Research Article

Cost-benefit analysis of a two similar cold standby missile system with failure due to bug in radar and tracking software and battery problem eyed as cause of missile defence failure

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Abstract

Lethal Software Defects: Patriot Missile Failure During the Gulf War, twenty-eight U.S. soldiers were killed and almost one hundred others were wounded when a nearby Patriot missile defense system failed to properly track a Scud missile launched from Iraq. The cause of the failure was later found to be a programming error in the computer embedded in the Patriot's weapons control system. On February 25, 1991, Iraq successfully launched a Scud missile that hit a U.S. Army barracks near Dhahran, Saudi Arabia. The 28 deaths by that one Scud constituted the single deadliest incident of the war, for American soldiers. Interestingly, the "Dhahran Scud", which killed more people than all 70 or so of the earlier Scud launches, was apparently the last Scud fired in the Gulf War. Unfortunately, the "Dhahran Scud" succeeded where the other Scuds failed because of a defect in the software embedded in the Patriot missile defense system. This same bug was latent in all of the Patriots deployed in the region. However, the presence of the bug was masked by the fact that a particular Patriot weapons control computer had to be continuously running for several days before the bug could cause the hazard of a failure to track a Scud. There is a nice concise write-up of the problem, with a prefatory background on how the Patriot system is designed to work, in the official post-failure analysis report by the U.S. General Accounting Office (GAO IMTEC-92-26) entitled "Patriot Missile Defense: Software Problem Led to System Failure at Dhahran, Saudi Arabia". The hindsight explanation is that: a software problem "led to an inbrtscurate tracking calculation that became worse the longer the system operated" and states that "at the time of the incident, the [Patriot] had been operating continuously for over 100 hours" by which time "the inbrtscuracy was serious enough to cause the system to look in the wrong place [in the radar data] for the incoming Scud." Battery problem eyed as cause of U.S. missile defense failure A failed U.S. missile defense test last week may be linked to a faulty battery that prevented an interceptor from separating from the rest of the rocket, an industry source said, citing initial findings in an investigation. "The initial look at the data indicates the problem was in the power suite, with the battery," said the source, who was not authorized to speak on the record. If that theory is proven, it would point to a component-manufacturing issue or quality control problem, the source said. We have taken units Failure due to Bug in Radar and Tracking Software and due to Battery problem with failure time distribution as exponential and repair time distribution as General. We have find out MTSF, Availability analysis, the expected busy period of the server for repair when the failure caused due to Failure due to Bug in Radar and Tracking Software in (0,t], expected busy period of the server for repair in(0,t], the expected busy period of the server for repair when failure caused due to Battery problem in (0,t], the expected number of visits by the repairman for failure of units due to Failure due to Bug in Radar and Tracking Software in (0,t], the expected number of visits by the repairman for Battery problem in (0,t] and Cost-Benefit analysis using regenerative point technique[1][3]. A special case using failure and repair distributions as exponential is derived and graphs have been drawn.

Keyword: Cold Standby, Failure due to Bug in Radar and Tracking Software, Failure caused due to Battery problem, MTSF, Availability, Busy period, Cost-Benefit Analysis

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Received Date: 19/09/2014 Accepted Date: 26/10/2014

How to site this article: Ashok Kumar Saini. Cost-benefit analysis of a two similar cold standby missile system with failure due to bug in radar and tracking software and battery problem eyed as cause of missile defence failure. *International Journal of Statistika and Mathemtika* Nov 2014 to Jan 2015; 12(1): 26-32. http://www.statperson.com (accessed 01 November 2014).

Access this article online	
Quick Response Code:	Website: www.statperson.com
	DOI: 29 October 2014

INTRODUCTION

During the Gulf War in the early 1990's, Operation Desert Storm used sophisticated technology to end the war in a quick and timely manner. Part of this technology was that of the Patriot missile air defence system. On the night of the 25th of February, 1991, a Patriot missile system operating in Dhahran, Saudi Arabia, failed to track and intercept an incoming Scud. The Iraqi missile impacted into an army barracks, killing 28 U.S. soldiers and injuring another 98. The cause of the missile system failing to defend against the incoming Scud was traced back to a bug in Patriot's radar and tracking software. The Patriot's weapon control computer performs crucial system functions for tracking and intercepting targets, as well as other control tasks. The system tracked and intercepted missiles in a number of stages:

1. The system was instructed to search for airborne objects with Scud missile characteristics (based on information such as velocity, latitude, longitude, azimuth, and altitude) on its radar.
2. A range gate, an electronic device in the radar, calculates an area in the air space for where the system should look next for the incoming missile. The missile is tracked by the system as it approaches.
3. The Patriot would launch one of it's own missiles once the incoming missile was in range.

A test of the only U.S. defense against long-range ballistic missiles failed last Friday, the third consecutive failure involving the interceptor system managed by Boeing Co, the Defense Department said. Neither of the previous failures involved battery problems, investigations showed. Pentagon officials have launched a detailed and extensive review of the failed test, but that could take months to complete, said one defense official. Officials have been tight-lipped about any initial findings. Boeing said it would continue to analyze test data along with military officials to better understand the outcome of the test. A company spokeswoman declined comment on whether the inceptor's battery was the suspected cause of the test failure, and had no immediate comment on the manufacturer of the battery. In this paper, we have Failure due to Bug in Radar and Tracking Software and failure due to Battery problem which are non-instantaneous in nature. Here, we investigate a two identical cold standby –a system in which offline unit cannot fail. The failure is due to Battery problem and due to Bug in Radar and Tracking Software. When there is Failure due to Bug in Radar and Tracking Software to less degree, that is, within specified limit, it operates as normal as before but if these are beyond the specified degree the operation of the unit is stopped to avoid excessive damage of the unit and as the Failure due to Bug in Radar and Tracking Software continues going on some characteristics of the unit change which we call failure of the unit. After failure due to Failure due to Bug in Radar and Tracking Software the failed unit undergoes repair immediately according to first come first served discipline.

ASSUMPTIONS

1. The system consists of two similar cold standby units. The failure time distributions of the operation of the unit stopped automatically, the Failure due to Bug in Radar and Tracking Software and failure caused due to Battery problem are exponential with rates λ_1, λ_2 and λ_3 whereas the repairing rates for repairing the failed system due to Bug in Radar and Tracking Software and due to Battery problem are arbitrary with CDF $G_1(t)$ and $G_2(t)$ respectively.
2. When there is Failure due to Bug in Radar and Tracking Software to less degree that is within specified limit, it operates as normal as before but if these are beyond the specified degree the operation of the unit is avoided and as the Failure due to Bug in Radar and Tracking Software continues goes on some characteristics of the unit change which we call failure of the unit.
3. The Failure due to Bug in Radar and Tracking Software actually failed the units. The Failure due to Bug in Radar and Tracking Software is non-instantaneous and it cannot occur simultaneously in both the units.
4. The repair facility works on the first fail first repaired (FCFS) basis.

5. The switches are perfect and instantaneous.
6. All random variables are mutually independent.

SYMBOLS FOR STATES OF THE SYSTEM

Superscripts: O, CS, SO, FBRTS, FBP

Operative, cold Standby, Stops the operation, Failed due to Failure due to Bug in Radar and Tracking Software, failed due to Battery problem respectively

Subscripts: nbrts, ubrts, bp, ur, wr, uR

No Failure due to Bug in Radar and Tracking Software, under Failure due to Bug in Radar and Tracking Software, Battery problem, under repair, waiting for repair, under repair continued respectively

Up states: 0, 1, 3;

Down states: 2, 4,5,6,7

States of the System

$0(O_{nbrts}, CS_{nbrts})$

One unit is operative and the other unit is cold standby and there is no Failure due to Bug in Radar and Tracking Software in both the units.

$1(SO_{ubrts}, O_{nbrts})$

The operation of the first unit stops automatically due to Failure due to Bug in Radar and Tracking Software and cold standby unit starts operating with no Failure due to Bug in Radar and Tracking Software.

$2(SO_{ubrts}, FBP_{nbrts,bp,ur})$

The operation of the first unit stops automatically Failure due to Bug in Radar and Tracking Software and the other unit fails due to Battery problem and undergoes repair.

$3(FBRTS_{ur}, O_{ubrts})$

The first unit fails due to Failure due to Bug in Radar and Tracking Software and undergoes repair and the other unit continues to be operative with no Failure due to Bug in Radar and Tracking Software.

$4(FBRTS_{ur}, SO_{ubrts})$

The one unit fails due to Failure due to Bug in Radar and Tracking Software and undergoes repair and the other unit also stops automatically Failure due to Bug in Radar and Tracking Software.

$5(FBRTS_{uR}, FBRTS_{wr})$

The repair of the first unit is continued from state 4 and the other unit failed due to Failure due to Bug in Radar and Tracking Software is waiting for repair.

$6(FBRTS_{uR}, SO_{ubrts})$

The repair of the first unit is continued from state 3 and unit fails Failure due to Bug in Radar and Tracking Software and operation of other unit stops automatically Failure due to Bug in Radar and Tracking Software.

$7(FBRTS_{wr}, FBP_{bp, uR})$

The repair of failed unit due to Battery problem is continued from state 2 and the first unit is failed due to Failure due to Bug in Radar and Tracking Software is waiting for repair.

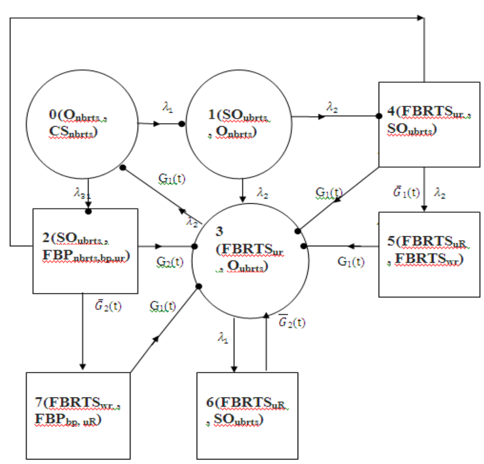

Figure 1: The State Transition Diagram

● Regeneration point ○ Up State □ Down State

TRANSITION PROBABILITIES

Simple probabilistic considerations yield the following expressions:

$p_{01} = \frac{\lambda 1}{\lambda 1+\lambda 3}$, $p_{02} = \frac{\lambda 3}{\lambda 1+\lambda 3}$

$p_{13} = \frac{\lambda 2}{\lambda 1+\lambda 2}$, $p_{14} = \frac{\lambda 1}{\lambda 1+\lambda 2}$

$p_{23}= \lambda_1 G_2^*(\lambda_2)$, $p_{23}^{(7)} = \lambda_2 G_2^*(\lambda_2)$, $p_{24}= \overline{G}_2^*(\lambda_2)$,

$p_{30}= G_1^*(\lambda_1)$, $p_{33}^{(6)}= \overline{G}_1^*(\lambda_1)$

$p_{43} = G_1^*(\lambda_2)$, $p_{43}^{(5)} = G_1^*(\lambda_2)$ (1)

we can easily verify that

$p_{01} + p_{02} = 1$, $p_{13} + p_{14} = 1$, $p_{23} + p_{23}^{(7)} + p_{24}= 1$, $p_{30} + p_{33}^{(6)} = 1$,

$p_{43}+ P_{43}^{(5)} = 1$ (2)

And mean sojourn time are

$\mu_0 = E(T) = \int_0^\infty P[T > t]dt = -1/\lambda_1$

Similarly

$\mu_1 = 1/\lambda_2$, $\mu_2 = \int_0^\infty e^{-}\lambda 1\overline{G}1(t)dt$,

$\mu_4 = \int_0^\infty e^{-}\lambda 2\overline{G}1(t)dt$ (3)

Mean Time To System Failure

We can regard the failed state as absorbing

$\theta_0(t) = Q_{01}(t)[s]\theta_1(t) + Q_{02}(t)$

$\theta_1(t) = Q_{13}(t)[s]\theta_3(t) + Q_{14}(t), \theta_3(t) = Q_{30}(t)[s]\theta_0(t) + Q_{33}^{(6)}(t)$ (4-6)

Taking Laplace-Stieltjes transforms of eq. (4-6) and solving for

$\theta_0(s) = N_1(s) / D_1(s)$ (7)

Where

$N_1(s) = Q_{01}^*(s) \ \{ Q_{13}^*(s)\, Q_{33}^{(6)*}(s) + Q_{14}^*(s) \} + Q_{02}^*(s)$

$D_1(s) = 1 - Q_{01}^*(s) \quad Q_{13}^*(s)\, Q_{30}^*(s)$

Making use of relations (1) and (2) it can be shown that $\theta_0(0)$ =1, which implies that $\theta_1(t)$ is a proper distribution.

$$\text{MTSF} = E[T] = \frac{d}{ds}\theta_0(s) \Big|_{s=0} = (D_1'(0) - N_1'(0)) / D_1(0)$$

$= (\mu_0 + p_{01}\,\mu_1 + p_{01}\, p_{13}\, \mu_3) / (1 - p_{01}\, p_{13}\, p_{30})$ (8)

Where

$\mu_0 = \mu_{01} + \mu_{02}$, $\mu_1 = \mu_{13} + \mu_{14}$, $\mu_2 = \mu_{23} + \mu_{23}^{(1)} + \mu_{24}$,

$\mu_3 = \mu_{30} + \mu_{33}^{(6)}$

$\mu_4 = \mu_{43} + \mu_{43}^{(5)}$

AVAILABILITY ANALYSIS

Let $M_i(t)$ be the probability of the system having started from state i is up at time t without making any other regenerative state. By probabilistic arguments, we have

The value of $M_0(t)= e^{-}\lambda_1 t\, e^{-}\lambda_3 t$, $M_1(t)= e^{-}\lambda_1 t\, e^{-}\lambda_2 t$

$M_3(t)= e^{-}\lambda_1\overline{G}_1(t)$. (9)

The point wise availability $A_i(t)$ have the following recursive relations

$A_0(t) = M_0(t) + q_{01}(t)[c]A_1(t) + q_{02}(t)[c]A_2(t)$

$A_1(t) = M_1(t) + q_{13}(t)[c]A_3(t) + q_{14}(t)[c]A_4(t)$,

$A_2(t) = \{q_{23}(t) + q_{23}^{(7)}(t)\}[c]A_3(t) + q_{33}^{(6)}(t)\ [c]A_3(t)$

$A_4(t) = \{q_{43}(t) + q_{43}^{(5)}(t)[c]A_3(t)$ (10 - 14)

Taking Laplace Transform of eq. (10-14) and solving for $\hat{A}_0(s)$

$\hat{A}_0(s) = N_2(s) / D_2(s)$ (15)

where

$N_2(s) = (1 - \hat{q}_{33}^{(6)}(s))\, \widehat{M}_0(s) + [\hat{q}_{01}(s)\{ \widehat{M}_1(s) + (\hat{q}_{13}(s) + \hat{q}_{14}(s)(\hat{q}_{43}(s) + \hat{q}_{43}^{(5)}(s)))\} + \hat{q}_{02}(s)\{\hat{q}_{23}(s)) + \hat{q}_{23}^{(1)}(s)) + \hat{q}_{24}(s)(\hat{q}_{43}(s) + \hat{q}_{43}^{(5)}(s))\}]\, \widehat{M}_3(s)$

$D_2(s) = (1 - \hat{q}_{33}^{(6)}(s)) - \hat{q}_{30}(s)\, [\hat{q}_{01}(s)\{\hat{q}_{13}(s) + \hat{q}_{14}(s)(\hat{q}_{43}(s) + \hat{q}_{43}^{(5)}(s))\} + \hat{q}_{20}(s)\{\hat{q}_{23}(s) + \hat{q}_{23}^{(7)}(s) + \hat{q}_{24}(s)(\hat{q}_{43}(s)) + \hat{q}_{43}^{(5)}(s))\}]$

The steady state availability

$A_0 = \lim_{t\to\infty}[A_0(t)] \quad = \lim_{s\to 0}[s\,\hat{A}_0(s)] \quad = \lim_{s\to 0} \frac{s\,N_2(s)}{D_2(s)}$

Using L' Hospitals rule, we get

$A_0 = \lim_{s\to 0} \frac{N_2(s)+s\;N_2{}'(s)}{D_2{}'(s)} = \quad \frac{N_2(0)}{D_2{}'(0)}$ (16)

Where

$N_2(0) = p_{30}\,\widehat{M}_0(0) + p_{01}\widehat{M}_1(0)\,\widehat{M}_3(0)\,)$

$D_2{}'(0) = \mu_3 + [\,\mu_0 + p_{01}\,(\,\mu_1 + p_{14}\,\mu_4 + p_{02}(\,\mu_2 + p_{24}\,\mu_4\,)]\,p_{30}$

The expected up time of the system in (0, t] is

$\lambda_u(t) = \int_0^{\propto} A_0\,(z)dz$ So that $\widehat{\lambda_u}\,(s) = \frac{\hat{A}_0\,(s)}{s} = \frac{N_2(S)}{SD_2(S)}$ (17)

The expected down time of the system in (0, t] is

$\lambda_d(t) = t - \lambda_u(t)$ So that $\widehat{\lambda_d}\,(s) \quad = \frac{1}{s^2} - \widehat{\lambda_u}\,(s)$ (18)

The expected busy period of the server when the operation of the unit stops automatically failed unit under Failure due to Bug in Radar and Tracking Software in (0,t]

$R_0(t) = q_{01}(t)[c]R_1(t) + q_{02}(t)[c]R_2(t)$

$R_1(t) = S_1(t) + q_{13}(t)[c]R_3(t) + q_{14}(t)[c]R_4(t),$

$R_2(t) = S_2(t) + q_{23}(t)[c]R_3(t) + q_{23}^{(7)}(t)[c]R_3(t) + q_{24}(t)[c]R_4(t)$

$R_3(t) = q_{30}(t)[c]R_0(t) + q_{33}^{(6)}(t)[c]R_3(t),$

$R_4(t) = S_4(t) + (q_{43}(t) + q_{43}^{(5)}(t))\,[c]R_3(t)$ (19-23)

Where

$S_1(t) = e^{-\lambda_1 t}\, e^{-\lambda_2 t}$, $S_2(t) = e^{-\lambda_1 t}\,\overline{G}_2(t)$, $S_4(t) = e^{-\lambda_1 t}\,\overline{G}_1(t)$ (24)

Taking Laplace Transform of eq. (19-23) and solving for $\widehat{R_0}(s)$

$\widehat{R_0}(s) \quad = N_3(s) / D_2(s)$ (25)

where

$N_3(s) = (1 - \hat{q}_{33}^{(6)}(s))\,[\,\hat{q}_{01}(s)(\,\hat{S}_1(s) + \hat{q}_{14}(s)\,\hat{S}_4(s) + \hat{q}_{02}(s)(\,\hat{S}_2(s) + \hat{q}_{24}(s)\,\hat{S}_4(s))]$ and $D_2(s)$ is already defined.

In the long run, $R_0 = \quad \frac{N_3(0)}{D_2{}'(0)}$ (26)

where $N_3(0) = p_{30}\,[\,p_{01}\,(\hat{S}_1(0) + p_{14}\,\hat{S}_4(0)\,) + p_{02}\,(\hat{S}_2(0) + p_{24}\,\hat{S}_4(0)\,)$ and $D_2{}'(0)$ is already defined.

The expected period of the system under Failure due to Bug in Radar and Tracking Software in (0, t] is

$\lambda_{rv}(t) = \int_0^{\propto} R_0\,(z)dz$ So that $\widehat{\lambda_{rv}}\,(s) = \frac{\hat{R}_0\,(s)}{s}$ (27)

The expected Busy period of the server for repair when failure is caused due to Failure due to Bug in Radar and Tracking Software in (0,t]

$B_0(t) = q_{01}(t)[c]B_1(t) + q_{02}(t)[c]B_2(t)$

$B_1(t) = q_{13}(t)[c]B_3(t) + q_{14}(t)[c]B_4(t),$

$B_2(t) = q_{23}(t)[c]\,B_3(t) + q_{23}^{(7)}(t)[c]B_3(t) + q_{24}(t)[c]\,B_4(t)$

$B_3(t) = T_3\,(t) + q_{30}(t)[c]\,B_0(t) + q_{33}^{(6)}(t)[c]B_3(t)$

$B_4(t) = T_4\,(t) + \{\,q_{43}(t) + q_{43}^{(5)}(t)\}\,[c]B_3(t)$ (28-32)

Where

$T_3\,(t) = e^{-\lambda_2 t}\,\overline{G}_1(t) \qquad T_4\,(t) = e^{-\lambda_1 t}\,\overline{G}_1(t)$ (33)

Taking Laplace Transform of eq. (28-32) and solving for $\widehat{B_0}\,(s)$

$\widehat{B_0}\,(s) \quad = N_4(s) / D_2(s)$ (34)

where

$N_4(s) = \widehat{T}_3(s)\,[\,\hat{q}_{01}(s)\,\{\,\hat{q}_{13}(s) + \hat{q}_{14}(s)\,(\,\hat{q}_{43}(s) + \hat{q}_{43}^{(5)}(s))\} + \hat{q}_{02}(s)\,\{\,\hat{q}_{23}(s) + \hat{q}_{23}^{(7)}(s) + \hat{q}_{24}(s)\,(\,\hat{q}_{43}(s) + \hat{q}_{43}^{(5)}(s))\}] + \widehat{T}_4(s)\,[\hat{q}_{01}(s)\,\hat{q}_{44}(s)(1 - \hat{q}_{33}^{(6)}(s)\,) + (\,\hat{q}_{02}(s)\,\hat{q}_{24}(s)(1 - \hat{q}_{33}^{(6)}(s)\,)]$

And $D_2(s)$ is already defined.

In steady state, $B_0 = \frac{N_4(0)}{D_2'(0)}$ (35)

where $N_4(0)= \hat{T}_3(0)+ \hat{T}_4(0)$ { p_{30} ($p_{01}p_{14}$ + p_{02} p_{24}) } and $D_2'(0)$ is already defined.

The expected busy period of the server for repair in (0, t] is

$\lambda_{ru}(t) = \int_0^{\infty} B_0(z)dz$ So that $\widehat{\lambda_{ru}}(s) = \frac{\hat{B}_0(s)}{s}$ (36)

The expected Busy period of the server for repair when failure caused due to Battery problem in (o, t]

$P_0(t) = q_{01}(t)[c]P_1(t) + q_{02}(t)[c]P_2(t)$

$P_1(t) = q_{13}(t)[c]P_3(t) + q_{14}(t)[c]P_4(t)$,

$P_2(t) = L_2(t)+ q_{23}(t)[c]P_3(t) + q_{23}^{(7)}(t)[c]P_3(t)+ q_{24}(t)[c]P_4(t)$

$P_3(t) = q_{30}(t)[c]P_0(t) + q_{33}^{(6)}(t)[c]P_3(t)$,

$P_4(t) = (q_{43}(t)+ q_{43}^{(5)}(t))$ [c]$P_3(t)$ (37-41)

Where $L_2(t) = e^{-\lambda_1 t}\,\overline{G}_2(t)$ (42)

Taking Laplace Transform of eq. (37-41) and solving $\widehat{P_0}(s) = N_5(s) / D_2(s)$ (43)

where $N_5(s) = \hat{q}_{02}(s)\,\hat{L}_2(s)\,(1 - \hat{q}_{33}^{(6)}(s))$ and $D_2(s)$ is defined earlier.

In the long run, $P_0 = \frac{N_5(0)}{D_2'(0)}$ (44)

where $N_5(0)= p_{30}\, p_{02}\, \hat{L}_2(0)$ and $D_2'(0)$ is already defined.

The expected busy period of the server for repair of the unit when failure due to battery problem in (0, t] is

$\lambda_{rs}(t) = \int_0^{\infty} P_0(z)dz$ So that $\widehat{\lambda_{rs}}(s) = \frac{\hat{P}_0(s)}{s}$ (45)

The expected number of visits by the repairman for repairing the when failure due to Failure due to Bug in Radar and Tracking Software in (0, t]

$H_0(t) = Q_{01}(t)[s]H_1(t) + Q_{02}(t)[s]H_2(t)$

$H_1(t) = Q_{13}(t)[s][1+H_3(t)] + Q_{14}(t)[s][1+H_4(t)]$,

$H_2(t) = [Q_{23}(t) + Q_{23}^{(7)}(t)]\,[s][1+H_3(t)] + Q_{24}(t)[s][1+ H_4(t)]$

$H_3(t) = Q_{30}(t)[s]H_0(t) + Q_{33}^{(6)}(t)[s]H_3(t)$,

$H_4(t) = (Q_{43}(t)+ Q_{43}^{(5)}(t))\,[s]H_3(t)$ (46-50)

Taking Laplace Transform of eq. (46-50) and solving for $H_0^*(s)$

$H_0^*(s) = N_6(s) / D_3(s)$ (51)

where

$N_6(s) = (1 - Q_{33}^{(6)*}(s))\{Q^*_{01}(s)(Q^*_{13}(s) + Q^*_{14}(s)) + Q^*_{02}(s)(Q^*_{24}(s) + Q^*_{23}(s)+Q^*_{23}{}^{(7)}(s))\}$

$D_3(s) = (1 - Q_{33}^{(6)*}(s)) - Q^*_{30}(s)[\,Q^*_{01}(s)\{\,Q^*_{13}(s) + Q^*_{14}(s)\,(Q_{43}{}^*(s) + Q^*_{43}{}^{(5)}(s))\} + Q^*_{02}(s)\{\,Q^*_{23}(s)+ Q^*_{23}{}^{(7)}(s)) + Q^*_{24}(s)(\,Q^*_{43}(s)) + Q^*_{43}{}^{(5)}(s))\,\}]$

In the long run, $H_0 = \frac{N_6(0)}{D_3'(0)}$ (52)

where $N_6(0)= p_{30}$ and $D'_3(0)$ is already defined.

The expected number of visits by the repairman for repairing when failure is caused due to Battery problem in (0, t]

$V_0(t) = Q_{01}(t)[s]V_1(t) + Q_{02}(t)[s][1+V_2(t)]$

$V_1(t) = Q_{13}(t)[s]V_3(t) + Q_{14}(t)[s]V_4(t)$,

$V_2(t) = Q_{24}(t)[s][1+V_4(t)] +[\,Q_{23}(t) + Q_{23}^{(7)}(t)[s][1+V_3(t)]$

$V_3(t) = Q_{30}(t)[s]V_0(t) + Q_{33}^{(6)}(t)[s]V_3(t)$ (53-57)

Taking Laplace-Stieltjes transform of eq. (53-57) and solving for $V_0{}^*(s)$

$V_0{}^*(s) = N_7(s) / D_4(s)$ (58)

where $N_7(s) = (1 - Q_{33}^{(6)*}(s))\{Q^*_{01}(s)(Q^*_{14}(s) + Q^*_{43}(s)) + Q^*_{02}(s)(\,Q^*_{24}(s)\,Q^*_{02}(s)(\,Q^*_{23}(s)+Q^*_{23}{}^{(7)}(s))\}$

and $D_4(s)$ is the same as $D_3(s)$

In the long run, $V_0 = \frac{N_7(0)}{D_4'(0)}$ (59)

where $N_7(0)= p_{30}\,[p_{01}\, p_{14}\, p_{43} + p_{02}\,]$ and $D'_3(0)$ is already defined.

COST BENEFIT ANALYSIS

The cost-benefit function of the system considering mean up-time, expected busy period of the system under Failure due to Bug in Radar and Tracking Software when the units stops automatically, expected busy period of the server for repair when failure due to Battery problem, expected number of visits by the repairman when failure is caused due to Failure due to Bug in Radar and Tracking Software, expected number of visits by the repairman for Battery problem. The expected total cost-benefit incurred in (0, t] is

C (t) = Expected total revenue in (0, t]

- expected total repair cost for failure due to Battery problem in (0,t]
- expected total repair cost for repairing the units when failure is caused due to Battery problem in (0,t]
- expected busy period of the system under Failure due to Bug in Radar and Tracking Software when the units automatically stop in (0,t]
- expected number of visits by the repairman for repairing when failure caused due to Battery problem in (0,t]
- expected number of visits by the repairman for repairing the units when failure is due to Failure due to Bug in Radar and Tracking Software in (0,t]

The expected total cost per unit time in steady state is

$C = \lim_{t\to\infty}(C(t)/t) \quad = \lim_{s\to 0}(s^2 C(s))$

$= K_1A_0 - K_2P_0 - K_3B_0 - K_4R_0 - K_5V_0 - K_6H_0$

Where

K_1: revenue per unit up-time,

K_2: cost per unit time for which the system failure due to Battery problem

K_3: cost per unit time for which the system is under unit repair failure due to Failure due to Bug in Radar and Tracking Software

K_4: cost per unit time for which the system is under Failure due to Bug in Radar and Tracking Software when units automatically stop.

K_5: cost per visit by the repairman for units repair when Failure due to Bug in Radar and Tracking Software,

K_6: cost per visit by the repairman for units repair when failure due to Battery problem.

CONCLUSION

After studying the system, we have analyzed graphically that when the failure rate due to operation of the unit stops automatically, due to Battery problem and, Failure due to Bug in Radar and Tracking Software rate increases, the MTSF and steady state availability decreases and the cost function decreased as the failure increases.

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Source of Support: None Declared
Conflict of Interest: None Declared