = q_{60}(t)[c]A_0(t) \\ A_7(t) &= (q_{72}(t) + q_{72}^{(8)}(t)) [c]A_2(t) + q_{74}(t)[c]A_4(t) \end{aligned}$ $A_{9}(t) = M_{9}(t) + q_{9,10}(t)[c]A_{10}(t), A_{10}(t) = q_{10,2}(t)[c]A_{2}(t) + q_{10,2}^{(11)}(t)[c]A_{2}(t)$ (7-14)Taking Laplace Transform of eq. (7-14) and solving for $\hat{A}_0(s)$ $\hat{A}_0(s) = N_2(s) / D_2(s)$ (15)where $N_{2}(s) = (1 - \hat{q}_{22}^{(3)}(s)) \{ \widehat{M}_{0}(s) + \hat{q}_{01}(s) \widehat{M}_{1}(s) + \hat{q}_{09}(s) \widehat{M}_{9}(s) \} + \widehat{M}_{2}(s) \{ \hat{q}_{01}(s) \hat{q}_{42}(s) + \hat{q}_{07}(s) (\hat{q}_{72}(s) + \hat{q}_{73}^{(8)}(s)) + \hat{q}_{09}(s) \hat{q}_{910}(s) (\hat{q}_{10,2}(s) + \hat{q}_{10,2}^{(11)}(s)) \}$ $D_{2}(s) = (1 - \hat{q}_{22}^{(3)}(s)) \{ 1 - \hat{q}_{46}^{(5)}(s) \hat{q}_{60}(s) (\hat{q}_{01}(s) \hat{q}_{44}^{(4)}(s) + \hat{q}_{07}(s) \hat{q}_{74}(s))$ $- \hat{q}_{20}(s) \{ \hat{q}_{01}(s) \hat{q}_{12}(s) + \hat{q}_{07}(s) (\hat{q}_{72}(s)) + \hat{q}_{72}^{(8)}(s) + \hat{q}_{09}^{(6)}(s) \hat{q}_{9,10}^{(6)}(s)$ $(\hat{q}_{10,2}^{(6)}(s) + \hat{q}_{10,2}^{(11)}(s)) \}$ The steady state availability $A_{0} = \lim_{t \to \infty} [A_{0}(t)] = \lim_{s \to 0} [s \hat{A}_{0}(s)] = \lim_{s \to 0} \frac{s N_{2}(s)}{D_{2}(s)}$ Using L' Hospitals rule, we get $A_0 = \lim_{s \to 0} \frac{\frac{N_2(s) + s N_2'(s)}{D_2'(s)}}{\frac{N_2(s) + s N_2'(s)}{D_2'(0)}} = \frac{N_2(0)}{D_2'(0)}$ (16)Where $N_2(0) = p_{20}(\widehat{M}_0(0) + p_{01}\widehat{M}_1(0) + p_{09}\widehat{M}_9(0)) + \widehat{M}_2(0)(p_{01}p_{12} + p_{07}(p_{72})) + p_{07}(p_{72}) + p_{17}(p_{72}) + p_{17}(p_{72})$ $+ p_{72}^{(8)} + p_{09}))$

$$\begin{split} & D_{2}(0) = p_{31}\{\mu_{0} + p_{01}, \mu_{1} + (p_{01}, p_{1} + p_{02}, p_{1}, p_{1} + p_{01}, \mu_{2} + p_{01}, \mu_{2} + p_{02}, \mu_{2} + \mu_{20} + \mu_{22} + \mu_$$

 $\widehat{P_0}(s) = N_5(s) / D_2(s)$ (48)where $N_2(s) = \hat{q}_{07}(s) \hat{L}_7(s) (1 - \hat{q}_{22}^{(3)}(s))$ and $D_2(s)$ is defined earlier. In the long run, $P_0 = \frac{N_5(0)}{D_2'(0)}$ (49)where $N_5(0) = p_{20} p_{07} \hat{L}_4(0)$ and $D_2(0)$ is already defined. The expected busy period of the server for repair of the unit when failure is due to landslide in (0, t] is $\lambda_{rs}(t) = \int_0^{\infty} P_0(z) dz$ So that $\widehat{\lambda_{rs}}(s) = \frac{\widehat{P}_0(s)}{s}$ (50)The expected number of visits by the repairman for repairing the non-identical units in (0, t] $H_0(t) = Q_{01}(t)[c]H_1(t) + Q_{07}(t)[c]H_7(t) + Q_{09}(t)[c]H_9(t)$ $H_{1}(t) = Q_{12}(t)[c][1+H_{2}(t)] + Q_{14}(t)[c][1+H_{4}(t)], H_{2}(t) = Q_{20}(t)[c]H_{0}(t) + Q_{22}^{(3)}(t)[c]H_{2}(t)$ $H_4(t) = Q_{46}^{(3)}(t)[c]H_6(t), H_6(t) = Q_{60}(t)[c]H_0(t)$ $H_{7}(t) = (Q_{72}(t) + Q_{72}^{(8)}(t)) [c]H_{2}(t) + Q_{74}(t)[c]H_{4}(t)$ $H_9(t) = Q_{910}(t)[c][1+H_{10}(t)], H_{10}(t) = (Q_{102}(t)[c] + Q_{102}^{(11)}(t))[c]H_2(t)$ (51-58)Taking Laplace Transform of eq. (51-58) and solving for $H_0^*(s)$ $H_0^*(s) = N_6(s) / D_3(s)$ (59)Where $N_{6}(s) = (1 - Q_{22}^{(3)*}(s)) \{ Q_{01}^{*}(s) (Q_{12}^{*}(s) + Q_{14}^{*}(s)) + Q_{09}^{*}(s) Q_{01}^{*}(s) \}$ $D_{3}(s) = (1 - Q_{22}^{(3)*}(s)) \{1 - (Q_{01}^{*}(s) Q_{14}^{*}(s) + Q_{07}^{*}(s) Q_{74}^{*}(s)) Q_{46}^{(5)*}(s) Q_{60}^{*}(s)\}$ $- Q_{20}^{*}(s) \{Q_{01}^{*}(s) Q_{12}^{*}(s) + Q_{07}^{*}(s)(Q_{72}^{*}(s)) + Q_{72}^{*}(s)) + Q_{72}^{*}(s) + Q_{09}^{*}(s) Q_{9,10}^{*}(s) (Q_{10,2}^{*}(s) + Q_{10,2}^{*}(s))\}$ $In the long run, H_{0} = \frac{N_{6}(0)}{D_{2}^{*}(0)}$ (60) $D_{3}'(0)$ where $N_6(0) = p_{20} (p_{01} + p_{09})$ and D'₃(0) is already defined. The expected number of visits by the repairman for repairing the unit when failure is due to caused by heavy rainfall in (0, t] $V_0(t) = Q_{01}(t)[c]V_1(t) + Q_{07}(t)[c]V_7(t) + Q_{09}(t)[c]V_9(t)$ $V_{1}(t) = Q_{12}(t)[c]V_{2}(t) + Q_{14}(t)[c]V_{4}(t), V_{2}(t) = Q_{20}(t)[c]V_{0}(t) + Q_{22}^{(3)}(t)[c]V_{2}(t)$ $V_4(t) = Q_{46}^{(3)}(t)[c]V_6(t), V_6(t) = Q_{60}(t)[c]V_0(t)$

 $V_{7}(t) = (Q_{72}(t)[1+V_{2}(t)] + Q_{72}^{(8)}(t)) [c]V_{2}(t) + Q_{74} (t)[c]V_{4}(t)$ $V_{9}(t) = Q_{9,10}(t)[c]V_{10}(t), V_{10}(t) = (Q_{10,2}(t) + Q_{10,2}^{(11)}(t))[c]V_{2}(t)$ (61-68) Taking Laplace-Stieltjes transform of eq. (61-68) and solving for $V_{0}^{*}(s)$ $V_{0}^{*}(s) = N_{7}(s) / D_{4}(s)$ (69) where $N_{7}(s) = Q^{*}_{07}(s) Q^{*}_{72}(s) (1 - Q_{22}^{(3)*}(s))$ and $D_{4}(s)$ is the same as $D_{3}(s)$ In the long run, $V_{0} = \frac{N_{7}(0)}{D_{4}'(0)}$ (70)

where $N_7(0) = p_{20} p_{07} p_{72}$ and $D'_3(0)$ is already defined.

COST-BENEFIT ANALYSIS

The gain- function of the system considering mean up-time, expected busy period of the system failure due to landslide caused by heavy rainfall when the units stops automatically, expected busy period of the server for repair of unit failure due to landslide caused by water level change, expected number of visits by the repairman for unit failure, expected number of visits by the repairman for failure due to landslide caused by heavy rainfall. The expected total cost-benefit incurred in (0, t] is

C (t) = Expected total revenue in (0, t]

- expected total repair cost for failure due to landslide caused by heavy rainfall in (0,t] when the units automatically stop in (0,t]
- expected total repair cost for repairing the units failure due to landslide caused by snowmelt in (0, t]
- expected busy period of the system when the units failure due to landslide caused by water level change in (0,t]
- expected number of visits by the repairman for repairing the unit fails due to landslide caused by heavy rainfall in (0,t]
- expected number of visits by the repairman for repairing of the non-identical units in (0,t]

The expected total cost per unit time in steady state is

 $C = \lim_{t \to \infty} (C(t)/t) = \lim_{s \to 0} (s^2 C(s))$ = K₁A₀ - K₂P₀ - K₃B₀ - K₄R₀ - K₅V₀ - K₆H₀ Where

K₁: revenue per unit up-time,

 K_2 : cost per unit time repair of the system when failure due to landslide caused by heavy rainfall when units automatically stop,

 K_3 : cost per unit time for which the system is under repair when failure due to landslide caused by snowmelt

K4: cost per unit time for which the system is under repair when failure due to due to water level change,

K₅: cost per visit by the repairman for repair when the failure is due to landslide caused by heavy rainfall,

K₆: cost per visit by the repairman for units repair.

CONCLUSION

After studying the system, we have analyzed graphically that when the failure rate due to landslide caused by heavy rainfall, failure rate due to landslide caused by Snowmelt and failure due to landslide caused by water level change increases, the MTSF and steady state availability decreases and the cost function decreased as the failure increases.

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