

# Benefit-Function of Two- Identical Cold Standby Helicopter System subject to Engine failure or Human errors

Ashok Kumar Saini

Associate Professor, Department of Mathematics, B. L. J. S. College, Tosham, Bhiwani, Haryana, INDIA.

Email: [drashokksaini2009@gmail.com](mailto:drashokksaini2009@gmail.com)

## Abstract

**Autorotation** is a state of flight where the main rotor system of a helicopter or similar aircraft turns by the action of air moving up through the rotor, as with an autogyro, rather than engine power driving the rotor. The term *autorotation* dates to a period of early helicopter development between 1915 and 1920, and refers to the rotors turning without the engine. In normal powered flight, air is drawn into the main rotor system from above and exhausted downward, but during autorotation, air moves up into the rotor system from below as the helicopter descends. Autorotation is permitted mechanically because of both a freewheeling unit, which allows the main rotor to continue turning even if the engine is not running, as well as curved main rotor blades such that when the collective pitch is fully down the inner part of the blade has negative pitch relative to the horizontal plane and can be spun up by the relative wind. It is the means by which a helicopter can land safely in the event of complete engine failure. Consequently, all single-engine helicopters must demonstrate this capability to obtain a type certificate. The longest autorotation in history was performed by Jean Boulet in 1972 when he reached a record altitude of 12,440 m (40,814 ft) in an Aérospatiale Lama. Because of a  $-63^{\circ}\text{C}$  temperature at that altitude, as soon as he reduced power the engine flamed out and could not be restarted. By using autorotation he was able to land the aircraft safely. In this paper we have taken **failure due to engine failure or failure due to human errors**. When the main unit fails due to **failure due to human errors** then cold standby system becomes operative. **Failure due to human errors** cannot occur simultaneously in both the units and after failure the unit undergoes very costly repair facility immediately. Applying the regenerative point technique with renewal process theory the various reliability parameters MTSF, Availability, Busy period, Benefit-Function analysis have been evaluated.

**Keywords:** Cold Standby, failure due to engine failure or failure due to human errors, first come first serve, MTSF, Availability, Busy period, Benefit -Function.

## \* Address for Correspondence:

Dr. Ashok Kumar Saini, Associate Professor, Department of Mathematics, BLJS College, Tosham, Bhiwani, Haryana, INDIA.

Email: [drashokksaini2009@gmail.com](mailto:drashokksaini2009@gmail.com)

Received Date: 12/11/2014 Accepted Date: 23/11/2014

## Access this article online

Quick Response Code:



Website:

[www.statperson.com](http://www.statperson.com)

DOI: 24 November  
2014

## INTRODUCTION

### Descent and landing

For a helicopter, "autorotation" refers to the descending maneuver where the engine is disengaged from the main rotor system and the rotor blades are driven solely by the upward flow of air through the rotor. The *freewheeling unit* is a

special clutch mechanism that disengages anytime the engine rpm is less than the rotor rpm. If the engine fails, the freewheeling unit automatically disengages the engine from the main rotor allowing the main rotor to rotate freely. The most common reason for autorotation is an engine malfunction or failure, but autorotation can also be performed in the event of a complete tail rotor failure, or following loss of tail-rotor effectiveness, since there is virtually no torque produced in an autorotation. If altitude permits, autorotations may also be used to recover from vortex ring state. In all cases, a successful landing depends on the helicopter's height and velocity at the commencement of autorotation. At the instant of engine failure, the main rotor blades are producing lift and thrust from their angle of attack and velocity. By immediately lowering collective pitch, which must be done in case of an engine failure, the pilot reduces lift and drag and the helicopter begins an immediate descent, producing an upward flow of air through the rotor system. This upward flow of air through the rotor provides sufficient thrust to maintain rotor rpm throughout the descent. Since the tail rotor is driven by the main rotor transmission during autorotation, heading control is maintained as in normal flight. Several factors affect the rate of descent in autorotation: density altitude, gross weight, rotor rpm, and forward airspeed. The pilot's primary control of the rate of descent is airspeed. Higher or lower airspeeds are obtained with the cyclic pitch control just as in normal flight. Rate of descent is high at zero airspeed and decreases to a minimum at approximately 50 to 90 knots, depending upon the particular helicopter and the factors previously mentioned. As the airspeed increases beyond the speed that gives minimum rate of descent, the rate of descent increases again. Even at zero airspeed, the rotor is quite effective as it has nearly the drag coefficient of a parachute despite having much lower solidity. When landing from an autorotation, the kinetic energy stored in the rotating blades is used to decrease the rate of descent and make a soft landing. A greater amount of rotor energy is required to stop a helicopter with a high rate of descent than is required to stop a helicopter that is descending more slowly. Therefore, autorotative descents at very low or very high airspeeds are more critical than those performed at the minimum rate of descent airspeed. Each type of helicopter has a specific airspeed at which a power-off glide is most efficient. The best airspeed is the one that combines the greatest glide range with the slowest rate of descent. The specific airspeed is different for each type of helicopter, yet certain factors (density altitude, wind) affect all configurations in the same manner. The specific airspeed for autorotations is established for each type of helicopter on the basis of average weather and wind conditions and normal loading. A helicopter operated with heavy loads in high density altitude or gusty wind conditions can achieve best performance from a slightly increased airspeed in the descent. At low density altitude and light loading, best performance is achieved from a slight decrease in normal airspeed. Following this general procedure of fitting airspeed to existing conditions, the pilot can achieve approximately the same glide angle in any set of circumstances and estimate the touchdown point. This optimum glide angle is usually 17-20 degrees.

### AUTOROTATIONAL REGIONS

During vertical autorotation, the rotor disc is divided into three regions—the driven region, the driving region, and the stall region. The size of these regions varies with the blade pitch, rate of descent, and rotor rpm. When changing autorotative rpm, blade pitch, or rate of descent, the size of the regions change in relation to each other. The driven region, also called the propeller region, is the region at the end of the blades. Normally, it consists of about 30 percent of the radius. It is the driven region that produces the most drag. The overall result is a deceleration in the rotation of the blade. The driving region, or autorotative region, normally lies between 25 to 70 percent of the blade radius, which produces the forces needed to turn the blades during autorotation. Total aerodynamic force in the driving region is inclined slightly forward of the axis of rotation, producing a continual acceleration force. This inclination supplies thrust, which tends to accelerate the rotation of the blade. Driving region size varies with blade pitch setting, rate of descent, and rotor rpm. The inner 25 percent of the rotor blade is referred to as the stall region and operates above its maximum angle of attack (stall angle) causing drag, which slows rotation of the blade. A constant rotor rpm is achieved by adjusting the collective pitch so blade acceleration forces from the driving region are balanced with the deceleration forces from the driven and stall regions. By controlling the size of the driving region, the pilot can adjust autorotative rpm. For example, if the collective pitch is raised, the pitch angle increases in all regions. This causes the point of equilibrium to move inboard along the blade's span, thus increasing the size of the driven region. The stall region also becomes larger while the driving region becomes smaller. Reducing the size of the driving region causes the acceleration force of the driving region and rpm to decrease. Stochastic behavior of systems operating under changing environments has widely been studied. Dhillon, B.S. and Natesan, J. (1983) studied outdoor power systems in fluctuating environment. Kan Cheng (1985) has studied reliability analysis of a system in a randomly changing environment. Jinhua Cao (1989) has studied a man machine system operating under changing environment subject to a Markov process with two states. The change in operating conditions viz. fluctuations of voltage, corrosive atmosphere, very low gravity etc. may make a system

completely inoperative. Severe environmental conditions can make the actual mission duration longer than the ideal mission duration. In this paper we have taken **failure due to engine failure or failure due to human errors**. When the main operative unit fails then cold standby system becomes operative. **Failure due to human errors failure** cannot occur simultaneously in both the units and after failure the unit undergoes repair facility of very high cost in case of **failure due to engine failure** immediately. The repair is done on the basis of first fail first repaired.

### ASSUMPTIONS

1.  $\lambda_1, \lambda_2$  are constant failure rates for **failure due to engine failure or failure due to human errors** respectively. The CDF of repair time distribution of Type I and Type II are  $G_1(t)$  and  $G_2(t)$ .
2. The failure due to **failure due to human errors** is non-instantaneous and it cannot come simultaneously in both the units.
3. The repair starts immediately after the failure due to **failure due to engine failure or failure due to human errors** works on the principle of first fail first repaired basis.
4. The repair facility does no damage to the units and after repair units are as good as new.
5. The switches are perfect and instantaneous.
6. All random variables are mutually independent.
7. When both the units fail, we give priority to operative unit for repair.
8. Repairs are perfect and failure of a unit is detected immediately and perfectly.
9. The system is down when both the units are non-operative.

### NOTATIONS

$\lambda_1, \lambda_2$  are the **failure rates due to engine failure or failure due to human errors** respectively.  $G_1(t), G_2(t)$  – repair time distribution Type -I, Type-II **failure due to engine failure or failure due to human errors** respectively.

$p, q$  - probability of **failure due to engine failure or failure due to human errors** respectively such that  $p+q=1$

$M_i(t)$  System having started from state  $i$  is up at time  $t$  without visiting any other regenerative state

$A_i(t)$  state is up state as instant  $t$

$R_i(t)$  System having started from state  $i$  is busy for repair at time  $t$  without visiting any other regenerative state.

$B_i(t)$  the server is busy for repair at time  $t$ .

$H_i(t)$  Expected number of visits by the server for repairing given that the system initially starts from regenerative state  $i$

#### Symbols for states of the System

##### Superscripts O, CS, EF, HEF

Operative, Cold Standby, **failure due to engine failure or failure due to human errors** respectively

##### Subscripts $n_{hef}, hef, ef, ur, wr, uR$

No failure due to human errors, failure due to engine failure, under repair, waiting for repair, under repair continued from previous state respectively

Up states – 0, 1, 2, 7, 8 ;

Down states – 3, 4, 5, 6

regeneration point – 0,1,2, 7, 8

#### States of the System

##### 0( $O_{n_{hef}}, CS_{n_{hef}}$ )

One unit is operative and the other unit is cold standby and there is no failure due to Human errors in both the units.

##### 1( $EF_{ef, ur}, O_{n_{hef}}$ )

The operating unit fails due to engine and is under repair immediately of very costly Type- I and standby unit starts operating with no failure due to Human errors.

##### 2( $HEF_{hef, ur}, O_{n_{hef}}$ )

The operative unit fails due to HEF resulting from failure due to Human errors and undergoes repair of type II and the standby unit becomes operative with no failure due to Human errors.

##### 3( $EF_{ef, uR}, HEF_{hef, wr}$ )

The first unit fails due to engine and under very costly Type-I repair is continued from state 1 and the other unit fails due to HEF resulting from Failure due to Human errors and is waiting for repair of Type -II.

##### 4( $EF_{ef, uR}, EF_{ef, wr}$ )

The repair of the unit is failed due to EF resulting from failure due to engine is continued from state 1 and the other unit failed due to RAF resulting from failure due to engine is waiting for repair of Type-I.

**5(HEF<sub>hef,ur</sub>, HEF<sub>hef,wr</sub>)**

The operating unit fails due to failure due to Human errors (HEF mode) and under repair of Type - II continues from the state 2 and the other unit fails also due to failure due to Human errors is waiting for repair of Type-II.

**6(HEF<sub>hef,ur</sub>, EF<sub>ef,wr</sub>)**

The operative unit fails due to HEF resulting from failure due to Human errors and under repair continues from state 2 of Type -II and the other unit is failed due to EF resulting from failure due to engine and under very costly Type-1

**7(O<sub>nhef</sub>, EF<sub>ef,ur</sub>)**

The repair of the unit failed due to EF resulting from failure due to engine failure is completed and there is no failure due to Human errors and the other unit is failed due to RAF resulting from failure due to engine is under repair of very costly Type-1

**8(O<sub>nhef</sub>, HEF<sub>hef,ur</sub>)**

The repair of the unit failed due to EF resulting from failure due to engine failure is completed and there is no failure due to Human errors and the other unit is failed due to HEF resulting from failure due to Human errors is under repair of Type-II.

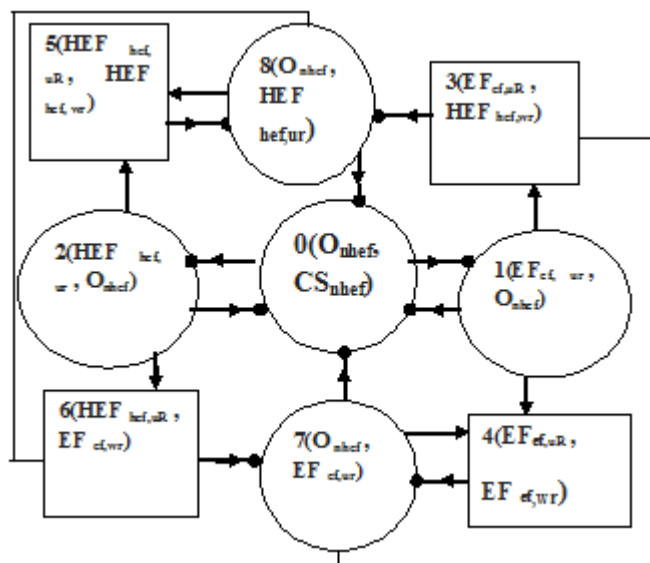


Figure 1: The State Space Diagram  
 ○ Up state    □ down state  
 ● Regeneration point

**TRANSITION PROBABILITIES**

Simple probabilistic considerations yield the following expressions:

$$\begin{aligned}
 p_{01} &= p, & p_{02} &= q, \\
 p_{10} &= pG_1^*(\lambda_1) + qG_1^*(\lambda_2) = p_{70}, \\
 p_{20} &= pG_2^*(\lambda_1) + qG_2^*(\lambda_2) = p_{80}, \\
 p_{11}^{(3)} &= p(1 - G_1^*(\lambda_1)) = p_{14} = p_{71}^{(4)} p_{28}^{(5)} = q(1 - G_2^*(\lambda_2)) = p_{25} = p_{82}^{(5)}
 \end{aligned}
 \tag{1}$$

We can easily verify that

$$\begin{aligned}
 p_{01} + p_{02} &= 1, & p_{10} + p_{17}^{(4)} (= p_{14}) + p_{18}^{(3)} (= p_{13}) &= 1, \\
 p_{80} + p_{82}^{(5)} + p_{87}^{(6)} &= 1
 \end{aligned}
 \tag{2}$$

And mean sojourn time is

$$\mu_0 = E(T) = \int_0^\infty P[T > t] dt$$

**Mean Time To System Failure**

$$\emptyset_0(t) = Q_{01}(t)[s] \emptyset_1(t) + Q_{02}(t)[s] \emptyset_2(t)$$

$$\emptyset_1(t) = Q_{10}(t)[s] \emptyset_0(t) + Q_{13}(t) +$$

$$\begin{aligned} \emptyset_2(t) = & Q_{20}(t)[s] \emptyset_0(t) + Q_{25}(t) + \\ & Q_{26}(t) \end{aligned} \tag{3-5}$$

We can regard the failed state as absorbing

Taking Laplace-Stiljes transform of eq. (3-5) and solving for

$$\emptyset_0^*(s) = N_1(s) / D_1(s) \tag{6}$$

where

$$N_1(s) = Q_{01}^* [ Q_{13}^* (s) + Q_{14}^* (s) ] + Q_{02}^* [ Q_{25}^* (s) + Q_{26}^* (s) ]$$

$$D_1(s) = 1 - Q_{01}^* Q_{10}^* - Q_{02}^* Q_{20}^*$$

Making use of relations (1) & (2) it can be shown that  $\emptyset_0^*(0) = 1$ , which implies that  $\emptyset_0(t)$  is a proper distribution.

$$\begin{aligned} \text{MTSF} = E[T] &= \frac{d}{ds} \emptyset_0^*(s) \Big|_{s=0} \\ &= (D_1'(0) - N_1'(0)) / D_1(0) \\ &= (\mu_0 + p_{01} \mu_1 + p_{02} \mu_2) / (1 - p_{01} p_{10} - p_{02} p_{20}) \end{aligned}$$

where

$$\begin{aligned} \mu_0 &= \mu_{01} + \mu_{02} \\ \mu_1 &= \mu_{01} + \mu_{17}^{(4)} + \mu_{18}^{(3)}, \\ \mu_2 &= \mu_{02} + \mu_{27}^{(6)} + \mu_{28}^{(5)} \end{aligned}$$

**Availability analysis**

Let  $M_i(t)$  be the probability of the system having started from state i is up at time t without making any other regenerative state. By probabilistic arguments, we have

$$M_0(t) = e^{-\lambda_1 t} e^{-\lambda_2 t}$$

$$M_1(t) = p G_1(t) e^{-(\lambda_1 + \lambda_2)t} = M_7(t)$$

$$M_2(t) = q G_2(t) e^{-(\lambda_1 + \lambda_2)t} = M_8(t)$$

The point wise availability  $A_i(t)$  have the following recursive relations

$$A_0(t) = M_0(t) + q_{01}(t)[c]A_1(t) + q_{02}(t)[c]A_2(t)$$

$$A_1(t) = M_1(t) + q_{10}(t)[c]A_0(t) + q_{18}^{(3)}(t)[c]A_8(t) + q_{17}^{(4)}(t)[c]A_7(t)$$

$$A_2(t) = M_2(t) + q_{20}(t)[c]A_0(t) + [q_{28}^{(5)}(t)[c]A_8(t) + q_{27}^{(6)}(t)[c]A_7(t)]$$

$$A_7(t) = M_7(t) + q_{70}(t)[c]A_0(t) + [q_{71}^{(4)}(t)[c]A_1(t) + q_{78}^{(3)}(t)[c]A_8(t)]$$

$$A_8(t) = M_8(t) + q_{80}(t)[c]A_0(t) + [q_{82}^{(5)}(t)[c]A_2(t) + q_{87}^{(6)}(t)[c]A_7(t)] \tag{7-11}$$

Taking Laplace Transform of eq. (7-11) and solving for  $\bar{A}_0(s)$

$$\bar{A}_0(s) = N_2(s) / D_2(s) \tag{12}$$

where

$$\begin{aligned} N_2(s) = & \bar{M}_0 (1 - \hat{q}_{78}^{(3)} - \hat{q}_{87}^{(6)}) - \hat{q}_{82}^{(5)} (\hat{q}_{27}^{(6)} \hat{q}_{78}^{(3)} + \hat{q}_{28}^{(5)} - \hat{q}_{71}^{(4)}) \\ & ( \hat{q}_{17}^{(4)} + \hat{q}_{87}^{(6)} \hat{q}_{18}^{(3)} ) + \hat{q}_{71}^{(4)} \hat{q}_{82}^{(5)} ( \hat{q}_{17}^{(4)} - \hat{q}_{27}^{(6)} \hat{q}_{18}^{(3)} ) + \hat{q}_{01} [ \bar{M}_1 (1 - \\ & \hat{q}_{78}^{(3)} \hat{q}_{87}^{(6)}) + \hat{q}_{71}^{(4)} ( \bar{M}_7 + \hat{q}_{78}^{(3)} \bar{M}_8 ) + \hat{q}_{18}^{(3)} ( \bar{M}_7 \hat{q}_{87}^{(6)} - \bar{M}_8 ) - \\ & \hat{q}_{82}^{(5)} ( \bar{M}_1 ( \hat{q}_{27}^{(6)} \hat{q}_{78}^{(3)} + \hat{q}_{28}^{(5)} ) + \hat{q}_{17}^{(4)} ( - \bar{M}_2 ( \hat{q}_{78}^{(3)} + \bar{M}_7 \hat{q}_{28}^{(5)} ) - \\ & \hat{q}_{18}^{(3)} ( \bar{M}_2 + \bar{M}_7 \hat{q}_{27}^{(6)} ) ) ] \hat{q}_{02} [ \bar{M}_2 (1 - \hat{q}_{78}^{(3)} \hat{q}_{87}^{(6)}) + \hat{q}_{27}^{(6)} ( \\ & \bar{M}_7 + \hat{q}_{78}^{(3)} \bar{M}_8 ) + \hat{q}_{28}^{(5)} ( \bar{M}_7 \hat{q}_{87}^{(6)} + \bar{M}_8 ) - \hat{q}_{71}^{(4)} ( \bar{M}_1 ( - \hat{q}_{27}^{(6)} - \hat{q}_{28}^{(5)} + \\ & \hat{q}_{87}^{(6)} ) + \hat{q}_{17}^{(4)} ( \bar{M}_2 + \hat{q}_{28}^{(5)} \bar{M}_8 ) - \hat{q}_{18}^{(3)} ( - \bar{M}_2 \hat{q}_{87}^{(6)} + \bar{M}_8 \hat{q}_{27}^{(6)} ) ) ] \\ & \hat{q}_{18}^{(3)} ( \bar{M}_2 + \bar{M}_7 \hat{q}_{27}^{(6)} ) ] \end{aligned}$$

$$\begin{aligned} D_2(s) = & (1 - \hat{q}_{78}^{(3)} - \hat{q}_{87}^{(6)}) - \hat{q}_{82}^{(5)} (\hat{q}_{27}^{(6)} \hat{q}_{78}^{(3)} + \hat{q}_{28}^{(5)}) - \hat{q}_{71}^{(4)} \\ & ( \hat{q}_{17}^{(4)} + \hat{q}_{87}^{(6)} \hat{q}_{18}^{(3)} ) + \hat{q}_{71}^{(4)} \hat{q}_{82}^{(5)} ( \hat{q}_{17}^{(4)} \hat{q}_{28}^{(5)} - \hat{q}_{18}^{(3)} ) + \hat{q}_{01} [ - \hat{q}_{10} (1 - \\ & \hat{q}_{78}^{(3)} \hat{q}_{87}^{(6)}) - \hat{q}_{71}^{(4)} ( \hat{q}_{70} + \hat{q}_{78}^{(3)} \hat{q}_{80} ) - \hat{q}_{18}^{(3)} ( \hat{q}_{70} \hat{q}_{87}^{(6)} - \hat{q}_{80} ) - \\ & \hat{q}_{82}^{(5)} ( - \hat{q}_{10} ( \hat{q}_{27}^{(6)} \hat{q}_{78}^{(3)} + \hat{q}_{28}^{(5)} ) + \hat{q}_{17}^{(4)} ( \hat{q}_{20} ( \hat{q}_{78}^{(3)} - \hat{q}_{70} \hat{q}_{28}^{(5)} ) + \end{aligned}$$

$$\begin{aligned} & \{ \hat{q}_{18}^{(3)}(\hat{q}_{20} + \hat{q}_{70}\hat{q}_{27}^{(6)}) \} \hat{q}_{02} [ -\hat{q}_{20}(1 - \hat{q}_{78}^{(3)}\hat{q}_{87}^{(6)}) - \hat{q}_{27}^{(6)}(\hat{q}_{70} + \hat{q}_{78}^{(3)}\hat{q}_{80}) - \hat{q}_{28}^{(5)}(\hat{q}_{70}\hat{q}_{87}^{(6)} + \hat{q}_{80}) - \hat{q}_{71}^{(4)}(\hat{q}_{10}(\hat{q}_{27}^{(6)} + \hat{q}_{28}^{(5)}\hat{q}_{87}^{(6)}) - \hat{q}_{17}^{(4)}(\hat{q}_{20} - \hat{q}_{28}^{(5)}\hat{q}_{80}) - \hat{q}_{18}^{(3)}(\hat{q}_{20}\hat{q}_{87}^{(6)} + \hat{q}_{80}\hat{q}_{27}^{(6)}) \} ] \\ & \text{(Omitting the arguments } s \text{ for brevity)} \end{aligned}$$

The steady state availability

$$A_0 = \lim_{t \rightarrow \infty} [A_0(t)] = \lim_{s \rightarrow 0} [s \hat{A}_0(s)] = \lim_{s \rightarrow 0} \frac{s N_2(s)}{D_2(s)}$$

Using L' Hospital's rule, we get

$$A_0 = \lim_{s \rightarrow 0} \frac{N_2(s) + s N_2'(s)}{D_2(s)} = \frac{N_2(0)}{D_2(0)}$$

(13)

The expected up time of the system in (0,t] is

$$\lambda_{up}(t) = \int_0^t A_0(z) dz \quad \text{So that } \bar{\lambda}_{up}(s) = \frac{\hat{A}_0(s)}{s} = \frac{N_2(s)}{s D_2(s)}$$

(14)

The expected down time of the system in (0,t] is

$$\lambda_{down}(t) = t - \lambda_{up}(t) \quad \text{So that } \bar{\lambda}_{down}(s) = \frac{1}{s^2} - \bar{\lambda}_{up}(s)$$

(15)

**The expected busy period of the server when there is HEF - Failure due to Human errors or EF- failure due to Failure due to engine in (0, t]**

$$\begin{aligned} R_0(t) &= q_{01}(t)[c]R_1(t) + q_{02}(t)[c]R_2(t) \\ R_1(t) &= S_1(t) + q_{10}(t)[c]R_0(t) + q_{18}^{(3)}(t)[c]R_8(t) + q_{17}^{(4)}(t)[c]R_7(t) \\ R_2(t) &= S_2(t) + q_{20}(t)[c]R_0(t) + q_{28}^{(5)}(t)R_8(t) + q_{27}^{(6)}(t)[c]R_7(t) \\ R_7(t) &= S_7(t) + q_{70}(t)[c]R_0(t) + q_{71}^{(4)}(t)R_1(t) + q_{78}^{(3)}(t)[c]R_8(t) \\ R_8(t) &= S_8(t) + q_{80}(t)[c]R_0(t) + q_{82}^{(5)}(t)R_2(t) + q_{87}^{(6)}(t)[c]R_7(t) \end{aligned}$$

(16-20)

Taking Laplace Transform of eq. (16-20) and solving for  $\bar{R}_0^*(s)$

$$\bar{R}_0^*(s) = N_3(s) / D_2(s)$$

(21)

where

$$\begin{aligned} N_3(s) &= \hat{q}_{01} [ \hat{S}_1(1 - \hat{q}_{78}^{(3)}\hat{q}_{87}^{(6)}) + \hat{q}_{71}^{(4)}(\hat{S}_7 + \hat{q}_{78}^{(3)}\hat{S}_8) + \hat{q}_{18}^{(3)}(\hat{S}_7 \\ & \hat{q}_{87}^{(6)} - \hat{S}_8) - \hat{q}_{01}\hat{q}_{82}^{(5)}(\hat{S}_1\hat{q}_{27}^{(6)}\hat{q}_{78}^{(3)} + \hat{q}_{28}^{(5)}) + \hat{q}_{17}^{(4)}(\hat{S}_2\hat{q}_{78}^{(3)} + \hat{S}_7\hat{q}_{28}^{(5)}) - \\ & \hat{q}_{18}^{(3)}(\hat{S}_2 + \hat{S}_7\hat{q}_{27}^{(6)}) + \hat{q}_{02}[\hat{S}_2(1 - \hat{q}_{78}^{(3)}\hat{q}_{87}^{(6)}) + \hat{q}_{27}^{(6)}(\hat{S}_7 + \hat{q}_{78}^{(3)}\hat{S}_8) + \hat{q}_{28}^{(5)}(\hat{S}_7\hat{q}_{87}^{(6)} + \hat{S}_8) - \hat{q}_{02}\hat{q}_{71}^{(4)}(\hat{S}_1 - \hat{q}_{27}^{(6)} - \hat{q}_{28}^{(5)}\hat{q}_{87}^{(6)}) \\ & \hat{q}_{17}^{(4)}(\hat{S}_2 + \hat{q}_{28}^{(5)}\hat{S}_8) - \hat{q}_{18}^{(3)}(-\hat{S}_2\hat{q}_{87}^{(6)} + \hat{S}_8\hat{q}_{27}^{(6)})] \end{aligned}$$

and

$D_2(s)$  is already defined.

(Omitting the arguments  $s$  for brevity)

$$\text{In the long run, } R_0 = \frac{N_3(0)}{D_2(0)}$$

(22)

The expected period of the system under HEF - failure due to Human errors or EF- Failure due to engine in (0, t] is

$$\lambda_{rv}(t) = \int_0^t R_0(z) dz \quad \text{So that } \bar{\lambda}_{rv}(s) = \frac{\bar{R}_0(s)}{s}$$

**The expected number of visits by the repairman for repairing the identical units in (0,t]**

$$\begin{aligned} H_0(t) &= Q_{01}(t)[s][1 + H_1(t)] + Q_{02}(t)[s][1 + H_2(t)] \\ H_1(t) &= Q_{10}(t)[s]H_0(t) + Q_{18}^{(3)}(t)[s]H_8(t) + Q_{17}^{(4)}(t)[s]H_7(t) \\ H_2(t) &= Q_{20}(t)[s]H_0(t) + Q_{28}^{(5)}(t)[s]H_8(t) + Q_{27}^{(6)}(t)[c]H_7(t) \\ H_7(t) &= Q_{70}(t)[s]H_0(t) + Q_{71}^{(4)}(t)[s]H_1(t) + Q_{78}^{(3)}(t)[c]H_8(t) \\ H_8(t) &= Q_{80}(t)[s]H_0(t) + Q_{82}^{(5)}(t)[s]H_2(t) + Q_{87}^{(6)}(t)[c]H_7(t) \end{aligned}$$

(23-27)

Taking Laplace Transform of eq. (23-27) and solving for  $H_0^*(s)$

$$H_0^*(s) = N_4(s) / D_3(s)$$

(28)

In the long run,  $H_0 = N_4(0) / D_3(0)$

(29)

## BENEFIT- FUNCTION ANALYSIS

The Benefit-Function analysis of the system considering mean up-time, expected busy period of the system under failure due to engine or Failure due to Human errors, expected number of visits by the repairman for unit failure.

The expected total Benefit-Function incurred in  $(0,t]$  is

$C(t)$  = Expected total revenue in  $(0,t]$

- expected busy period of the system under failure due to engine or failure due to Human errors for repairing the units in  $(0,t]$

- expected number of visits by the repairman for repairing of identical the units in  $(0, t]$

The expected total cost per unit time in steady state is

$$C = \lim_{t \rightarrow \infty} (C(t)/t) = \lim_{s \rightarrow 0} (s^{-2} C(s))$$

$$= K_1 A_0 - K_2 R_0 - K_3 H_0$$

where

$K_1$  - revenue per unit up-time,

$K_2$  - cost per unit time for which the system is under repair of type- I or type- II

$K_3$  - cost per visit by the repairman for units repair.

## CONCLUSION

After studying the system, we have analyzed graphically that when the failure rate due to engine or Failure due to Human errors increases, the MTSF and steady state availability decreases and the Benefit-function decreased as the failure increases.

## REFERENCES

1. Dhillon, B.S. and Natesen, J, Stochastic Analysis of outdoor Power Systems in fluctuating environment, Microelectron. Reliab. ,1983; 23, 867-881.
2. Kan, Cheng, Reliability analysis of a system in a randomly changing environment, Acta Math. Appl. Sin. 1985, 2, pp.219-228.
3. Cao, Jinhua, Stochastic Behaviour of a Man Machine System operating under changing environment subject to a Markov Process with two states, Microelectron. Reliab. ,1989; 28, pp. 373-378.
4. Barlow, R.E. and Proschan, F., Mathematical theory of Reliability, 1965; John Wiley, New York.
5. Gnedanke, B.V., Belyayar, Yu.K. and Soloyer , A.D. , Mathematical Methods of Relability Theory, 1969 ; Academic Press, New York.

Source of Support: None Declared  
Conflict of Interest: None Declared