Research Article

The growth rate of partial maxima on a subsequence for samples from two independent populations

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Abstract In this paper, we obtain the almost sure limit set of the random vector consisting of properly normalized partial maxima from two independent populations over a subsequence.
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INTRODUCTION

Let $\{X_n, n \geq 1\}$ be a sequence of independent random variables. For each integer $n \geq 1$, let the distribution function ofX_n be either F_1 or F_2 where F_1 and F_2 are two specified d.f 's. The number of r.v's with d.f F_j among $X_1, X_2 \ldots\ldots\ldots X_n$ is $t_j(n)$where

t_1 and t_2 are specified positive integers (assumed non-random) with $t_1(n) + t_2(n) = n$ and $t_j(n) \to \infty, n \to \infty$ $j = 1,2$.

Let $F_j(x)$ have a density $f_j(x)$ which is positive for large x and for such x put $g_j(x) = \frac{(1-F_j(x))}{f_j(x)} loglog\left(\frac{1}{1-F_j(x)}\right)$ $j = 1,2$.

Let $Y_n = \max(X_1, X_2 \ldots\ldots\ldots X_n)$. Note that $Y_n = \max(Y_{1,n}, Y_{2,n})$ where Y_{jn} is the maximum of $t_j(n)$ observations that follow the law F_j. Define $b_j(n)$ by

$1 - F_j\left(b_j(n)\right) = \frac{1}{n}$ $j = 1,2$.

If $\lim_{x\to\infty} \frac{g_j(x)}{x} = c_j$ $(0 \leq c_j \leq \infty), j = 1,2$, then by De Haan and Hordijk (1972) we have the following results.

$$\lim_{n\to\infty} sup \frac{Y_{j,n}}{b_j(t_j(n))} = e^{c_j} \text{ a.s} \quad (1.1)$$

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$\lim_{n\to\infty} inf \frac{Y_{j,n}}{b_j(t_j(n))} = 1$ a.s. (1.2)
where for all $x > 0$,
$1 - F_j(b_j\left(t_j(n,x)\right) = \frac{(log t_j(n))^{r_{j,n}(x)}}{t_j(n)}$ (1.3)
and for all $x > 0$,
$\lim_{n\to\infty} r_{j,n}(x) = -\frac{logx}{c_j}$. (1.4)
Let $n_k = [\exp k^h]$ where h>1. Then by Husler, J. (1985) we have the following theorem.

Theorem 1.1
(Husler, J., 1985): Let $c = \frac{1}{h}$. If $\lim_{n\to\infty} \frac{g_j(x)}{x} = c_j \left(0 \le c_j < \infty\right), j = 1,2.$ then $\lim_{n\to\infty} sup \frac{Y_{j,n_k}}{b_j(t_j(n_k))} = e^{cc_j}$ a.s and $\lim_{n\to\infty} inf \frac{Y_{j,n_k}}{b_j(t_j(n_k))} = 1$ a.s.

Nayak. S.S (1986) obtained the almost sure limit set of properly normalized $\left(\frac{Y_{1,n}}{b_1(t_1(n))}, \frac{Y_{2,n}}{b_2(t_2(n))}\right)$. In this paper, we obtain the limit set of the above random vector overn_k. Some more related results on limit point may be found in Nayak, S.S (1985), Hebbar, H.V.(1980) and Wichuna (1974). Throughout the paper, the letter D with a suffix denotes a positive constant which may have different values in different appearances. “Infinitely often” is denoted by i. o.

PRELIMINARIES

Lemma 2.1: Let $\{x_n, n \ge 1\}$ be a sequence of real numbers such that $\lim_{n\to\infty} nx_n = 0$
Then If $\lim_{n\to\infty} \frac{1-(1-x_n)^n}{nx_n} = 1.$

Proof: Using Binomial theorem we have

$$1 - (1 - x_n)^n = \sum_{r=1}^{n} \binom{n}{r} (-1)^{r+1} {x_n}^r$$
$$= \sum_{r=1}^{n} \frac{(-1)^{r+1}}{r!} (nx_n)^r \left(1 - \frac{1}{n}\right)\left(1 - \frac{2}{n}\right) \ldots\ldots\ldots\ldots\ldots \left(1 - \frac{r-1}{n}\right)$$

Hence $\left|\frac{1-(1-x_n)^n}{nx_n} - 1\right| \le \frac{nx_n}{1-nx_n} \to 0 \; as \; n \to \infty.$

Lemma 2.2 (Ortega and Wschebor, 1984): Let $\{A_n, n \ge 1\}$ be a sequence of events. If
(i) $\sum P(A_n) = \infty$ and

$$(ii) \lim_{n\to\infty} \inf \sum_{1\le j} \sum_{\le k \le n} \frac{P\left(A_j \cap A_k\right) - P\left(A_j\right)(A_k}{\left[\sum_{j=1}^{n} P\left(A_j\right)\right]^2} < 0 \; then \; P(A_n \; i.o.) = 1.$$

LIMIT POINTS

Theorem 3.1: Assume that $\lim_{n\to\infty} \frac{t_j(n)}{n} = a_j \left(0 < a_j < 1\right) j = 1,2 \; and$
$\lim_{n\to\infty} \frac{g_j(x)}{x} = c_j \left(0 \le c_j < \infty\right) j = 1,2$. Then the set of all almost sure limit points of $\left(\frac{Y_{1,n_k}}{b_1(t_1(n_k))}, \frac{Y_{2,n_k}}{b_2(t_2(n_k))}\right)$ is $S = \left\{(x,y): 1 \le x \le e^{cc_1}, 1 \le y \le e^{cc_2}, \frac{logx}{c_1} + \frac{logy}{c_2} \le c\right\}$ where $c = \frac{1}{h}$ and $n_k = [\exp k^h]$, h>1.
The proof of the theorem depends on the following two lemmas.

Lemma 3.1: Assume the conditions of theorem 3.1. For all $\varepsilon > 0$ and x, y satisfying $1 < x < e^{cc_1}, 1 < y < e^{cc_2}, \frac{logx}{c_1} + \frac{logy}{c_2} < c$ we have

$$P\left(x - \varepsilon < \frac{Y_{1,n_k}}{b_1(t_1(n_k))} < x + \varepsilon, y - \varepsilon < \frac{Y_{2,n_k}}{b_2(t_2(n_k))} < y + \varepsilon \; i.o) = 1\right)$$

Proof: Let

$$E_k = \left\{x - \varepsilon < \frac{Y_{1,n_k}}{b_1(t_1(n_k))} < x + \varepsilon, y - \varepsilon < \frac{Y_{2,n_k}}{b_2(t_2(n_k))} < y + \varepsilon\right\}$$

Note that $P(E_k)$ is an increasing function of ε. Hence it is enough to prove that

$P(E_k\ i.o)$=1 for $0 < \varepsilon < \min(x, y)\ where 1 < x < e^{cc_1}, 1 < y < e^{cc_2}\ and\ \frac{logx}{c_1} + \frac{logy}{c_2} < c$. since $Y_{1,n_k}\ and\ Y_{2,n_k}$are independent, we have $P(E_k) = P(A_{1k}(x)P(A_{2k}(y))$ where
$A_{jk}(x) = \{a_{jk}(x-\varepsilon) < Y_{jn_k} < a_{jk}(x+\varepsilon)\}$ and $a_{jk}(x) = b_j\left(t_j(n_k)\right)x. j = 1,2.$
By (1.3) and (1.4) and the fact that $\lim_{k\to\infty}\frac{t_j(n_k)}{n_k} = a_j(0 < a_j < 1)$ it follows that
$t_j\{1 - F_j(a_{jk}(x)\} \to 0$ as $k \to \infty$. Hence by Lemma 2.1,
$1 - F_j^{t_j(n_k)}\left(a_{jk}(x)\right) \sim t_j(n_k)\{1 - F_j(a_{jk}(x)\}, k \to \infty.$
Hence as $k \to \infty$, we have

$$\frac{1 - F_1^{t_1(n_k)}\left(a_{1k}(x+\varepsilon)\right)}{1 - F_1^{t_1(n_k)}\left(a_{1k}(x-\varepsilon)\right)} \sim \frac{1 - F_1\left(a_{1k}(x+\varepsilon)\right)}{1 - F_1\left(a_{1k}(x-\varepsilon)\right)}$$
$$\sim (logt_1(n_k))^{r_{1n_k}(x+\varepsilon) - r_{1n_k}(x-\varepsilon)}$$
$$\sim (logn_k)^{r_{1n_k}(x+\varepsilon) - r_{1n_k}(x-\varepsilon)}$$
$\to 0$ as $k \to \infty$.
This gives ,

$$P(A_{1k}(x)) = F_1^{t_1(n_k)}\left(a_{1k}(x+\varepsilon)\right) - F_1^{t_1(n_k)}\left(a_{1k}(x-\varepsilon)\right)$$
$$= 1 - F_1^{t_1(n_k)}\left(a_{1k}(x-\varepsilon)\right) - \{1 - F_1^{t_1(n_k)}\left(a_{1k}(x+\varepsilon)\right)\}$$
$$\sim 1 - F_1^{t_1(n_k)}\left(a_{1k}(x-\varepsilon)\right) \text{ as } k \to \infty$$
$$\sim t_1(n_k)\{1 - F_1\left(a_{1k}(x-\varepsilon)\right)\} \text{ as } k \to \infty \text{ (by lemma 2.1)}$$
$$= (logt_1(n_k))^{r_{1n_k}(x-\varepsilon)}$$
$$\sim (logn_k)^{r_{1n_k}(x-\varepsilon)} \text{ as } k \to \infty.$$

Similarly $P(A_{2k}(y)) \sim (logn_k)^{r_{2n_k}(y-\varepsilon)}$ as $k \to \infty$.
Hence we have $P(E_k) \sim (logn_k)^{r_{1n_k}(x-\varepsilon) + r_{2n_k}(y-\varepsilon)}$ as $k \to \infty$.
$> Dk^{-h\left[\frac{1}{c_1}\log(x-\varepsilon) + \frac{1}{c_2}\log(y-\varepsilon) + \varepsilon_1\right]}\ for\ all\ k \geq k_1$
Where $0 < \varepsilon_1 < \frac{1}{h} - \frac{1}{c_1}\log(x-\varepsilon) - \frac{1}{c_2}\log(y-\varepsilon)$. Hence
$\sum_k P(E_k) = \infty$ (3.1)
Let u and v be two large positive integers such that $u < v\ and\ a_{ju}(x) > 0\ and$
$a_{jv}(x) > 0, x > 1$. We have
$P(E_u \cap E_v) = P\left(A_1(u, v, x)\right)P\left(A_2(u, v, y)\right)$
where
$A_j(u, v, x)\left\{a_{1u}(x-\varepsilon) < Y_{jn_u} < a_{1u}(x+\varepsilon), a_{1v}(x-\varepsilon) < Y_{jn_u} < a_{1v}(x+\varepsilon)\right\}$, j=1,2.
Now
$P\left(A_1(u, v, x)\right) =$
$P(a_{1u}(x-\varepsilon) < Y_{1n_u} < a_{1u}(x+\varepsilon), a_{1v}(x-\varepsilon) < max(Y_{1n_u}, Y_{n_v - n_u}) < a_{1v}(x+\varepsilon))$
where $Y_{1n_v - n_u}$ is the maximum of $t_1(n_v) - t_2(n_u)$ observations from F_1.
Note that $Y_{1n_u} and\ Y_{1n_v - n_u}$are independent .
Let $A = \{a_{1u}(x-\varepsilon) < Y_{1n_u} < a_{1u}(x+\varepsilon)\}$
$and\ B = \{\ a_{1v}(x-\varepsilon) < \max\ (Y_{1n_u}, Y_{1n_v - n_u}) < a_{1v}(x+\varepsilon)\}$.
Then $B = \left(\{Y_{1n_u} > a_{1v}(x-\varepsilon), Y_{1n_v - n_u} \leq a_{1v}(x-\varepsilon)\} \cup \{Y_{1n_u} \leq a_{1v}(x-\varepsilon), Y_{1n_v - n_u} > a_{1v}(x-\varepsilon)\} \cup \{Y_{1n_u} > a_{1v}(x-\varepsilon), Y_{1n_v - n_u} > a_{1v}(x-\varepsilon)\} \cap \{Y_{1n_u} < a_{1v}(x+\varepsilon), Y_{1n_v - n_u} < a_{1v}(x+\varepsilon)\}\right)$
$= \{\ a_{1v}(x-\varepsilon) < Y_{1n_u} < a_{1v}(x+\varepsilon), Y_{1n_v - n_u} < a_{1v}(x-\varepsilon)\}$
$\cup \{Y_{1n_u} < a_{1v}(x-\varepsilon),\ a_{1v}(x-\varepsilon) < Y_{1n_v - n_u} \leq a_{1v}(x+\varepsilon)\}$
$\cup \{\ a_{1v}(x-\varepsilon) < Y_{1n_u} < a_{1v}(x+\varepsilon),\ a_{1v}(x-\varepsilon) < Y_{1n_v - n_u} < a_{1v}(x+\varepsilon)\}$
$= B_1 \cup B_2 \cup B_3$ say.
Then $A_1(u, v, x) = P(A \cap B)$
$= P\left(A \cap (B_1 \cup B_2 \cup B_3)\right)$

$= P(C_1 \cup C_2 \cup C_3)$
Where $C_1 = A \cap B_1, C_2 = A \cap B_2$ *and* $C_3 = A \cap B_3$.
Note that $C_1 \cap C_2 = \emptyset$ $C_2 \cap C_3 = \emptyset$ *and* $C_1 \cap C_3 = \emptyset$. Hence
$P\big(A_1(u, v, x)\big) = P(C_1) + P(C_2) + P(C_3)$
$= P(A \cap B_1) + P(A \cap B_2) + P(A \cap B_3)$
$=P\left(a_{1v}(x-\varepsilon) < Y_{1n_u} < a_{1u}(x+\varepsilon)\right) P\left(Y_{1n_v-n_u} < a_{1v}(x-\varepsilon)\right)+$
$P\left(a_{1u}(x-\varepsilon) < Y_{1n_u} < min\big(a_{1u}(x+\varepsilon), a_{1v}(x-\varepsilon)\big)\right) P\left(Y_{1n_v-n_u} > a_{1v}(x-\varepsilon)\right)+$
$P\left(a_{1v}(x-\varepsilon) < Y_{1n_u} < a_{1u}(x+\varepsilon)\right)\ P\left(Y_{1n_v-n_u} > a_{1v}(x-\varepsilon)\right)$
$< P\left(a_{1v}(x-\varepsilon) < Y_{1n_u} < a_{1v}(x+\varepsilon)\right) +$
$P\left(a_{1u}(x-\varepsilon) < Y_{1n_u} < a_{1u}(x+\varepsilon)\right)\ P\left(Y_{1n_v-n_u} > a_{1v}(x-\varepsilon)\right).$
Now $P\left(a_{1v}(x-\varepsilon) < Y_{1n_u} < a_{1v}(x+\varepsilon)\right)$
$= 1\text{-}F_1^{t_1(n_u)}\big(a_{1v}(x-\varepsilon)\big)\text{-}(1\text{-}F_1^{t_1(n_u)}\big(a_{1v}(x+\varepsilon)\big)).$
Consider
$\frac{1-F_1^{t_1(n_u)}\left(a_{1v}(x+\varepsilon)\right)}{1-F_1^{t_1(n_u)}\left(a_{1v}(x-\varepsilon)\right)} \sim \frac{1-F_1\left(a_{1v}(x+\varepsilon)\right)}{1-F_1\left(a_{1v}(x-\varepsilon)\right)}$ as u,v→ ∞ by lemma 2.1.
$= (logn_v)^{r_{1v}(x+\varepsilon)-r_{1n_v}(x-\varepsilon)} \to 0$ as u,v→ ∞ from (1.3) and (1.4).
Thus as u,v→ ∞, $P\left(a_{1v}(x-\varepsilon) < Y_{1n_u} < a_{1v}(x+\varepsilon)\right) \sim 1\text{-}F_1^{t_1(n_u)}\big(a_{1v}(x-\varepsilon)\big)$
$\sim t_1(n_u)\left(1 - F_1\left(a_{1v}(x-\varepsilon)\right)\right)$ by lemma 2.1. (3.2)
Also $P\left(a_{1u}(x-\varepsilon) < Y_{1n_u} < a_{1u}(x+\varepsilon)\right) = F_1{}^{t_1(n_u)}\big(a_{1u}(x+\varepsilon)\big) - F_1{}^{t_1(n_u)}\big(a_{1u}(x-\varepsilon)\big)$
$= 1 - F_1{}^{t_1(n_u)}\big(a_{1u}(x-\varepsilon)\big) - \left(1 - F_1{}^{t_1(n_u)}\big(a_{1u}(x+\varepsilon)\big)\right)$
$\sim t_1(n_u)\{1 - F_1\big(a_{1u}(x-\varepsilon)\big)\}, u \to \infty$ (3.3)
since $\frac{1-F_1\left(a_{1u}(x+\varepsilon)\right)}{1-F_1\left(a_{1u}(x-\varepsilon)\right)} = \sim (logt_1(n_u))^{r_{1n_u}(x+\varepsilon)-r_{1u}(x-\varepsilon)} \to 0\ , u \to \infty$.

Note that as u,v→ ∞ ,

$$\frac{t_1(n_u)}{t_1(n_v)} \sim \frac{n_u}{n_v} \le \frac{n_u}{n_{u+1}} \sim e^{u^h-(u+1)^h} < \frac{1}{(u+1)^h - u^h}$$

$\sim \frac{1}{hu^{h-1}} \to 0$ *as* $h > 1$.
This gives, $\{t_1(n_v) - t_1(n_u)\}\{1 - F_1 a_{1v}(x-\varepsilon)\} = \frac{t_1(n_v)-t_1(n_u)}{t_1(n_v)}(logt_1n_v)^{r_{1v}(x-\varepsilon)}$
$\to 0$ *as* u,v→ ∞ from (1.3) and (1.4).
Hence by lemma 2.1, we have u,v→ ∞,
$P\left(Y_{1n_v-n_u} > a_{1v}(x-\varepsilon)\right) = 1 - F_1{}^{t_1(n_v)-t_1(n_u)}\big(a_{1v}(x-\varepsilon)\big)$
$\sim t_1(n_v)\{1 - F_1 a_{1v}(x-\varepsilon)\}$ (3.4)
From (3.2), (3.3) and (3.4) we have

$$\frac{P(a_{1v}(x-\varepsilon)<Y_{1n_u}<a_{1v}(x+\varepsilon)}{P(a_{1u}(x-\varepsilon)<Y_{1n_u}<a_{1u}(x+\varepsilon)P(Y_{1n_v-n_u}>a_{1v}(x-\varepsilon))}$$
$$= \frac{1}{t_1(n_v)\{1-F_1 a_{1u}(x-\varepsilon)\}}$$
$$= \frac{t_1(n_u)}{t_1(n_v)(logt_1n_u)^{r_{1u}(x-\varepsilon)}} \text{ from (1.3).}$$
$$\sim \frac{n_u}{n_v(logn_u)^{r_{1u}(x-\varepsilon)}}$$
$$< D_2 \frac{n_u}{n_{u+1}u^{hr_1 n_u^{(x-\varepsilon)}}}$$
$$< D_3 \frac{1}{\{(u+1)^h - u^h\}^l u^{\frac{-hlog(x-\varepsilon)}{c_1}-\varepsilon_1}}$$

$< D_4 \frac{1}{u^{l(h-1)-\frac{hlog(x-\varepsilon)}{c_1}-\varepsilon_1}} \to 0$ *as* u,v→ ∞

where $0 < \varepsilon_1 < l(h-1) - \frac{hlog(x-\varepsilon)}{c_1}$ and $l > \frac{h}{h-1} \frac{log(x-\varepsilon)}{c_1}$.

Hence for $\varepsilon_2 > 0$ *and* large u and v we have

$P(A_1(u,v,x)) < (1+\varepsilon_2)t_1(n_u)t_1(n_v)\{1-F_1a_{1u}(x-\varepsilon)\}\{1-F_1a_{1v}(x-\varepsilon)\}$

Similarly for large u and v we have

$P(A_1(u,v,y)) < (1+\varepsilon_2)t_1(n_u)t_1(n_v)\{1-F_2a_{1u}(y-\varepsilon)\}\{1-F_2a_{1v}(y-\varepsilon)\}$

Hence $P(E_u \cap E_v) \sim P(E_u)P(E_v)$ for large u and v . This implies

$$\lim_{n\to\infty} \inf \sum_{1\le j} \sum_{\le k \le n} \frac{P(A_j \cap A_k) - P(A_j)(A_k}{\left[\sum_{j=1}^{n} P(A_j)\right]^2} < 0 \; then \; P(A_n \; i.o.) = 1. \tag{3.5}$$

From (3.1) and (3.5) and lemma 2.2, we have

Hence $P(E_k, i, o) = 1$

Lemma 3.2 : For all $\varepsilon > 0$ *and all* (x,y) such that

$1 < x < e^{cc_1}$,$1 < y < e^{cc_2}$, $\frac{logx}{c_1} + \frac{logy}{c_2} \ge c$, we have

$P\big(Y > (x+\varepsilon)b_1(t_1(n_k)), Y_{2,n_k} > (y+\varepsilon)b_2(t_2(n_k)) i.o\big) = 0$

where $c = \frac{1}{h}$.

Proof : we have $P\left(Y_{1,n_k} > (x+\varepsilon)b_1(t_1(n_k)), Y_{2,n_k} > (y+\varepsilon)b_2(t_2(n_k))\right)$

$\sim (logn_k)^{r_{1n_k}(x+\varepsilon)+r_{2n_k}(y+\varepsilon)}$

$\sim k^{hr_{1n_k}(x+\varepsilon)+hr_{2n_k}(y+\varepsilon)}$

$< k^{-h\left[\frac{\log(x+\varepsilon)}{c_1}+\frac{\log(y+\varepsilon)}{c_2}-\varepsilon_3\right]}, k \ge k_2$

where $0 \le \varepsilon_3 \le \frac{\log(x+\varepsilon)}{c_1} + \frac{\log(y+\varepsilon)}{c_2} - \frac{1}{h}$.

Hence $\sum P\left(y_{1,n_k} > (x+\varepsilon)b_1(t_1(n_k)), y_{2,n_k} > (y+\varepsilon)b_2(t_2(n_k))\right) < \infty$.

By Borel -Cantilli Leema the proof is complete.

Proof of theorem 3.1:

From theorem 1.1 and lemma 3.2, it follows that the limit set is contained in S. From Lemma 3.1, we get that every point of

$S^* = \left\{1 < x < e^{cc_1}, 1 < y < e^{cc_2}, \frac{logx}{c_1} + \frac{logy}{c_2} < c\right\}$ is a limit point. By continuity Considerations, we get that S is the required limit set.

Corollary 3.1: Under the conditions of theorem 3.1, every point of $[1, e^{cc_j}]$ is a point of

$$\frac{Y_{j,n_k}}{b_j\left(t_j(n_k)\right)} \; j = 1,2.$$

REFERENCES

1. De Haan,L. and Hordijk, A. (1972): The rate of growth of sample maxima. Ann. Math, Statist., 43, 1185-1196.
2. Hebbar, H.V. (1980): Almost Sure Limit Points of Maxima of Stationary Gaussian Sequences. Ann. Prob. Vol. 8, No. 2, pp. 393-399.
3. Husler, S. (1985): The Growth Rate of Partial Maxima on Subsequences. Statistics Probab ility Letters. 3, pp. 245-249.
4. Nayak, S.S. (1985): Almost Sure Limit Points of Independent Copies of Sample Maxima. Stoch. Process. Appl., 20, pp. 353-360.
5. Nayak, S.S., (1986). Almost sure limit points of maxima of samples from s independent populations. Sankhya Ser. A, pt.3 48, 273-287.
6. Ortega, J. and wschebor, M. (1983): On the Sequence of Partial Maxima of Some Random Sequences. Stoch. Proc. Appl. 16, pp. 85-98.
7. Wichura, M. (1974): On the Functional Form of the Law Of The Iterated Logarithm for the Partial Maxima of Independent Identically Distributed Random Variables. Ann. Prob., Vol. 2, No. 2, pp. 202-230.

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