Original Article

Study of BFS and DFS in logistic map

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Abstract The map $f(x) = m\,x(1-x)$, is known as the LOGISTIC map where m is the control parameter. This map have been extensively studied by many researchers, and rich contents of bifurcations and chaos have been explored. Logistic map is one of the well known maps and has became a standard map for studying bifurcations and chaos of discrete dynamical systems. In this paper we have verified the B.F.S. and D.F.S algorithm and their scenario in the logistic map $f(x) = m\,x(1-x), 0 < x < 1 \; and \; 0 < m \leq 4$
Keywords: Bifurcation, accumulation point, B.F.S, D.F.S.

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INTRODUCTION

Logistic Map

The map $f(x) = m\,x(1-x)$, is known as the LOGISTIC map, which helps to study the population growth subject to the resources. Here m is the control parameter. This map have been extensively studied by many researchers, and rich contents of bifurcations and chaos have been explored. Logistic map is one of the well known maps and has became a standard map for studying bifurcations and chaos of discrete dynamical systems. We list some references in[1-9]. In general, x and m of logistic map are restricted in $0 \leq x \leq 1 \; and \; 0 < m \leq 4$ so that each x in the interval [0,1] is mapped onto the same interval [0,1]. It is known that there are stable fixed points $\boldsymbol{x^* = 0 \; and \; x^* = 1 - 1/m}$ in the interval of our interest. After that, we have period-doubling bifurcation at m = 3, 3.4494897, 3.54409... . These numerical results are well known and can be reproduced through computer programs. In logistic map the accumulation point is given by m =3.566945672.....

SOME BASIC CONCEPTS

Fixed Point

Let $\boldsymbol{f}: \boldsymbol{X} \longrightarrow \boldsymbol{X}$ be a dierentiable map where X is an interval on the real line. point $x^\star \in \boldsymbol{X}$ is called a fixed point of X is $f(\boldsymbol{x}^*) = \boldsymbol{x}^*$ [1,5]. In this paper, our mathematical model is $f(x) = m\,x(1-x)$, where x ∈ [0,1] and μ ∈ (1,4]. Clearly solution of $f(x) = x$ gives the fixed points of f. A fixed point x is said to be a

1. Stable fixed point or attractor if. $|f'(x)| < 1$,

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2. Unstable fixed point or repeller if. $|f'(x)| > 1$,
3. Superattractive or superstable if $f'(x) = 0$

Bifurcation and Bifurcation Point:

The word 'Bifurcation' literally means splitting into two parts. It is often desirable to know how the fixed points of a system change when a parameter of the system is changed. Normally a gradual variation of a parameter in the system corresponds to the gradual variation of the solutions of the problem. However, there exist a large number of problems for which the number of solutions changes abruptly and the structure of solution manifolds varies dramatically when a parameter passes through some critical values (fixed values). This qualitative change in the structural behaviour of the system is called bifurcation, an originally French word introduced by Poincare [1899], and these parameter values are called bifurcation values or bifurcation points $f'(x) = -1$ indicates bifurcation for unimodel map.[2,4,8]

Bifurcation diagram

Qualitative changes in the system dynamics is called bifurcations, and the parameter values at which they occur are called bifurcation points. In our study for the logistic map f(x) = mx (1-x), for the values of parameter m just below 3.0 the orbits converge to a stable fixed point. When the value of m exceeds 3.0, the fixed point becomes unstable, and the orbits converge to a stable period-2 orbit, which is created at m=3.0. Therefore, we say that m=3.0 is a bifurcation point of the map f(x).The bifurcation occurs at m=3.0 is called period-doubling bifurcation, which is one of many types of bifurcations that can occur in dynamical systems.

Chaos

Generally there is no generally accepted definition of Chaos. From a practical point of view, chaos can be defined as bounded steady state behaviour that is not an equilibrium point, not periodic, and not quasi periodic. The trajectories are, indeed bounded. They are not periodic and they do not have uniform distribution characteristic of quasi- periodic solutions. A noise like spectrum is a characteristic of chaotic systems. Another important fact about the chaotic systems is that the limit set for chaotic behaviour is not a simple geometrical object like circle or torous, but is related to fractals. [5,6] In short chaos can be defined as efectively unpredictable long time behaviour arising in a deterministic dynamical system because of sensitivity to the initial conditions. The key to long-term unpredictability is a property known as sensitivity to (or sensitive dependence on) initial conditions. Examples of such systems include the atmosphere, the solar system, plate tectonics, turbulent fluids, economic and population growth.[4,5] For Devaney, chaos is seen as mixing of unpredictability and regular behaviors: a system is chaotic in the sense of Devaney if it is transitive, sensitive to initial conditions and has a dense set of periodic points[2].

Breadth-first search (BFS)

B.F.S is an algorithm for traversing or searching tree or graph data structures. It starts at the tree root (or some arbitrary node of a graph, sometimes referred to as a `search key') and explores the neighbor nodes first, before moving to the next level neighbours. BFS was invented in the late 1950s by **E. F. Moore** who used it to find the shortest path out of a maze, and discoverd independently by **C. Y. Lee** as a wire routing algorithm (published 1961).

Depth-first search (DFS)

D.F.S is an algorithm for traversing or searching tree or graph data structures. One starts at the root (selecting some arbitrary node as the root in the case of a graph) and explores as far as possible along each branch before backtracking.
A version of depth-first search was investigated in the 19th century by French mathematician **Charles Pierre Trémaux** as a strategy for solving mazes.

OUR FINDINGS

The node values are: -
When m =3, m1=0.75,
For m=3.449, m2=0.86225.
For m=3.544, m3=.886.
For m=3.5699, m4=.892475
For m=3.84, m5=0.96 etc.

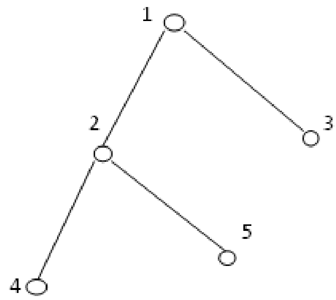

Figure 1: Complete tree of the nodes

ALGORITHEM

(A).B.F.S.: The general idea behind b.f.s beginning at a starting vertex (1) as follows.

Step I. First we process the starting vertex (1). Then we process all the neighbours of (1).And so on.

Step II: Naturally we need to keep track of the neighbours of vertex, and we need to guarantee that no vertex is processed twice.

Step III: This accomplished by using a QUEUE to hold vertices that are waiting to be processed, and by a field STATUS which tells us the current status of a vertex.

FOR Breadth first search :

The sequence of nodes are:

(1). 0.75, (2) 0.86225 (3),0.886, (4) 0.892475, (5) 0.96

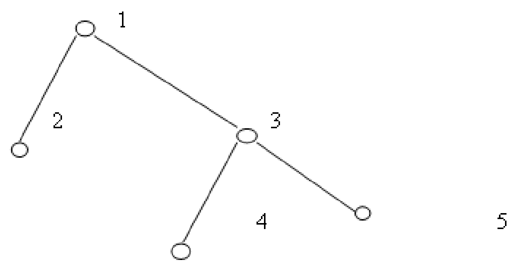

Figure 2: B.F.S tree of the nodes

(B) D.F.S: The general idea behind d.f.s. Beginning at a starting vertex (1) is as follows. **Step I.** First we process the starting vertex (1).Then we process each vertex along a path which begins at (1), that is, we process a neighbour of (1) then a neighbour of a neighbour of (1),and so on **Step II**. After coming to a dead end, that is to a vertex with no unprocessed neighbour. we backtrack on the path until we can continue along another path and so on. **Step III**. THE backtracking is accomplished by using a STACK to hold the initial vertices of future possible paths. We also need a field STATUS which tells us the current STATUS of any vertex so that no vertex is processed more than once. For Depth first search:

The sequence of nodes are: (*1*). 0.75, (2) 0.886, (3). 0.86225, (4).0.96, (5). 0.892475

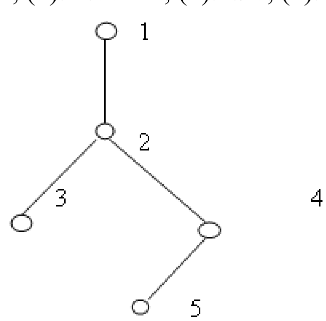

Figure 3: D.F.S tree of the nodes

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