# Behaviour of three non-identical unit system with hypo-exponential repair time distributions 

V. K. Shivgotra ${ }^{1 *}$, JP singh Joorel ${ }^{2}$, Pawan Kumar ${ }^{3}$<br>${ }^{1,2,3}$ Department of Statistics, University of Jammu, Jammu and Kashmir, INDIA. Email:vijayshivgotra@yahoo.com, joorel@rediffmail.com, pkk skumar@yahoo.co.in


#### Abstract

A three non-identical unit system having Hypo-exponential repair time distributions and Negative exponential failure time distributions has been proposed and analysed by using Regenerative Point technique. Various reliability characteristics of interest have been obtained and some of them have been studied graphically. Keywords:hypo-exponential, time distributions.


*Address for Correspondence:
Dr.V. K. Shivgotra1, Department of Statistics, University of Jammu, Jammu and Kashmir, INDIA
Email:vijayshivgotra@yahoo.com
Received Date: 08/11/2015 Revised Date: 16/12/2015 Accepted Date: 24/01/2016


## INTRODUCTION

Numerous researchers viz ${ }^{1,7}$ in reliability theory have analyzed two unit/complexsystems models under different sets of assumptions. In Most of the papers authors have assumed that the failure and repair time distributions are exponential. Some authors including have also used the linearly increasing failure rate and hypo-exponential repair time distribution in the analysis of two unit systems. Gupta and Bansal ${ }^{5}$ have used the linearly increasing failure rate to analyze a three unit standby system. Gupta R and Krishan $\mathrm{R}^{3}$ carried out on the comparison on two stochastic model each having two unit with fixed preparation time and hypo exponential repair time distribution. The concept of correlated failure and repair time distribution have also been used by various authors including ${ }^{2,4,6}$ in analyzing two unit system model and complex system model under different set of assumptions and concept of administrative delay, preventive maintenance and repair machine etc. The purpose of the present paper is to analyze a three unit, non - identical, system with failure times of each unit as negative exponential with different parameters whereas the repair times of each unit as hypoexponential with different parameters. Using regenerative point technique, the various reliability measures of interest such as reliability, mean time to system failure, steady state availability, expected busy period of the repairman, expected number of repairs and net expected profit incurred by the system in steady state are obtained.

## SYSTEM DESCRIPTION AND ASSUMPTIONS

The system is composed of three non - identical units, namely, A, B and C. For the successful operation of the system, unit $A$ and at least one of the units $B$ and $C$ should work. The failure time distributions are negative exponential while the repair time distributions are hypo-exponential with different parameters. A single repairman is available with the system

[^0]to repair the failed units. The unit A gets priority in repair over the units B and C, while FCFS service discipline is followed for units B and C. After repair, a unit works as good as new.
Notations and States of the System
$\alpha_{i},(i=1,2,3)$ failure rate of unit A, B and C respectively.
$G_{i}(t),(i=1,2,3)$ cdf of repair time of Unit A, B and C respectively.
$\mathrm{G}_{1}(\mathrm{t})=\frac{1}{\beta_{2}-\beta_{1}}\left[\beta_{2}\left(1-e^{-\beta_{1} t}\right)-\beta_{1}\left(1-e^{-\beta_{2} t}\right)\right], \quad \beta_{1}, \beta_{2}>0$
$\mathrm{G}_{2}(\mathrm{t})=\frac{1}{\gamma_{2}-\gamma_{1}}\left[\gamma_{2}\left(1-e^{-\gamma_{1} t}\right)-\gamma_{1}\left(1-e^{-\gamma_{2} t}\right)\right], \quad \gamma_{1}, \gamma_{2}>0$
$\mathrm{G}_{3}(\mathrm{t})=\frac{1}{\delta_{2}-\delta_{1}}\left[\delta_{2}\left(1-e^{-\delta_{1} t}\right)-\delta_{1}\left(1-e^{-\delta_{2} \mathrm{t}}\right)\right], \quad \delta_{1}, \delta_{2}>0$
$p_{i j}$ : steady state probability of direct transition from state " i " to " j "
$p_{i j}^{(k)}$ : steady state transition probability from state " i " to " j " via " k ".
$\mu_{i}$ : mean sojourn time in state i.
© : symbol for ordinary convolution.
$\Theta$ : symbol for Stieltjes convolution.
$A_{o} / B_{o} / C_{o}$ : Unit $\mathrm{A}, \mathrm{B}$ and C are operative.
$A_{r} / B_{r} / C_{r} \quad$ : Unit A, B, and C are under repair.
$A_{g} / B_{g} / C_{g}$ :Unit A, B and C are good.
$A_{w} / B_{w} / C_{w}$ : Unit A, B and C are waiting for repairs.
The possible states of the system are:
$\mathrm{S}_{0} \equiv\left(A_{o}, B_{o}, C_{o}\right) \mathrm{S}_{1} \equiv\left(A_{o}, B_{r}, C_{o}\right) \mathrm{S}_{2} \equiv\left(A_{o}, B_{o}, C_{r}\right)$
$\mathrm{S}_{3} \equiv\left(A_{r}, B_{g}, C_{g}\right) \mathrm{S}_{4} \equiv\left(A_{r}, B_{w}, C_{g}\right) \mathrm{S}_{5} \equiv\left(A_{r}, B_{g}, C_{w}\right)$
$\underline{\mathrm{S}_{6}} \equiv\left(A_{g}, B_{w}, C_{r}\right) \underline{\mathrm{S}_{7}} \equiv\left(A_{g}, B_{r}, C_{w}\right)$
The underlined states are non regenerative. Transition diagram along with all transition is shown in fig. 1 Transition Probabilities and Sojourn Times
The non-zero elements of the transition probability matrix $P=\left(p_{i j}\right)$ are as follows

\[

$$
\begin{aligned}
& p_{01}=\frac{\alpha_{2}}{\alpha_{1}+\alpha_{2}+\alpha_{3}}, p_{02}=\frac{\alpha_{3}}{\alpha_{1}+\alpha_{2}+\alpha_{3}}, p_{03}=\frac{\alpha_{1}}{\alpha_{1}+\alpha_{2}+\alpha_{3}} \\
& p_{10}=\frac{\gamma_{1} \gamma_{2}}{\left(\alpha_{1}+\alpha_{3}+\gamma_{1}\right)\left(\alpha_{1}+\alpha_{3}+\gamma_{2}\right)} \\
& p_{14}=\frac{\alpha_{1}}{\left(\alpha_{1}+\alpha_{3}\right)}\left[1-\frac{\gamma_{2} \gamma_{1}}{\left(\alpha_{1}+\alpha_{3}+\gamma_{1}\right)\left(\alpha_{1}+\alpha_{3}+\gamma_{2}\right)}\right] \\
& p_{20}=\frac{\delta_{2} \delta_{1}}{\left(\alpha_{1}+\alpha_{2}+\delta_{1}\right)\left(\alpha_{1}+\alpha_{2}+\delta_{2}\right)} \\
& p_{25}=\frac{\alpha_{1}}{\left(\alpha_{1}+\alpha_{3}\right)}\left[1-\frac{\delta_{1} \delta_{2}}{\left(\alpha_{1}+\alpha_{3}+\delta_{1}\right)\left(\alpha_{1}+\alpha_{3}+\delta_{2}\right)}\right] \\
& \mathrm{p}_{30}=\int_{0}^{\infty} \mathrm{d} \mathrm{G}_{1}(\mathrm{t})=\mathrm{p}_{41}=\mathrm{p}_{52}=1, \mathrm{p}_{61}=\int_{0}^{\infty} \mathrm{d} \mathrm{G}_{3}(\mathrm{t})=1, \mathrm{p}_{72}=\int_{0}^{\infty} \mathrm{dG}_{2}(\mathrm{t})=1 \\
& \mathrm{p}_{12}^{(7)}=\frac{\alpha_{3}}{\left(\alpha_{1}+\alpha_{3}\right)}\left[1-\frac{\gamma_{1} \gamma_{2}}{\left(\alpha_{1}+\alpha_{3}+\gamma_{1}\right)\left(\alpha_{1}+\alpha_{3}+\gamma_{2}\right)}\right] \\
& \mathrm{p}_{21}^{(6)}=\frac{\alpha_{2}}{\left(\alpha_{1}+\alpha_{3}\right)}\left[1-\frac{\delta_{1} \delta_{2}}{\left(\alpha_{1}+\alpha_{3}+\delta_{1}\right)\left(\alpha_{1}+\alpha_{2}+\delta_{2}\right)}\right]
\end{aligned}
$$
\]

It can be easily verified that

$$
\begin{align*}
& p_{01}+p_{02}+p_{03}=1, p_{10}+p_{14}+p_{12}^{(7)}=1, p_{20}+p_{25}+p_{21}^{(6)}=1 \text { and } \\
& p_{30}=p_{41}=p_{52}=p_{61}=p_{72}=1 \tag{4.1-4.4}
\end{align*}
$$

The mean sojourn times in various states are:

$$
\begin{aligned}
& \mu_{0}=\int_{0}^{\infty} \mathrm{P}\left(\mathrm{~T}_{0}>\mathrm{t}\right) \mathrm{dt}=\int_{0}^{\infty} \mathrm{e}^{-\left(\alpha_{1}+\alpha_{2}+\alpha_{3}\right) t} d t=\frac{1}{\alpha_{1}+\alpha_{2}+\alpha_{3}} \\
& \mu_{1}=\frac{1}{\alpha_{1}+\alpha_{3}}-\left[1-\frac{\gamma_{1} \gamma_{2}}{\left(\alpha_{1}+\alpha_{3}+\gamma_{1}\right)\left(\alpha_{1}+\alpha_{3}+\gamma_{2}\right)}\right] \\
& \mu_{2}=\frac{1}{\left(\alpha_{1}+\alpha_{2}\right)}-\left[1-\frac{\delta_{1} \delta_{2}}{\left(\alpha_{1}+\alpha_{3}+\delta_{1}\right)\left(\alpha_{1}+\alpha_{3}+\delta_{2}\right)}\right] \\
& \mu_{3}=\int_{0}^{\infty} \bar{G}_{1}(t) d t=\mu_{4}=\mu_{5}=\frac{1}{\beta_{1}} \\
& \mu_{6}=\int_{0}^{\infty} \overline{G_{3}}(t) d t=\frac{1}{\delta_{1}} \text { and } \mu_{7}=\int_{0}^{\infty} \bar{G}_{2}(t) d t=\frac{1}{\gamma_{1}} .
\end{aligned}
$$

Analysis of Reliability and Mean Time to System Failure (MTSF)
Let the random variable $\mathrm{T}_{\mathrm{i}}$ denotes the time to system failure when the system starts from state $S_{i} \in E(i=0,1,2)$. Then, the reliability of the system is given by $R_{i}(t)=P\left[T_{i}>t\right]$ To determine $R_{i}(t)$, we regard the failed states of the system as absorbing. By probabilistic arguments we have

$$
R_{0}(\mathrm{t})=Z_{0}(\mathrm{t})+\mathrm{q}_{01}(\mathrm{t}) \odot R_{1}(\mathrm{t})+\mathrm{q}_{02} \odot R_{2}(\mathrm{t})
$$

$$
\begin{align*}
& R_{1}(\mathrm{t})=Z_{1}(\mathrm{t})+\mathrm{q}_{10}(t) \odot R_{0}(\mathrm{t}) \\
& R_{2}(\mathrm{t})=Z_{2}(\mathrm{t})+\mathrm{q}_{20}(t) \odot R_{0}(\mathrm{t}) \tag{5.1-5.3}
\end{align*}
$$

where
$Z_{0}(t)=e^{-\left(\alpha_{1}+\alpha_{2}+\alpha_{3}\right) t}, Z_{1}(t)=e^{-\left(\alpha_{1}+\alpha_{3}\right) t} G_{2}(t)$ and $Z_{2}(t)=e^{-\left(\alpha_{1}+\alpha_{2}\right) t} G_{3}(t)$
Taking, Laplace transform of the relation (1-3) and simplifying them for $R_{0}^{*}(s)$, we obtain
$R_{0}^{*}(s)=\frac{Z_{0}^{*}+q_{01}^{*} Z_{1}^{*}+q_{02}^{*} Z_{2}^{*}}{1-q_{01}^{*} q_{10}^{*}-q_{02}^{*} q_{20}^{*}}$
we have omitted the argument s for brevity. Taking inverse Laplace transform of (4) one can get the reliability of the system starting from $S_{0}$.
The mean time to system failure (MTSF) can be obtained by using the following relation
$\operatorname{MTSF}=E(T o)=\int_{0}^{\infty} \mathrm{R}_{0}(\mathrm{t}) \mathrm{dt}=\lim _{\mathrm{s} \rightarrow 0} \mathrm{R}_{0}^{*}(\mathrm{~s})$
and hence, its value is given by
MTSF $=\frac{\mu_{0}+p_{01} \mu_{1}+p_{02} \mu_{2}}{1-p_{01} p_{10}-p_{02} p_{20}}(5.5)$
where, we have used $q_{i j}^{*}(0)=p_{i j}$ and $Z_{i}^{*}(0)=\mu_{i}$.
Availability Analysis
To obtain recursive relations among point wise availability define $\mathrm{A}_{i}(\mathrm{t})$ as the probability that the system is up at epoch' $t$ ' when it initially starts from regenerative state $S_{i}$. Elementary probabilistic arguments yield the following recursive relations:

$$
\begin{align*}
& \mathrm{A}_{0}(\mathrm{t})=\mathrm{Z}_{0}(\mathrm{t})+\mathrm{q}_{01}(\mathrm{t}) \odot A_{1}(t)+q_{02}(t) \odot A_{2}(t)+q_{03}(t) \odot A_{3}(t) \\
& \mathrm{A}_{1}(\mathrm{t})=\mathrm{Z}_{1}(\mathrm{t})+\mathrm{q}_{10}(\mathrm{t}) \odot A_{0}(t)+q_{14}(t) \odot A_{4}(t)+q_{12}^{(7)}(t) \odot A_{2}(t) \\
& \mathrm{A}_{2}(\mathrm{t})=\mathrm{Z}_{2}(\mathrm{t})+\mathrm{q}_{20}(\mathrm{t}) \odot \mathrm{A}_{0}(\mathrm{t})+\mathrm{q}_{25}(\mathrm{t}) \odot \mathrm{A}_{5}(\mathrm{t})+\mathrm{q}_{21}^{(6)}(\mathrm{t}) \odot \mathrm{A}_{1}(\mathrm{t}) \\
& \mathrm{A}_{3}(\mathrm{t})=\mathrm{q}_{30}(\mathrm{t}) \odot A_{0}(t) \\
& \mathrm{A}_{4}(\mathrm{t})=\mathrm{q}_{41}(\mathrm{t}) \odot A_{1}(t) \\
& \mathrm{A}_{5}(\mathrm{t})=\mathrm{q}_{52}(\mathrm{t}) \odot A_{2}(\mathrm{t}) \tag{6.1-6.6}
\end{align*}
$$

where, $\mathrm{Z}_{0}(\mathrm{t}), \mathrm{Z}_{1}(\mathrm{t})$ and $\mathrm{Z}_{2}(\mathrm{t})$ have already been defined section 5 .
Taking Laplace transform of the relations (6.1-6.6) and simplifying them for $\mathrm{A}_{0}^{*}(\mathrm{~s})$, we get

$$
\begin{equation*}
A_{0}^{*}(s)=\frac{N_{1}(s)}{D_{1}(s)} \tag{6.7}
\end{equation*}
$$

where

$$
\begin{align*}
\mathrm{N}_{1}(\mathrm{~s})= & \mathrm{Z}_{0}^{*}\left[\left(1-\mathrm{q}_{14}^{*} \mathrm{q}_{41}^{*}\right)\left(1-\mathrm{q}_{25}^{*} \mathrm{q}_{52}^{*}\right)-\mathrm{q}_{21}^{(6)^{*}} \cdot \mathrm{q}_{12}^{(7)^{*}}\right]+\mathrm{Z}_{1}^{*}\left[\mathrm{q}_{01}^{*}\left(1-\mathrm{q}_{25}^{*} \mathrm{q}_{52}^{*}\right)\right. \\
& \left.+\mathrm{q}_{02}^{*} \mathrm{q}_{21}^{(6)^{*}}\right]+\mathrm{Z}_{2}^{*}\left[\mathrm{q}_{02}^{*}\left(1-\mathrm{q}_{14}^{*} \mathrm{q}_{41}^{*}\right)+\mathrm{q}_{01}^{*} \cdot \mathrm{q}_{12}^{(7)^{*}}\right] \tag{6.8}
\end{align*}
$$

and
$D_{1}(s)=\left[\left(1-q_{03}^{*} q_{30}^{*}\right)\left(1-q_{14}^{*} q_{41}^{*}\right)\left(1-q_{25}^{*} q_{52}^{*}\right)-q_{05}^{*} q_{10}^{*}\left(1-q_{25}^{*} q_{52}^{*}\right)-q_{02}^{*} q_{20}^{*}\right.$
$\left.\left(1-q_{14}^{*} q_{41}^{*}\right)-q_{21}^{(6)^{*}} q_{12}^{(7)^{*}}-q_{02}^{*} q_{10}^{*} q_{21}^{(6)^{*}}-q_{01}^{*} q_{20}^{*} q_{12}^{(7)^{*}}+q_{03}^{*} q_{30}^{*} q_{12}^{(7)^{*} q^{(6)}}{ }^{(6)^{*}}\right]$
The steady state system availability is given by
$A_{0}=\lim _{t \rightarrow \infty} \mathrm{~A}_{0}(\mathrm{t})=\lim _{s \rightarrow 0} \mathrm{sA}_{0}^{*}(\mathrm{~s})=\frac{\mathrm{N}_{1}(0)}{\mathrm{D}_{1}^{\prime}(0)}$
where
$N_{1}(0)=\left[\left(\mathrm{p}_{10} \mu_{0}+\mathrm{p}_{01} \mu_{1}\right)\left(\mathrm{p}_{20}+\mathrm{p}_{21}^{(6)}\right)\right]+\mathrm{p}_{12}^{(7)}\left(\mathrm{p}_{20} \mu_{0}+\mathrm{p}_{02} \mu_{2}\right)$
$+\mu_{1}\left[p_{02} p_{21}^{(6)}+p_{01}\left(p_{20}+p_{21}^{(6)}\right)\right]$
and
$\mathrm{D}_{1}^{\prime}(0)=\left(\mu_{0}+\mu_{3} \mathrm{p}_{03}\right)\left[\mathrm{p}_{10}\left(\mathrm{p}_{20}+\mathrm{p}_{21}^{(6)}\right)+\mathrm{p}_{20} \mathrm{o}_{12}^{(7)}\right]+\left(\mu_{1}+\mu_{4} \mathrm{p}_{14}\right)\left[p_{01}\left(p_{20}+p_{21}^{(6)}\right)+\mathrm{p}_{02} p_{21}^{(6)}\right]$
$+\left(\mu_{2}+\mu_{5} p_{25}\right)\left[p_{02}\left(p_{10}+p_{12}^{(7)}\right)\right]+\mu_{2} p_{01} p_{12}^{(7)}+\mu_{5} p_{02} p_{12}^{(7)} p_{25}$
The expected uptime of the system during $(0, t)$ is given by
$\mu_{u p}(\mathrm{t})=\int_{0}^{\mathrm{t}} \mathrm{A}_{0}(u) d u$
So that $\mu_{\text {up }}^{*}(s)=A_{0}^{*}(s) / s$
Busy Period Analysis
Define $B_{i}(t)$ as the probability that the system having started from regenerative state $S_{i}, S_{i} \in E$ at time $t=0$ is under repair at instant 't'. By simple probabilistic arguments, we have the following recursive relations:
$B_{0}(t)=\sum_{\mathrm{j}=1}^{3} q_{0 j}(t) \oplus \mathrm{B}_{\mathrm{j}}(\mathrm{t})$
$B_{1}(t)=\mathrm{Z}_{1}(\mathrm{t})+\mathrm{q}_{10}(\mathrm{t}) \subset \mathrm{B}_{0}(\mathrm{t})+\mathrm{q}_{12}^{(7)}(\mathrm{t}) \subset \mathrm{B}_{2}(\mathrm{t})+\mathrm{q}_{14}(\mathrm{t}) \subset \mathrm{B}_{4}(\mathrm{t})$
$\mathrm{B}_{2}(\mathrm{t})=\mathrm{Z}_{2}(\mathrm{t})+\mathrm{q}_{20}(\mathrm{t}) \odot B_{0}(t)+q_{21}^{(6)}(t) \odot B_{1}(t)+q_{25}(t) \Subset B_{5}(t)$
$\mathrm{B}_{3}(\mathrm{t})=\mathrm{Z}_{3}(\mathrm{t})+\mathrm{q}_{30}(\mathrm{t}) \odot B_{0}(\mathrm{t})$
$\mathrm{B}_{4}(\mathrm{t})=\mathrm{Z}_{4}(\mathrm{t})+\mathrm{q}_{41}(\mathrm{t}) \odot B_{1}(t)$
$\mathrm{B}_{5}(\mathrm{t})=\mathrm{Z}_{5}(\mathrm{t})+\mathrm{q}_{52}(\mathrm{t})$ © $B_{2}(\mathrm{t})$
Taking Laplace transform of the relations (7.1-7.6) and solving the resulting set of equations for $B_{0}^{*}(s)$, we get

$$
\begin{equation*}
B_{0}^{*}(s)=\frac{\mathrm{N}_{2}(\mathrm{~s})}{\mathrm{D}_{2}(\mathrm{~s})} \tag{7.7}
\end{equation*}
$$

where

$$
\begin{align*}
& N_{2}(s)=\mathrm{Z}_{1}^{*}\left[\mathrm{q}_{01}^{*}\left(1-\mathrm{q}_{25}^{*} \mathrm{q}_{52}^{*}\right)+\mathrm{q}_{02}^{*} \mathrm{q}_{21}^{(6)^{*}}\right]+\mathrm{Z}_{2}^{*}\left[\mathrm{q}_{02}^{*}\left(1-\mathrm{q}_{14}^{*} \mathrm{q}_{41}^{*}\right)+\mathrm{q}_{01}^{*} \mathrm{q}_{12}^{(7)^{*}}\right]+\mathrm{Z}_{3}^{*} q_{03}^{*}\left[\left(1-q_{14}^{*} q_{41}^{*}\right)\right. \\
& \left.\left(1-\mathrm{q}_{25}^{*} \mathrm{q}_{52}^{*}\right)-\mathrm{q}_{12}^{(7)^{*}{ }^{(6)}} \mathrm{q}_{21}^{()^{*}}\right]+\mathrm{Z}_{4}^{*} \mathrm{q}_{14}^{*}\left[\mathrm{q}_{01}^{*}\left(1-\mathrm{q}_{25}^{*} \mathrm{q}_{52}^{*}\right)+\mathrm{q}_{02}^{*} \mathrm{q}_{21}^{(6)^{*}}\right] \\
& +\mathrm{Z}_{5}^{*} \mathrm{q}_{25}^{*}\left[\mathrm{q}_{02}^{*}\left(1-\mathrm{q}_{14}^{*} \mathrm{q}_{41}^{*}\right)+\mathrm{q}_{02}^{*} \mathrm{q}_{12}^{(7)^{*}}\right] \tag{7.8}
\end{align*}
$$

and $D_{2}(s)$ is same as given by equation (6.9)
In the long- run, the probability that the repairman is busy is given by

$$
B_{0}=\lim _{\mathrm{t} \rightarrow \infty} B_{0}(t)=\lim _{\mathrm{s} \rightarrow 0} s B_{0}^{*}(s)=\frac{\mathrm{N}_{2}(0)}{\mathrm{D}_{1}^{\prime}(0)}
$$

where
$N_{2}(0)=\left(1-\mathrm{p}_{25}\right)\left[\mathrm{p}_{01}\left(\mu_{1}+\mathrm{p}_{14} \mu_{4}\right)+\left(1-\mathrm{p}_{14}\right) \mathrm{p}_{03} \mu_{3}\right]+\mathrm{p}_{21}^{(6)}\left[\mathrm{p}_{02} \mu_{1}-\mathrm{p}_{12}^{(7)} \mathrm{p}_{03} \mu_{3}\right.$
$\left.+\mathrm{p}_{02} \mathrm{p}_{14} \mu_{4}\right]+\mathrm{p}_{02}\left(1-\mathrm{p}_{14}\right)\left(\mu_{2}+\mathrm{p}_{25} \mu_{5}\right)+\mathrm{p}_{12}^{(7)}\left(\mathrm{p}_{01} \mu_{1}+p_{02} p_{25} \mu_{5}\right)$
and $\mathrm{D}_{1}^{\prime}(0)$ is same as given by equation (6. 12)
The expected busy period of the repairman during ( $0, t$ ] is given by
$\mu_{b}(t)=\int_{0}^{t} \mathrm{~B}_{0}(\mathrm{u}) \mathrm{du}$
so that $\mu_{\mathrm{b}}^{*}(s)=B_{0}^{*}(s) / s$
Expected Number of Repairs
Define $V_{i}(t)$ as the expected number of repairs of the unit during the interval $(0, \mathrm{t}]$ when the system initially starts from regenerative state $S_{i}$. Using elementary probabilistic arguments we have the following relations:

$$
\begin{align*}
& V_{0}(t)=\mathrm{Q}_{01}(\mathrm{t}) \Theta \mathrm{V}_{1}(\mathrm{t})+\mathrm{Q}_{02}(\mathrm{t}) \Theta \mathrm{V}_{2}(\mathrm{t})+\mathrm{Q}_{03}(\mathrm{t}) \Theta \mathrm{V}_{3}(\mathrm{t}) \\
& V_{1}(t)=\mathrm{Q}_{10}(\mathrm{t}) \Theta\left[1+\mathrm{V}_{0}(\mathrm{t})\right]+\mathrm{Q}_{14}(\mathrm{t}) \Theta \mathrm{V}_{4}(\mathrm{t})+\mathrm{Q}_{12}^{(7)}(\mathrm{t}) \Theta\left[1+\mathrm{V}_{2}(\mathrm{t})\right] \\
& \left.V_{2}(\mathrm{t})=\mathrm{Q}_{20}(t) \Theta\left[1+\mathrm{V}_{0}(\mathrm{t})\right]+\mathrm{Q}_{21}^{(6)}(\mathrm{t}) \Theta\left[1+\mathrm{V}_{1}(\mathrm{t})\right]+\mathrm{Q}_{25}(\mathrm{t}) \Theta \mathrm{V}_{5}(\mathrm{t})\right] \\
& V_{3}(t)=\mathrm{Q}_{30}(\mathrm{t}) \Theta\left[1+\mathrm{V}_{0}(\mathrm{t})\right] \\
& V_{4}(t)=\mathrm{Q}_{41}(\mathrm{t}) \Theta\left[1+\mathrm{V}_{1}(\mathrm{t})\right] \\
& V_{5}(\mathrm{t})=\mathrm{Q}_{52}(\mathrm{t}) \Theta\left[1+\mathrm{V}_{2}(\mathrm{t})\right] \tag{8.1-8.6}
\end{align*}
$$

Taking Laplace Stieltjes transform of the relations (1-6), the solution for $\widetilde{V}_{0}(s)$ can be written in the following form:

$$
\begin{equation*}
\widetilde{V}_{0}(s)=\frac{\mathrm{N}_{3}(\mathrm{~s})}{\mathrm{D}_{3}(\mathrm{~s})} \tag{8.7}
\end{equation*}
$$

where

$$
\begin{aligned}
& N_{3}(s)=\left(\widetilde{\mathrm{Q}}_{10}+\widetilde{\mathrm{Q}}_{12}^{(7)}\right)\left[\widetilde{\mathrm{Q}}_{01}\left(1-\widetilde{\mathrm{Q}}_{25} \widetilde{\mathrm{Q}}_{52}\right)+\widetilde{\mathrm{Q}}_{02} \widetilde{\mathrm{Q}}_{21}^{(6)}\right]+\left(\widetilde{Q}_{21}+\widetilde{\mathrm{Q}}_{21}^{(6)}\right)\left[\widetilde{\mathrm{Q}}_{02}\left(1-\widetilde{\mathrm{Q}}_{14} \widetilde{\mathrm{Q}}_{41}\right)\right. \\
& \left.+\widetilde{\mathrm{Q}}_{01} \widetilde{\mathrm{Q}}_{12}^{(7)}\right]+\widetilde{\mathrm{Q}}_{30} \widetilde{\mathrm{Q}}_{03}\left[\left(1-\widetilde{\mathrm{Q}}_{14} \widetilde{\mathrm{Q}}_{41}\right)\left(1-\widetilde{\mathrm{Q}}_{25} \widetilde{\mathrm{Q}}_{52}\right)-\widetilde{\mathrm{Q}}_{12}^{(7)} \widetilde{\mathrm{Q}}_{21}^{(6)}\right] \\
& \left.+\widetilde{\mathrm{Q}}_{14} \widetilde{\mathrm{Q}}_{41} \widetilde{\mathrm{Q}}_{01}\left(1-\widetilde{\mathrm{Q}}_{25} \widetilde{\mathrm{Q}}_{52}\right)+\widetilde{\mathrm{Q}}_{02} \widetilde{\mathrm{Q}}_{21}^{(6)}\right]+\widetilde{\mathrm{Q}}_{25} \widetilde{\mathrm{Q}}_{52}\left[\widetilde{\mathrm{Q}}_{02}\left(1-\widetilde{\mathrm{Q}}_{14} \widetilde{\mathrm{Q}}_{41}\right)+\widetilde{\mathrm{Q}}_{01} \widetilde{\mathrm{Q}}_{12}^{(7)}\right]
\end{aligned}
$$

and $D_{3}(\mathrm{~s})$ can be obtained on replacing $\mathrm{q}_{\mathrm{ij}}^{*}(\mathrm{~s})$ by $\mathrm{Q}_{\mathrm{ij}}^{*}(\mathrm{~s})$ in (6.9).
In steady state the expected number of repairs per unit of time is given by

$$
\begin{align*}
& V_{0}=\lim _{t \rightarrow \infty} \frac{\mathrm{~V}_{0}(\mathrm{t})}{\mathrm{t}}=\lim _{s \rightarrow 0} s . \widetilde{\mathrm{V}}_{0}(\mathrm{~s})=\frac{\mathrm{N}_{4}(0)}{\mathrm{D}_{1}^{\prime}(0)}  \tag{8.8}\\
& \qquad \begin{aligned}
\mathrm{N}_{4}(0)= & \left(1-\mathrm{p}_{14}\right)\left[\mathrm{p}_{01}\left(1-\mathrm{p}_{25}\right)+\mathrm{p}_{02} \mathrm{p}_{21}^{(6)}\right]+\left(1-\mathrm{p}_{25}\right)\left[\mathrm{p}_{02}\left(1-\mathrm{p}_{14}\right)+\mathrm{p}_{01} \mathrm{p}_{12}^{(7)}\right] \\
& \quad+\mathrm{p}_{03}\left[\left(1-p_{14}\right)\left(1-\mathrm{p}_{25}\right)-\mathrm{p}_{12}^{(7)} \mathrm{p}_{21}^{(6)}\right]+\mathrm{p}_{14}\left[\mathrm{p}_{01}\left(1-\mathrm{p}_{25}\right)+\mathrm{p}_{02} \mathrm{p}_{21}^{(6)}\right] \\
& \quad+\mathrm{p}_{25}\left[\mathrm{p}_{02}\left(1-\mathrm{p}_{14}\right)+\mathrm{p}_{01} \mathrm{p}_{12}^{(7)}\right.
\end{aligned}
\end{align*}
$$

## Profit function Analysis

Considering the mean up time, expected busy period of the repairman and expected number of repairs per unit of time, the net expected total profits in interval $(0, t]$ are given by

$$
\begin{aligned}
& P_{1}(t)=k_{0} \mu_{u p}(t)-k_{1} \mu_{b}(t) \\
& P_{2}(t)=k_{0} \mu_{u p}(t)-k_{2} V_{0}(t)
\end{aligned}
$$

The expected total profit per unit time in steady state is
$\mathrm{P}_{1}=\mathrm{k}_{0} \mathrm{~A}_{0}-\mathrm{k}_{1} \mathrm{~B}_{0}$
$\mathrm{P}_{2}=\mathrm{k}_{0} \mathrm{~A}_{0}-\mathrm{k}_{2} \mathrm{~V}_{0}$
where, $k_{0}$ is revenue per unit time, $k_{1}$ is cost of repair per unit time and $k_{2}$ is the per unit repair cost.

## GRAPHICAL STUDY OF THE SYSTEM BEHAVIOR

For more concrete study of the system behavior, we plot the graph of MTSF and Availability with respect to failure parameter $\alpha_{1}$ for three different values of repair parameter $\gamma_{1}=0.1,0.2,0.3$ respectively, when other parameters are kept fixed as $\alpha_{2}=\alpha_{3}=0.5, \beta_{1}=0.3, \beta_{2}=0.5, \gamma_{2}=0.6, \delta_{1}=0.5, \delta_{2}=0.8$. Fig. 2 shows the variation in MTSF in respect of $\alpha_{1}$ for three different value of repair parameter $\gamma_{1}$. It can be seen from the graph that MTSF decreases with increase in failure parameter $\alpha_{1}$ and increases with the increase in the repair parameter.Fig. 3 reveals the change in the behaviour of availability with respect to varying value of $\alpha_{1}$ for three different value of $\gamma_{1}=0.1,0.2,0.3$. From graph, it is seen that the availability decreases uniformly with the increase in the failure rate and increases, as the repair parameter is increased. In fig. 4 the smooth and dotted curves represent the change in the profit function $\mathrm{P}_{1}$ and $\mathrm{P}_{2}$ for varying values of failure parameter $\alpha_{1}$ for three different values of repair parameter $\gamma_{1}=0.1,0.2,0.3$, when other parameters are kept as $k_{0}=1200, k_{1}=150, k_{2}=250 \quad \mathrm{k}_{0}=1200, \alpha_{2}=\alpha_{3}=0.5, \quad \beta_{1}=0.3, \beta_{2}=0.5$, $\gamma_{2}=0.6, \delta_{1}=0.5, \delta_{2}=0.9$ respectively. From graph it is observed that both the profit function decrease with the increase in the failure rate $\alpha_{1}$ and increase with the increase repair rate $\gamma_{1}$. It is also seen that both the profit function $\mathrm{P}_{1}$ and $\mathrm{P}_{2}$ decreases rapidly with increase of repair rate $\alpha_{1}$, also the rate of decrement for both profit function $\mathrm{P}_{1}$ and $\mathrm{P}_{2}$ is almost same, but profit function $P_{1}$ is always better than profit function $P_{2}$.



## REFERENCES

1. Dillon B. S. (1992). Stochastic Modeling of k-out of-n units family of systems Int. J. System Sciences, Vol. 23, No.28, 1277-1287.
2. Gupta R., CK Goyal and A Tomar (2010). A two dissimilar unit parallel system with administrative delay in Repair and Correlated Lifetimes International Transactions in Mathematical Sciences and Computer, 3(1) 103-112.
3. Gupta R. and Krishan R.(1999). On profit comparison of two stochastic models each pertaining to a two unit standby system with fixed preparation time and hyper- exponential repair time distributions. Int. J. System Sciences, Vol.30, No.12, 1309-1317
4. Gupta R.,and Shivakar (2010). Cost Benefit of Two unit parallel system with correlated failure and Repair Rate IAPQR TransactiosnV.35, No 2 117-140.
5. Gupta R. and S. Bansal (1991). Cost analysis of a three unit standby system subjects to random shocks and linearly increasing failure rates. RESS 33, 249-253.
6. Kumar P and Neha Kumari (2014). Reliability Analysis of a complex systemwith repair machine and CorrelatedFailure and Repair Time. AIJRSTEM 14-565 148-155.
7. Kumar A. and Randar M. C.(1993). Reliability analysis of a complex redundant system.Microelectron. Reliab., 33, 459-462.

## Source of Support: None Declared Conflict of Interest: None Declared


[^0]:    How to site this article: V K Shivgotra, JP singh Joorel, Pawan Kumar. Behaviour of three non-identical unit system with hypoexponential repair time distributions. International Journal of Statistika and Mathemtika February to April 2016; 18(1): 08-15. http://www.statperson.com (accessed 04 February 2016).

