Original Article

Fuzzy approach to solve multi objective linear Fractional transportation programming problem (MOLTFPP)

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Abstract In this paper we have developed an algorithm to solve multi objective linear fractional transportation programming problem (MOLFTPP) using fuzzy compromise approach. Here each of the linear fractional transportation problems is solved using simplex method. Then each of the fractional objective function is expanded about optimal solution vector by Taylor's Series method and converted it into approximate linear transportation programming problem, using partial differentiation. Finally we have solved this problem as Multi Objective Linear Transportation Programming Problem using Fuzzy compromise programming approach. The coefficients of decision variables are obtained by MATLAB programme. Also compromise solution is also obtained using *linprog* function of MATLAB.
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INTRODUCTION

In standard transportation problem allocation of goods is to be made from origin of goods to destinations such that either total cost is to be minimised or total profit is to be maximised. But if allocations are to be made such that ratio of total profit to total cost is to be maximised then in this type of situation the objective function is nothing but ratio of two linear functions hence the name of the problem is linear fractional transportation problem (LFTP). When there are several linear fractional objective functions then the problem is called multi objective linear fractional transportation programming problem (MOLFTPP). Fractional programming problem can be converted into linear programming problem (LPP) by using variable transformation give by Charnes and Cooper[1]. MOFLPP can be converted into MOLPP using Taylors series method[2], with variable transformation method Chakraborty and Gupta[3] converted MOLFPP to MOLPP using Fuzzy set theoretic approach. Surapati Pramanik and Durga Banerjee[4] gave solution to chance constrained multi objective linear plus fractional programming problem. Singh *et al.*[5] developed an algorithm for solving MOFLPP with the help of Taylor series. Pitram singh, *et al.*[6] gave approach for multi objective linear plus fractional programming problem. MOLPP is solved using fuzzy compromise approach by Lushu Li and K Lai[7]. Multi objective linear transportation problem is solved by Doke and Jadhav[8] using fuzzy compromise programming approach. Doke and Jadhav [9] gave fuzzy approach to solve multi objective fractional programming problem. In this paper we will solve linear fractional transportation programming problem (LFTPP) by Simplex Method. Then convert each of the LFTP into linear programming problem using Taylor Series method. Finally we will solve the problem using MOLTPP using fuzzy compromise approach. Fuzzy programming approach involves solving LPP with many constraints and such problem is to be solved for many combinations thus for quick calculations MATLAB is used to find compromise solution.

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FORMULATION OF LFTPP

In general a problem is specified as under

1. There are m supply points from which goods are to be transported. Supply available at i^{th} point is at most b_i units (i = 1,2,...,m).
2. There are n demand centres where good are required. Minimum requirement of j^{th} demand centre is a_j units (j = 1,2,...,n).
3. Profit matrix is P = $\| p_{ij} \|$mxn, where p_{ij} is profit of transporting one unit from i^{th} origin to j^{th} destination.
4. Cost matrix is D = $\| d_{ij} \|$ mxn, where d_{ij} is cost of transporting one unit from i^{th} origin to j^{th} destination.
5. p_0 and d_0 be the fixed profit and cost respectively.

Suppose variable x_{ij} denotes number units to be transported from i^{th} origin to j^{th} destination. The objective function is,

Maximize or Minimize $Q(x) = \frac{P(x)}{D(x)} = \frac{\sum_{i=1}^{m} \cdot \sum_{j=1}^{n} pijxij + p0}{\sum_{i=1}^{m} \cdot \sum_{j=1}^{n} dijxij + d0}$ (1)

Subject to constraint $\sum_{j=1}^{n} xij \leq bi$ i = 1, 2,3,....,m, (2)

$\sum_{i=1}^{m} xij \geq aj$ j=1,2,3,....,n, (3)

$x_{ij} \geq 0$ for all i = 1,2,.....m and j=1,2,...n (4)

Q (x) is called objective function which is fractional. Note that P(x) and D(x) are linear and (2), (3) and (4) are also linear. Hence the name of the problem is Linear fractional transportation Programming problem (LFTPP).

Here $D(x) > 0$, for every X = (xij) ∈ S, Where S is feasible set defined by constraint (2),(3) and (4). Lastly we assume that $b_i > 0$ and $a_j > 0$, i=1,2,...,m and j=1,2,...,n.

Total supply is not less than total demand i.e. $\sum_{i=1}^{m} bi \geq \sum_{j=1}^{n} aj$ (5)

The problem is to find X =(xij) ∈ S, satisfying (2-4) and having optimal value of (1).

This LFTPP has following properties,

1. The problem has a feasible solution, i.e. feasible set S ≠ Ø.
2. The set of feasible solution is bounded.
3. The problem is always solvable.

MULTI OBJECTIVE LINEAR FRACTIONAL TRANSPORTATION PROBLEM

Suppose $Q_1(x) = \frac{P1(x)}{Q1(x)}$, $Q_2(x) = \frac{P2(x)}{Q2(x)}, \ldots \ldots$, $Q_k(x) = \frac{Pk(x)}{Qk(x)}$

are k linear fractional transportation problems such that objective functions to be optimised simultaneously subject to constraint (2), (3),(4) and (5) and $D_k(x) \geq 0$ for every k then the problem is called Multi Objective Linear Fractional Transportation Programming Problem (MOLFTPP).

Definitions

1. If total demand equals to total supply, i.e. $\sum_{i=1}^{m} bi = \sum_{j=1}^{n} aj$, (6) then LFTP is said to be balanced transportation problem.
2. LFTP is said to be in canonical form if
   $\sum_{j=1}^{n} xij = bi$ i = 1, 2,3,....,m, (7)
   $\sum_{i=1}^{m} xij = aj$ j=1,2,3,....,n, (8)
   $Xij \geq 0$ for all i and j (9)
3. There is exactly one equality redundant constraint in (7) and (8). When any one of the constraint in (7) and (8) are dropped, what remains is exactly m+n-1 linearly independent equations.
4. A solution of LFTPP is said to be basic solution if it satisfies conditions (7) and (8).In basic solution there are m+n-1 values of variables in basis.
5. A basic solution of the LFTPP is said to basic feasible (BFS) if all values in basis are non negative i.e. it satisfies condition (9).
6. The basic solution is degenerate if at least one of its basic variable equal to zero. Otherwise it is said to be non degenerate.

Theorems

1. LFTPP is solvable if and only if it is balanced.
2. If all a_i and b_j in LFTPP are positive integers, then every basic solution of LFTPP is integer vector. Hence if all a_i and b_j of LFTPP are positive integers and the problem is in the canonical then LFTPP has an optimal solution x* with all elements as positive integers.

Criteria for Optimality of LFPP

Following are the steps involved to find optimal solution of LFTPP

1. Suppose u_i', v_j',and u_i'',v_j'' are simplex multipliers associated with the numerator P(x) and denominator D(x) respectively. Elements u_i' and u_i'' corresponds to m supply constraints (7) and v_j' and v_j'' corresponds to n demand constraints (8). These values are obtained using following equations. $u_i' + v_j' = p_{ij}$ and $u_i'' + v_j'' = d_{ij}$ such that cell (i,j) is in the basis. Note that in each of the set there are m+n variables and only m+n-1 equations hence to solve these systems put any one of the variable as some arbitrary value say 0.
2. Using these variables find reduced costs for the cells which are not in basis as given below, $\Delta'ij = u_i' + v_j' - p_{ij}$ $\Delta''ij = u_i'' + v_j'' - d_{ij}$ $i = 1,2,\dots,m$ and $j = 1,2,\dots,n$ $\Delta ij = \Delta'ij$ - Q(x) $\Delta''ij$ Where Q(x) is value of objective function at current solution.
3. A basic feasible solution X =(xij) is a basic optimal solution of LFTPP (1) subject to (7) –(9) if and only if $\Delta ij(x) \geq 0$ for all i and j.

Algorithm To Solve MOLFTPP

1. Consider MOLFTPP, Maximize { $Q_1(x), Q_2(x), \dots, Q_k(x)$ } subject to constraint (7-9)
2. Find optimal solution of each of the linear fractional transportation problem as single objective function subject to constraint, using simplex method.
3. Suppose X_l* is optimal solution of of $Q_l(x)$ for l =1,2,...,k
4. Expand objective function $Q_l(x)$ about X^*_l =(xij) using Taylor's theorem and ignoring second and higher order terms convert $Q_l(X)$ into linear function. Consider $Q_l(x) = \frac{Pl(x)}{Ql(x)}$ and X^*_l be the optimal solution of $Q_l(x)$, then using Taylor Series approach differentiate Q_l (X) with respect to each of the variable partially. $Q_l(x) \sim \cong Q_l(x^*_l)$ + $(x_{11}-x_{11l})$ $\{\frac{\partial Ql(Xl)}{\partial x11}\}$ at X_l^* + $(x_{12}-x_{12l})$$\{\frac{\partial Ql(Xl)}{\partial x12}\}$ at X_l^* +....+ $(x_{mn}-x_{mnl})$ $\{\frac{\partial Ql(Xl)}{\partial xmn}\}$ at X_l^*+ O(h^2). Using this expansion each of the objective function becomes linear function. To avoid complexity of notations we write, $Q_l(x) \sim = Z_l(x)$, l =1,2,3,....,k. The problem is to Maximize (Z1(x),Z2(x),....,Zk(x)) subject to (6-9) i.e.Multi Objective Linear Transportation Programming Problem. (MOLTPP).It can be solved by using fuzzy compromise approach as stated in next section.

Fuzzy compromise approach for MOLTPP.

Consider MOLTPP

Maximize Z (x) = [Z_1 (x), Z_2, …, Z_k (x)] ---------------- (10)

Subject to (6-9) and X∈ S, where S is set of feasible solutions.

Solution to (10) is often conflicting as several objectives cannot be optimized simultaneously. To find compromise solutions first solve each of the objective function as marginal or single objective function. In this paper we have converted Linear Fractional Transportatiion Programming problem to Linear Programming problem using Taylor series approach. Suppose x_k * is optimal solution of k^{th} objective function. Find values of each objective at optimal solution of k^{th} objective function for all k= 1,2,…,K. Thus we have matrix of evaluation of objectives.

To find Marginal evaluation for single objective. For each particular objective we define marginal evaluation function $\varphi_k : X \rightarrow [0,1]$ as given below

$$\varphi_k(x) = \begin{cases} 1 \ if \ Zk \leq Uk \\ \frac{Zk-Uk}{Lk-Uk} \ if \ U\boldsymbol{k} < Zk(x) < Lk \\ 0 \ if \ Zk(x) \geq Lk \end{cases}$$

Where U_k = Min Z_k (x) k=1,2,…,K L_k = Max Z_k (x) k=1,2,…,K

According to fuzzy sets, φ_k is fuzzy subset describing fuzzy concept of optimum for objective Z_k on feasible solution space S. To find compromise solution maximize an aggregation operator

$\mu(x) = \varphi_w[\varphi 1(x), \varphi 2(x), \dots, \varphi k(x)]$

Max $\mu(x) = M^{(\alpha)}_w[\varphi 1(x), \varphi 2(x), \dots, \varphi k(x)]$

$\mu(x) = [\sum wi\, \varphi^{\alpha}_i(x)]^{(1/\alpha)}$

If α = 1 then Maximize $\mu(x) = \sum [wi\, \varphi_i(x)]$ i.e. weighted A.M. of Fuzzy sets

This procedure gives weighted arithmetic mean to find global evaluation of multiple objectives.

Example:

Consider following two objective functions,

$$Q_1(x) = \frac{10x11+14x12+8x13+12x14+8x21+12x22+14x23+8x24+9x31+6x32+15x33+9x34}{15x11+12x12+16x13+8x14+10x21+6x22+13x23+12x24+13x31+15x32+12x33+10x34}$$

$$Q_2(x) = \frac{14x11+9x12+11x13+9x14+12x21+9x22+6x23+15x24+6x31+9x32+12x33+10x34}{12x11+14x12+7x13+17x14+6x21+11x22+13x23+10x24+9x31+15x32+14x33+16x34}$$

Maximize (Q1(x), Q2(x))
Subject to $x_{11}+x_{12}+x_{13} +x_{14} = 15$
$x_{21}+x_{22}+x_{23} + x_{24} = 25$
$x_{31}+x_{32}+x_{33}+ x_{34} = 20$
$x_{11} + x_{21} +x_{31} = 15$
$x_{12} + x_{22} +x_{32} = 25$
$x_{13} + x_{23} + x_{33} = 5$
$x_{14} + x_{24} +x_{34} = 15$
$x_{ij} \geq 0$ for i = 1,2...,m and j = 1,2,....,n

Step1: Solve $Q_1(x)$ and Q_2 (x) using simplex method.
Optimal solution of Q_1 (x) is X_1 = [0;0;0;15;0;25;0;0;15;0;5;0]; and optimal value is 1.3143 and Optimal solution of Q_2 X_2 = [5;5;5;0;10;;0;0;15;0;20;0;0], and optimal value is 1.0296.

Step 2: Following MATLAB program finds coefficients of x_{ij}'s to convert it into linear function using Taylor Series. It also finds compromise solution for given combination of two objective functions. Thus Z_1 (X) and Z_2 (X) are as under.

$Z_1(X) = 1.3143 - 0.185\ x_{11} - 0.0034\ x_{12} – 0.0248\ x_{13} + 0.0028\ x_{14} – 0.0098\ x_{21} + 0.0078\ x_{22} - 0.0059x_{23} -0.0148\ x_{24} - 0.0154\ x_{31} – 0.0261\ x_{32} -0.0015\ x_{33} – 0.0154\ x_{34}$

$Z_1(X) = 1.0296 + 0.0024\ x_{11} - 0.0080\ x_{12} + 0.0056\ x_{13} - 0.0126\ x_{14} + 0.0086\ x_{21} - 0.0034\ x_{22} - 0.0109x_{23} + 0.0070\ x_{24} - 0.0048\ x_{31} – 0.0095\ x_{32} -0.0036\ x_{33} – 0.0096\ x_{34}$

Step 3: A MATLAB programme.

```
% A program to Solve multi objective fractional transportation problem.
% Enter initial value of ALPHA and first profit matrix P1 and cost matrix D1
ALPHA = 0.0;
for n = 1:10
P1 = [10;14;8;12;8;12;14;8;9;6;15;9];
D1 = [15;12;16;8;10;6;13;12;13;15;12;10];
% Enter optimal solution of first objective function
X1 = [0;0;0;15;0;25;0;0;15;0;5;0];
% Convert fractional objective function as linear using Taylors Series
C1P = P1'*(D1'*X1);
C1D = D1'*(P1'*X1);
Z1 = (C1P-C1D)/((D1'*X1)^2);
% Enter second profit matrix P2 and cost matrix D2
P2 = [14;9;11;9;12;9;6;15;6;9;12;10];
D2 = [12;14;7;17;6;11;13;10;9;15;14;16];
% Enter optimal solution of second objective function
X2 = [5;5;5;0;10;0;0;15;0;20;0;0];
% Convert fractional objective function as linear using Taylors Series
C2P = P2'*(D2'*X2);
C2D = D2'*(P2'*X2);
Z2 = (C2P -C2D)/((D2'*X2)^2);
% Otain values of objective function at each of the solution.
U1 = (P1'*X1) /(D1'*X1); L1 = (P1'*X2 )/(D1'*X2);
U2 = (P2'*X2)/(D2'*X2); L2 = (P2'*X1)/(D2'*X1);
% Define Fuzzy set functions for each of the linear objective functions.
PHI1 = Z1/(U1 - L1);
PHI2 = Z2/(U2 -L2);
% Enter constraint Matrices Aeq, Beq, A, B.
Aeq = [ 1,1,1,1,0,0,0,0,0,0,0,0;
0,0,0,0,1,1,1,1,0,0,0,0;
0,0,0,0,0,0,0,0,1,1,1,1;
1,0,0,0,1,0,0,0,1,0,0,0;
0,1,0,0,0,1,0,0,0,1,0,0;
```

```
0,0,1,0,0,0,1,0,0,0,1,0;
0,0,0,1,0,0,0,1,0,0,0,1;
1,1,1,1,1,1,1,1,1,1,1,1];
Beq =[ 15;25;20;15;25;5;15;60];
A = [ 0,0,0,0,0,0,0,0,0,0,0,0;
0,0,0,0,0,0,0,0,0,0,0,0;
0,0,0,0,0,0,0,0,0,0,0,0;
0,0,0,0,0,0,0,0,0,0,0,0;
0,0,0,0,0,0,0,0,0,0,0,0;
0,0,0,0,0,0,0,0,0,0,0,0;
0,0,0,0,0,0,0,0,0,0,0,0;
0,0,0,0,0,0,0,0,0,0,0,0];
B = [0;0;0;0;0;0;0;0];
% Enter non negativity constraint
% Upper bound for each of the variable is 60
lb = [0;0;0;0;0;0;0;0;0;0;0;0];
ub = [60;60;60;60;60;60;60;60;60;60;60;60];
% Define aggregation operator to find compromise solution
% Linprog minimizes objective function but here we need to maximize.
% Multiply each of the objective function by (-1)
% Find compromise solution for each of the alpha.
ALPHA = ALPHA +.1;
F = (-1)* ALPHA*PHI1 + (-1)*(1-ALPHA)*PHI2;
CS = linprog(F,A,B,Aeq,Beq,lb,ub);
Q1 = (P1'*CS)/(D1'*CS);
Q2 = (P2'*CS)/(D2'*CS);
ALPHA
CS'
Q1
Q2
end;
```

Step 4: Five different solutions for MOLFTPP obtained using above algorithm

Alpha	0.1 and 0.2	0.3	0.4 and 0.5	0.6	0.7 t0 1.0
x_{11}	5	5	0	15	0
x_{12}	5	10	15	0	0
x_{13}	5	0	0	0	0
x_{14}	0	0	0	0	15
x_{21}	10	10	15	0	0
x_{22}	0	0	10	25	25
x_{23}	0	0	0	0	0
x_{24}	15	15	0	0	0
x_{31}	0	0	0	0	15
x_{32}	20	15	0	0	0
x_{33}	0	5	5	5	5
x_{34}	0	0	15	15	0
$Q_1(X)$	0.6038	0.7303	1.1000	1.1282	1.3143
$Q_2(X)$	1.0296	0.9929	0.8531	0.8431	0.6939

Note that optimal solution of $Q_1(x)$ and PHI1(x) are same, also optimal solution of $Q_2(x)$ and PHI2(x) are same.

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