Original Article

Estimation for generalized life distribution under progressive type-II censoring scheme

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Abstract

In this paper we consider a 'family of life time distributions' which includes several life time distributions as specific cases. We derive maximum likelihood and Bayes estimators of the parameters included in this family and reliability function under squared-error loss function using progressive type-II censoring scheme. We use iterative procedures to obtain maximum likelihood estimates and Gibbs sampler for Bayesian estimation. We perform simulation study to show the real life applications of the problem.
Keywords: Bayes estimator; Gibbs sampler; progressive type-II censoring; reliability function; squared-error loss function.

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INTRODUCTION

Life-testing experiments are often terminated before the failure of all the units put to test. Such experiments are called censored experiments. The most common censoring schemes, which are frequently used in literature, are Type-I and Type-II censoring schemes. In type-I scheme the time of termination of experiment is prefixed whereas in type-II censoring scheme, some units are put on test and the test is terminated after a prefixed number of failures [see Sinha (1985)]. An extension of type-II censoring scheme is a progressive type-II censoring scheme which provides the flexibility of removal of items before the termination point [see Balakrishnan and Aggarwala (2000)]. The scheme is described as follows. Let n units are put to test and the numbers $R_1, R_2,\ldots, R_m$ are prefixed in advance, such that, at the times of first failure R_1 units are removed randomly from the experiment out of surviving n-1 units; at the time of second failure R_2 items are removed out of $n-2-R_1$ surviving units; the process continue till the m^{th} failure at which remaining R_m units are removed. The observed failures $x_1, x_2,\ldots, x_m$ are called progressively type-II censored order statistics with progressive censoring scheme $(R_1, R_2,\ldots, R_m)$. when $R_1= R_2=,\ldots, R_{m-1}=0,$ then $R_m=n-m$, the case corresponds to the conventional Type-II right censoring scheme. When $R_1= R_2=,\ldots, R_m=0$, then $n=m,$ we get the complete sample. Several authors have considered the problems of estimation of reliability under progressive type-II censoring scheme. Mann (1971) derived the best linear invariant estimator for Weibull parameters. Balasooriya and Balakrishnan (2000) worked out reliability sampling plans for lognormal distribution. Viveros and Balakrishnan (1994) have done pioneer work on progressive censoring. They have obtained a conditional method of inference to derive exact confidence intervals for several life characteristics such as location, scale, quantiles, and reliability with Type II progressively censored data. Tse and Yuen (1998) considered the expected experiment times for Weibull distributed lifetimes under type-II progressive censoring, with the numbers of removals being random. He carried out a detailed numerical study of the expected time

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for different combinations of model parameters. Wu and Chang (2002) discussed the estimation problem for the two-parameter exponential distribution under progressive Type II censoring with random removals, where the number of units removed at each failure time has a binomial distribution. Balakrishnan *et al*. (2003) have obtained the likelihood equations based on a progressively Type-II censored sample from a Gaussian distribution Wu and Wu (2004) have considered the estimation of two parameter Pareto distribution under Type-II progressive censoring with random removals, where the number of units removed at each failure time follows a binomial or a uniform distribution. Ng, *et al*. (2004) have done a remarkable work on optimal progressive censoring plans for the Weibull Distribution. They have computed the expected Fisher information and the asymptotic variance-covariance matrix of the maximum likelihood estimates based on a progressively type II censored sample by direct calculation as well as the missing-information principle. Bayesian Estimation in reliability and life testing was introduced by Bhattacharya (1967). He considered the estimation of the reliability function for exponential distribution under squared error loss function (SELF) and type-II censoring scheme. Since then plenty of paper have been published in this field under the assumption of SELF. One may refer to Marts and Waller (1982) for some citations. Soliman (2005) have considered the maximum likelihood, and Bayesian estimation for some lifetime parameters of the Burr-XII model based on progressive type-II censored data. He obtained Bayes Estimators using the symmetric (Squared Error) loss function, and asymmetric (LINEX, General Entropy) loss functions. Kundu (2008) discusses the Bayesian estimation of unknown parameters of the progressively censored Weibull distribution. Balakrishnan and Dembinska (2008) considered Progressively Type-II right censored order statistics for discrete distributions. Cheng, *et al*. (2010) discussed the ML estimation for the exponential and Weibull distributions by considering progressive Type-I interval censored data. Cramer and Iliopoulos (2010) extended the model of progressive Type-II censoring scheme by introducing an adoption process. It allows us to choose the next censoring number taking into account both the previous censoring numbers and previous failure times. After deriving some distributional results, they showed that MLEs coincide with those in deterministic progressive Type-II censoring. Cramer and Lenz (2010) established the association of progressively Type-II censored order statistics from a sample of associated random variables $X_1,\ldots, X_n$. They also discuss some bivariate dependence properties for independent but not necessarily identically distributed $X_1,\ldots, X_n$. Wu and Huang (2010) investigate a decision problem under the warranty which is a combination of free-replacement, and pro-rata policies. They use a Bayesian approach to determine the optimal warranty lengths. The Rayleigh distribution is employed to describe the product lifetime. Chen (2011) estimated the parameters of a location scale distribution family. As a special case, they used the method for estimating the parameters of a normal distribution and Cauchy distribution. Salem and Abo-Kasem (2011) discussed Bayes and non-Bayesian estimation for two-parameter exponentiated Weibull distribution under progressive hybrid censoring scheme. Din and Amein (2011) and Jones (1953) has derived approximate MLE to estimate the location parameter based on order statistics. Krishna and Malik (2011), presented the Maxwell distribution as a life time model and supports its usefulness in the reliability theory through real data examples. They developed estimation procedures for the mean life, reliability and failure rate functions for this distribution. Rastogi and Tripathi (2012), estimated unknown parameters and reliability function of a two parameter Burr type XII distribution is considered on the basis of a progressively type II censored sample. Mubarak (2012) discussed estimation problem for the Frechet distribution under progressive Type II censoring with random removals, where the number of units removed at each failure time has a binomial distribution. Rest of the paper is organized as fallows. In Section 2, we discuss the 'generalized life distributions' and provide procedure to obtain the ML estimates of the unknown parameters and reliability function using numerical procedure. In Section 4, we obtain Bayes estimators using Gibbs sampler. Finally, in Section 5, we present simulation study in support of theoretical results.

THE GENERALIZED FAMILY OF DISTRIBUTIONS

Let the random variable (rv) *X* follows the distribution presented by the *pdf*

$$f(x \mid \delta,\theta)=\frac{g^{\delta-1}(x)g'(x)}{\theta^{\delta}\Gamma(\delta)}\exp\left(-\frac{g(x)}{\theta}\right); x>a; g(x),\theta,\delta>0. \qquad (1)$$

where '*a*' is known and δ and θ are the parameters. Here, $g(x)$ is real-valued, strictly increasing function of x with $g(a)=0$ and $g`(x)$ stands for the derivative of x. The distribution (1) known as the 'generalized life distributions' since it covers the following life distributions as specific cases [see Chaturvedi ansd Singh (2003)]:

I. For $g(x)=x$, $a=0$ and $\delta=1$, we get the one parameter exponential distribution [see Johnson and Kotz (1970, p.166)].
II. For $g(x)=x$, $a=0$, (1) becomes the gamma distribution and for δ taking integer values, it is known as Erlang distribution [see Johnson and Kotz (1970, p.166)].
III. For $g(x)=x^p$ ($p>0$) and $a=0$, (1) gives the generalized gamma distribution [see Johnson and Kotz (1970, p.197)].

IV. For $g(x)=x^p$ ($p>0$), $\delta = 1$ and $a = 0$, (1) represents Weibull distribution [see Johnson and Kotz (1970, p.250)].
V. For $g(x)=x^2$, $\delta=1/2$ and $a=0$, (1) comes out to be half-normal distribution [see Davis (1952)].
VI. For $g(x)=x^2$, $\delta=1$ and $a=0$, (1) is Rayleigh distribution [see Sinha (1986)]. For $g(x)=x^2/2$, $\delta=\alpha/2$ and $a=0$, (1) turns out to be chi-distribution [see Patel, Kapadia and Owen (1976, p.173)].Taking $\alpha=3$, it becomes Maxwel's distribution [see Tyagi and Bhattacharya (1989a, b)].
VII. For $g(x)=\log(1+x^b)$ ($b>0$), $\delta=1$ and $a=0$, we obtain from (1) Burr distribution [see Burr(1942) and Cislak and Burr (1968)].
VIII. For $g(x)=\log(x)$, $\delta=1$ and $a=1$, (1) leads us to Pareto distribution [see Johnson and Kotz (1970, p.233)].

For the model (1), reliability function $R(t)$ at a specified mission time t (>0) is

$$R(t\mid\delta,\theta)=P(X>t)$$
$$=\frac{1}{\Gamma(\delta)}\Gamma\left(\delta,\frac{g(t)}{\theta}\right). \tag{2}$$

where, $\Gamma(a,b)$ is incomplete gamma function defined as follows

$$\Gamma(a,b)=\int_b^\infty y^{a-1}\exp(-y)dy .$$

MAXIMUM LIKELIHOOD ESTIMATION

Suppose that n independent items are put to test where the lifetime distribution of each item is given by (1). The progressive censoring scheme ($R_1, R_2, \dots, R_m$) is followed, where each $R_i>0$, $\sum_{i=1}^m R_i+m=n$ and a progressively ordered sample $x_1,x_2,\dots,x_m$ (denoted by $\underline{d}$ henceforth) is obtained. The likelihood function of the observed data $\underline{d}$ is [see Balakrishnan and Aggarwala (2000)]

$$L(\delta,\theta\mid\underline{d})=c_o\prod_{i=1}^m f(x_i;\delta,\theta)\left(R(x_i;\delta,\theta)\right)^{R_i}, \tag{3}$$

where $c_o=n(n-R_1-1)(n-R_1-R_2-2)\dots(n-R_1-R_2-\dots-R_{m-1}-m+1)$. Now using (1) and (2), we obtain from (3), that

$$L(\delta,\theta\mid\underline{d})=c_o\frac{1}{\theta^{m\delta}\left(\Gamma(\delta)\right)^{\sum_{i=1}^m(R_i+1)}}\prod_{i=1}^m\left\{\left(g^{\delta-1}(x_i)g'(x_i)\right)\left(\Gamma\left(\delta,\frac{g(x_i)}{\theta}\right)\right)^{R_i}\right\}\exp\left(-\frac{1}{\theta}\sum_{i=1}^m g(x_i)\right). \tag{4}$$

In order to obtain the MLE of δ and θ, we take likelihood of (4) and differentiate it partially w.r.t. δ and θ, to get likelihood equations as follows.

$$\log L(\delta,\theta\mid\underline{d})=\log(c_o)-m\delta\log(\theta)-\sum_{i=1}^m(R_i+1)\log(\Gamma(\delta))+(\delta-1)\sum_{i=1}^m\log(g(x_i))+\sum_{i=1}^m\log(g'(x_i))$$
$$+\left(-\frac{1}{\theta}\sum_{i=1}^m g(x_i)\right)-\sum_{i=1}^m R_i\log(\Gamma(\delta))+\sum_{i=1}^m R_i\log\left(\Gamma\left(\delta,\frac{g(x_i)}{\theta}\right)\right).$$

$$\frac{\partial}{\partial\theta}\log L=-\frac{m\delta}{\theta}+\frac{1}{\theta^2}\sum_{i=1}^m g(x_i)-+\sum_{i=1}^m R_i\left(\Gamma\left(\delta,\frac{g(x_i)}{\theta}\right)\right)^{-1}\Gamma'\left(\delta,\frac{g(x_i)}{\theta}\right)=0, \tag{5}$$

Where $\Gamma'=\frac{\partial}{\partial\theta}\Gamma$ and

$$\frac{\partial}{\partial\delta}\log L=-m\log(\theta)-\sum_{i=1}^m(R_i+1)\frac{\partial}{\partial\delta}\log(\Gamma(\delta))+\sum_{i=1}^m\log(g(x_i))+\sum_{i=1}^m R_i\left(\Gamma\left(\delta,\frac{g(x_i)}{\theta}\right)\right)^{-1}\Gamma'\left(\delta,\frac{g(x_i)}{\theta}\right)=0. \tag{6}$$

From (5) and (6), we observe that the MLEs of δ and θ, cannot be obtained in closed forms. Therefore, we use iteration method to evaluate these MLEs. The computational procedure is given in Section 4.

Remarks: using the invariance property of MLE, the MLE of reliability and hazard rate functions at time t, can be obtain as follows.

$$\hat{R}(t\mid\delta,\theta)=\frac{1}{\Gamma(\hat{\delta})}\Gamma\left(\hat{\delta},\frac{g(t)}{\hat{\theta}}\right). \tag{7}$$

Similarly, we can obtain the ML estimate of hazard rate which is given by

$$\hat{h}(t) = \frac{\hat{f}(t \mid \hat{\delta}, \hat{\theta})}{\hat{R}(t \mid \hat{\delta}, \hat{\theta})}.$$

BAYESIAN ESTIMATION

Let us consider the inverted gamma prior with parameter (μ, υ) for θ given by

$$\pi(\theta) = \frac{\mu^{\upsilon}}{\Gamma(\upsilon)\theta^{\upsilon+1}} \exp\left(\frac{-\mu}{\theta}\right); \qquad \theta, \mu, \upsilon > 0$$

and a constant prior for δ

$$\pi(\delta) = \frac{1}{c}.$$

Considering δ and θ to be independent, the joint prior density of δ and θ comes out to be

$$\pi(\delta, \theta) = \frac{\mu^{\upsilon}}{c\theta^{\upsilon+1}\Gamma(\upsilon)} \exp\left(-\frac{\mu}{\theta}\right). \tag{8}$$

Merging the prior distribution (8) with likelihood (4), via Bayes theorem, we get the joint posterior distribution of δ and θ given by

$$\pi(\delta, \theta | \underline{d}) = K^{-1} \frac{1}{\theta^{m\delta+\nu+1}\left(\Gamma(\delta)\right)^{\sum_{i=1}^{m}(R_i+1)}} \frac{\mu^{\upsilon}}{\Gamma(\upsilon)} \prod_{i=1}^{m} \left\{ \left(g^{\delta-1}(x_i)\right) \left(\Gamma\left(\delta, \frac{g(x_i)}{\theta}\right)\right)^{R_i} \right\} \exp\left(-\frac{1}{\theta}\left(\sum_{i=1}^{m} g(x_i) + \mu\right)\right), \tag{9}$$

where

$$K = \int_{\delta}\int_{\theta} \frac{1}{\theta^{m\delta+\nu+1}\left(\Gamma(\delta)\right)^{\sum_{i=1}^{m}(R_i+1)}} \frac{\mu^{\upsilon}}{\Gamma(\upsilon)} \prod_{i=1}^{m} \left\{ \left(g^{\delta-1}(x_i)\right) \left(\Gamma\left(\delta, \frac{g(x_i)}{\theta}\right)\right)^{R_i} \right\} \exp\left(-\frac{1}{\theta}\left(\sum_{i=1}^{m} g(x_i) + \mu\right)\right) d\theta d\theta.$$

Now, we derive Bayes estimators of δ , θ, R(t) and h(t). Now, the posterior expectation of any parametric function of $\omega = \omega(\delta, \theta)$ can be obtained as follows.

$$E(\omega \mid d) = \int \omega\, \pi(\omega | \underline{d}) d\omega. \tag{10}$$

From (10), we observe that the marginal distributions of δ and θ cannot be obtained in closed form, which is essential in order to obtain Bayes estimates of individual parameters. We therefore use Gibbs sampler to obtain the samples from the marginal posterior distribution to draw further inferences for parameters. To implement Gibbs sampler, the full conditionals up to proportionality for δ and θ obtained from (10) are given, respectively by

$$\pi(\delta | \underline{d}) \propto \frac{1}{\theta^{m\delta}\left(\Gamma(\delta)\right)^{\sum_{i=1}^{m}(R_i+1)}} \prod_{i=1}^{m} \left\{ \left(g^{\delta-1}(x_i)\right) \left(\Gamma\left(\delta, \frac{g(x_i)}{\theta}\right)\right)^{R_i} \right\} \tag{11}$$

and

$$\pi(\theta | \delta) \propto \frac{1}{\theta^{m\delta+\nu+1}} \prod_{i=1}^{m} \left\{ \left(\Gamma\left(\delta, \frac{g(x_i)}{\theta}\right)\right)^{R_i} \right\} \exp\left(-\frac{1}{\theta}\left(\sum_{i=1}^{m} g(x_i) + \mu\right)\right). \tag{12}$$

Thus, sample values for θ and δ can be obtained using Gibbs sampler from above full conditionals and hence the posterior inferences can be drawn.

SIMULATION STUDY AND CONCLUSION

In this section we present simulation study to show how one can apply the derived results for data analysis. We consider $g(x)=x^p$ in the considered family of distributions. Thus we get Weibull distribution. For the values of the parameters $\delta = 1.5$ and $\theta = 2$, we generate the progressively type-II censored sample using software *R*. We first obtain the MLEs of δ and θ using (5) and (6) through iterative procedure. Then using these MLEs, get MLEs of R(t) and h(t). We consider six different patterns of progressive censoring schemes and evaluate the values of ML estimators of both the parameters, R(t) and h(t) and their respective root mean squared errors (RMSEs). For Bayesian study, we have chosen the values of prior parameters to be $\mu = 2$ and $\upsilon = 3$. For these values, the Bayes estimates are evaluated using Gibbs Sampler. We provide the average value MLE's and Bayes estimates along with their mean square error mean (MSE's) based on 2000 repeated samples. The estimates of parameters are given in Table 1 and estimates of reliabilities and hazard rates in Table 2. We have also presented analysis of a data set, generated for the values of parameters $\delta = 1$ and $\theta = 1.2$ for four different schemes. This data set is given in Table 3 and corresponding values of estimates for these samples are given in Table 4. The MCMC plots for data set of scheme S_1 is given in Figure 1, cumsum plots in Figure 2 and the reliability curve in Figure 3.

Table 1: Average values of ML and Bayes estimate of δ and θ their MSEs (in Bracket) for n=50 and m=20 and 40 with different progressively scheme

Scheme	$\hat{\theta}$	$\hat{\delta}$	$\tilde{\theta}$	$\tilde{\delta}$
$S_{50:20}^{(1)} = (0*19, 30)$	2.1071 (0.2051)	1.5700 (0.0676)	1.8841 (0.1541)	1.4991 (0.0104)
$S_{50:20}^{(2)} = (0*10, 3*10)$	2.0921 (0.2381)	1.6349 (0.0950)	1.9160 (0.1682)	1.5069 (0.0120)
$S_{50:20}^{(3)} = (0*5, 2*15)$	2.0295 (0.2509)	1.6093 (0.1172)	1.9173 (0.1763)	1.5048 (0.0162)
$S_{50:20}^{(4)} = (0*3, 2*15, 0*2)$	2.095 (0.2560)	1.6269 (0.1017)	1.9062 (0.1748)	1.5079 (0.0164)
$S_{50:20}^{(5)} = (3*10, 0*10)$	2.0197 (0.2893)	1.5708 (0.1367)	1.8948 (0.1842)	1.5014 (0.0179)
$S_{50:20}^{(6)} = (30, 0*19)$	2.1216 (0.5160)	1.5749 (0.1783)	1.9085 (0.1843)	1.4991 (0.0196)
$S_{50:40}^{(1)} = (0*39, 10)$	1.9530 (0.1406)	1.5675 (0.0379)	2.0683 (0.0919)	1.5041 (0.0011)
$S_{50:40}^{(2)} = (0*30, 1*10)$	2.0697 (0.1770)	1.5537 (0.0424)	1.9602 (0.0962)	1.5006 (0.0021)
$S_{50:40}^{(3)} = (0*15, 1*10, 0*15)$	1.8446 (0.1787)	1.5890 (0.0416)	1.9862 (0.0954)	1.5009 (0.0023)
$S_{50:40}^{(4)} = (0*10, (0,1)*10, 0*10)$	2.0696 (0.1820)	1.5519 (0.0517)	1.9587 (0.0952)	1.5013 (0.0026)
$S_{50:20}^{(5)} = (1*10, 0*30)$	2.0535 (0.1922)	1.5540 (0.0521)	1.9420 (0.0975)	1.5023 (0.0025)
$S_{50:20}^{(6)} = (10, 0*39)$	2.0588 (0.1940)	1.5450 (0.0780)	1.9444 (0.0991)	1.4998 (0.0093)
$S_{50:50}^{(0)} = (0*50)$	2.0968 (0.1360)	1.5543 (0.0309)	2.0096 (0.0907)	1.5019 (0.0010)

Table 2: Average values of Bayes estimate of $R(t)$ and $h(t)$ their MSEs (in Bracket) for n=50 and m=20 and 40 with different progressively scheme

Scheme	$\hat{R}(t)$	$\hat{h}(t)$	$\tilde{R}(t)$	$\tilde{h}(t)$
$S_{50:20}^{(1)} = (0*19, 30)$	0.4866 (0.0082)	0.9148 (0.0324)	0.4657 (0.0059)	0.9351 (0.0215)
$S_{50:20}^{(2)} = (0*10, 3*10)$	0.4745 (0.0084)	1.0225 (0.0556)	0.4721 (0.0066)	0.9218 (0.0506)
$S_{50:20}^{(3)} = (0*5, 2*15)$	0.4770 (0.0085)	1.0169 (0.0737)	0.4717 (0.0066)	0.9245 (0.0517)
$S_{50:20}^{(4)} = (0*3, 2*15, 0*2)$	0.4754 (0.0087)	1.0106 (0.0716)	0.4695 (0.0066)	0.9317 (0.0538)
$S_{50:20}^{(5)} = (3*10, 0*10)$	0.4809 (0.0073)	0.9390 (0.0908)	0.4675 (0.0066)	0.9314 (0.0550)
$S_{50:20}^{(6)} = (30, 0*19)$	0.4914 (0.0090)	0.9021 (0.1022)	0.4697 (0.0067)	0.9243 (0.0559)
$S_{50:40}^{(1)} = (0*39, 10)$	0.4956 (0.0035)	0.8842 (0.2076)	0.4830 (0.0030)	0.8843 (0.0112)
$S_{50:40}^{(2)} = (0*30, 1*10)$	0.4963 (0.0036)	0.8736 (0.0231)	0.4843 (0.0030)	0.8791 (0.0209)
$S_{50:40}^{(3)} = (0*15, 1*10, 0*15)$	0.4735 (0.0037)	0.8575 (0.0270)	0.4653 (0.0031)	0.8953 (0.0216)

$S_{50:40}^{(4)} = (0*10,(0,1)*10,0*10)$	0.4967 (0.0037)	0.8714 (0.0288)	0.4840 (0.0030)	0.8796 (0.0220)
$S_{50:20}^{(5)} = (1*10,0*30)$	0.4935 (0.0038)	0.8811 (0.0299)	0.4807 (0.0033)	0.8889 (0.0277)
$S_{50:20}^{(6)} = (10,0*39)$	0.4949 (0.0038)	0.8709 (0.0328)	0.4816 (0.0033)	0.8845 (0.0296)
$S_{50:50}^{(0)} = (0*50)$	0.4991 (0.0033)	0.8651 (0.0123)	0.4836 (0.0023)	0.8807 (0.0110)

Table 3: Simulated Sample *n*=20; m=10; θ =1.2, δ =1.

Schemes	Sample Values
S_1=0*9, 10	0.0489, 0.1114, 0.2331, 0.2777, 0.4168, 0.5117, 0.6816, 1.1553, 1.2957, 1.3803
S_2=(0,2)*5	0.0486, 0.0887, 0.2549, 0.4263, 0.5327, 0.8009, 1.1653, 1.3448, 1.7187, 3.3346.
S_3=2*5, 0*5	0.0425, 0.0635, 0.1704, 0.6761, 0.8705, 1.0735, 1.2228, 1.4304, 3.03786, 7.3639.
S_4=10, 0*9	0.0279, 0.5459, 0.6237, 0.6364, 1.2113, 1.7375, 2.5363, 3.0543, 4.3937, 10.53060.

Table 4: Various estimates for simulated sample

S	$\hat{\theta}$	$\hat{\delta}$	$\hat{R}(t)$	$\hat{h}(t)$	$\tilde{\theta}$	$\tilde{\delta}$	$\tilde{R}(t)$	$\tilde{h}(t)$
S_1=0*9, 10	1.9706	0.9345	0.5352	0.4673	1.8815	1.1715	0.5011	0.6487
S_2=(0,2)*5	1.8115	0.8940	0.5097	0.4820	1.7722	0.9693	.4953	0.5459
S_3=2*5, 0*5	1.7847	0.8985	0.5042	0.4921	1.7692	0.9683	0.4942	0.5469
S_4=10, 0*9	2.1926	0.8859	0.5736	0.3939	2.1914	0.9548	0.5656	0.4355

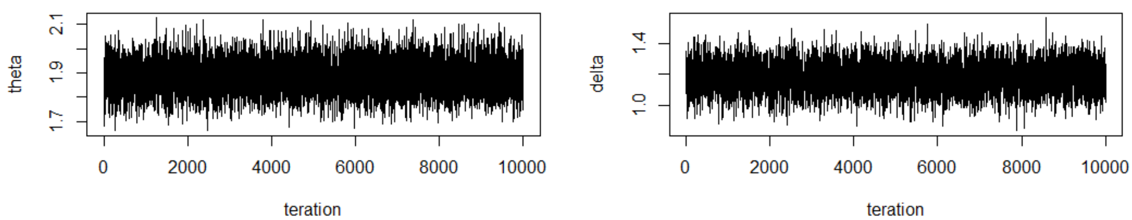

Figure 1: MCMC plot of θ and δ using scheme S_1..

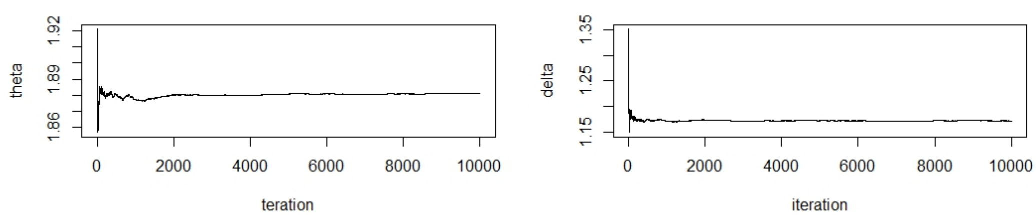

Figure 2: cumsum plot of θ and δ using scheme S_1..

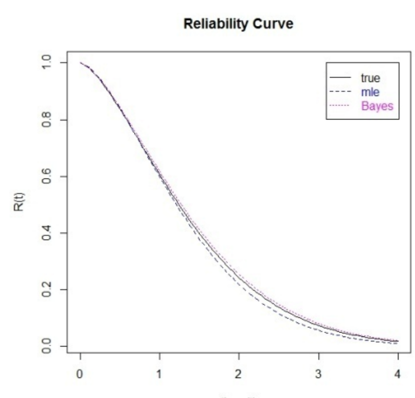

Figure 3: Reliability curve based on ML and Bayes estimates using scheme S_1.

REFERENCES

1. Balakrishnan, N. and Aggarwala, R. (2000): Progressive Censoring: Theory, methods and application, Birkhauser.
2. Balakrishnan, N., Kannan, N., Lin, C. T., and Ng, H. T. (2003). Point and interval estimation for Gaussian distribution, based on progressively Type-II censored samples. Reliability, IEEE Transactions on, 52(1), 90-95.
3. Balakrishnan, N., and Dembińska, A. (2008). Progressively Type-II right censored order statistics from discrete distributions. Journal of Statistical Planning and Inference, 138(4), 845-856.
4. Balasoorya, U. and Balakrishnan, N.(2000): Reliability sampling plans for log-normal distribution, Based on progressively censored samples, IEEE Transaction in reliability,49,199-203.
5. Bhattacharya, S.K.(1967): Bayesian approach to life testing and reliability estimation, journal of American statistical association, 62, 48-62.
6. Cheng, C., Chen, J., and Li, Z. (2010). A new algorithm for maximum likelihood estimation with progressive Type-I interval censored data. Communications in Statistics—Simulation and Computation, 39(4), 750-766.
7. Cramer, E., and Iliopoulos, G. (2010). Adaptive progressive Type-II censoring. Test, 19(2), 342-358.
8. Cramer, E., and Lenz, U. (2010). Association of progressively Type-II censored order statistics. Journal of Statistical Planning and Inference,140(2), 576-583.
9. El-Din, M. M., and Amein, M. M. (2011). Estimation of Parameters of the Exponential-Bernoulli Distribution Based on Progressively Censored Data. Applied Mathematical Sciences, 5(58), 2883-2890.
10. Jones, H. L. (1953). Approximating the mode from weighted sample values. Journal of the American Statistical Association, 48(261), 113-127.
11. Krishna, H., and Malik, M. (2012). Reliability estimation in Maxwell distribution with progressively type-II censored data. Journal of Statistical Computation and Simulation, 82(4), 623-641.
12. Kundu, D. (2008). Bayesian inference and life testing plan for the Weibull distribution in presence of progressive censoring. Technometrics, 50(2), 144-154.
13. Mann, M.R.(1971): Best linear invariant estimation for Wiebull parameters under Progressive Censoring, 13,521-533.
14. Martz, H.F. and Waller, R.A. (1982):Bayesian Reliability estimation, John Wiley Son. Inc. New Yark.
15. Mubarak, M. (2012). Parameter estimation based on the Frechet Progressive type II censored data with binomial removals. International Journal of Quality, Statistics, and Reliability.
16. Ng, H. K. T., Chan, P. S., and Balakrishnan, N. (2004). Optimal progressive censoring plans for the Weibull distribution. Technometrics, 46(4), 470-481.
17. Rastogi, M. K., and Tripathi, Y. M. (2012). Estimating the parameters of a Burr distribution under progressive type II censoring. Statistical Methodology,9(3), 381-391.
18. Salem, A. M., and Abo-Kasem, O. E. (2011). Estimation for the parameters of the exponentiated Weibull distribution based on progressive hybrid censored samples. Int J Contemp Math Sci, 6, 1713-1724.
19. Sinha. S. K. (1985). Reliability and Life Testing, Wiley Eastern.
20. Soliman, A. (2005). Estimation of parameters of life from progressively censored data using Burr-XII model. Reliability, IEEE Transactions on, 54(1), 34-42.
21. Tse, S. K., and Yuen, H. K. (1998). Expected experiment times for the Weibull distribution under progressive censoring with random removals. Journal of Applied Statistics, 25(1), 75-83.
22. Viveros, R., and Balakrishnan, N. (1994). Interval estimation of parameters of life from progressively censored data. Technometrics, 36(1), 84-91.
23. Wu, C. C., Wu, S. F., and Chan, H. Y. (2004). MLE and the estimated expected test time for Pareto distribution under progressive censoring data. International journal of information and management sciences, 15(3), 29-42.
24. Wu, S. J., and Chang, C. T. (2002). Parameter estimations based on exponential progressive type II censored data with binomial removals. International journal of information and management sciences, 13(3), 37-46.
25. Wu, S. J., and Huang, S. R. (2010). Optimal warranty length for a Rayleigh distributed product with progressive censoring. IEEE Transactions on Reliability, 59(4), 661-666.
26. Zhenmin Chen (2011) A simple method for estimating parameters of the location–scale distribution family, Journal of Statistical Computation and Simulation, 81:1, 49-58,

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