Original Article

Cost-benefit analysis of two-identical cold standby system subject to failure due to heavy rainfall and strong winds

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Abstract In this paper we have taken failure due to Heavy rainfall and strong winds. When the main unit fails due to Heavy rainfall then cold standby system becomes operative. Failure due to strong winds cannot occur simultaneously in both the units and after failure the unit undergoes Type-1 or Type-2 or Type-3 repair facility immediately. Applying the regenerative point technique with renewal process theory the various reliability parameters MTSF, Availability, Busy period, Benefit-Function analysis have been evaluated.
Keywords: Cold Standby, failure due to Heavy rainfall and strong winds, first come first serve, MTSF, Availability, Busy period, Benefit -Function.

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INTRODUCTION

Heavy rain fall and strong winds make the human resource handicapped due to failure of Function for example in our college annual function 2015-16 was badly affected due to heavy rainfall and strong winds. Similar was the case on 15th August, 2015, the Independence Day in Tosham, District Bhiwani, Haryana when the chief guest came on Dias for delivering his speech heavy rainfall and strong winds make the Function totally disturbed. Stochastic behavior of systems operating under changing environments has widely been studied. Dhillon, B.S. and Natesan, J. (1983) studied an outdoor power systems in fluctuating environment. Kan Cheng (1985) has studied reliability analysis of a system in a randomly changing environment. Jinhua Cao (1989) has studied a man machine system operating under changing environment subject to a Markov process with two states. The change in operating conditions viz. fluctuations of voltage, corrosive atmosphere, very low gravity etc. May make a system completely inoperative. Severe environmental conditions can make the actual mission duration longer than the ideal mission duration. In this paper we have taken failure due to Heavy rainfall and failure due to strong winds. When the main operative unit fails then cold standby system becomes operative. Failure due to strong winds cannot occur simultaneously in both the units and after failure the unit undergoes repair facility of Type- 2 by ordinary repairman or Type-3 by disaster management repairman in case of failure due to strong winds immediately. The repair is done on the basis of first fail first repaired.

ASSUMPTIONS

1. λ_1, λ_2 are constant failure rates for Heavy rainfall and failure due to strong winds respectively. The CDF of repair time distribution of Type 1, Type 2 and disaster management repairmen Type-3 are $F_1(t)$, $F_2(t)$ and $F_3(t)$.
2. The failure due to strong winds is non-instantaneous and it cannot come simultaneously in both the units.
3. The repair starts immediately after failure due to Heavy rainfall and failure due to strong winds and works on the principle of first fail first repaired basis.
4. The repair facility does no damage to the units and after repair units are as good as new.
5. The switches are perfect and instantaneous.
6. All random variables are mutually independent.

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7. When both the units fail, we give priority to operative unit for repair.
8. Repairs are perfect and failure of a unit is detected immediately and perfectly.
9. The system is down when both the units are non-operative.

SYMBOLS FOR STATES OF THE SYSTEM

Superscripts: O, CS, FHR, FSW.

Operative, Cold Standby, failure due to Heavy rainfall and failure due to strong winds respectively

Subscripts: nfhr, fhr, fsw, ur, wr, uR

No failure due to heavy rainfall, failure due to heavy rainfall, failure due to strong winds, under repair, waiting for repair, under repair continued from previous state respectively

Up states: 0, 1, 2, 3, 8,9; Down states: 4, 5, 6, 7; Regeneration point: 0,1,2, 3, 8, 9

States of the System

0(O_{nfhr}, CS_{nfhr}) One unit is operative and the other unit is cold standby and there is no failure due to Heavy rainfall of both the units.

1($FHR_{fhr,\ urI}$, O_{nfhr}) The operating unit failed due to Heavy rainfall is under repair immediately of Type- 1 and standby unit starts operating with no failure due to strong winds

2($FSW_{fsw,\ urII}$, O_{nfhr}) The operative unit failed due to strong winds and undergoes repair of type-2 and the standby unit becomes operative with no failure due to heavy rainfall.

3($FSW_{fsw,\ urIII}$, O_{nfhr}) The first unit failed due to strong winds and under Type-3 disaster management repairman and the other unit is operative with no failure due to.

4($FHR_{\ fhr,uR1}$, $FHR_{\ fhr,wrI}$) The unit failed due to FHR resulting from failure due to Heavy rainfall under repair of Type- 1 continued from state 1and the other unit failed due to FHR resulting from failure due to Heavy rainfall is waiting for repair of Type-1.

5($FHR_{\ fhr,uR1}$, $FSW_{fsw,\ wrII}$) The unit failed due to FHR resulting from failure due to Heavy rainfall is under repair of Type-1 continued from state 1and the other unit fails due to insufficient control gain is waiting for repair of Type- 2.

6($FSW_{fsw,\ uRII}$, $FHR_{\ fhr,\ wrI}$) The operative unit failed due to insufficient control gain is under repair continues from state 2 of Type –2 and the other unit failed due to FHR resulting from failure due to Heavy rainfall is waiting under repair of Type-1

7($FSW_{fsw,\ uRII}$, $FHR_{fhr,wrII}$) The one unit failed due to insufficient control gain is continued to be under repair of Type 2 and the other unit failed due to FHR resulting from failure due to Heavy rainfall is waiting for repair of Type-2

8($FHR_{fhr,urIII}$, $FSW_{fsw,\ wrII}$) The one unit failed due to Heavy rainfall is under disaster management repair of Type-3 and the other unit failed due to strong winds is waiting for repair of Type-2.

9($FHR_{fhr,urIII}$, $FSW_{fsw,\ wrI}$) The one unit failed due to Heavy rainfall is under disaster management repair of Type-3 and the other unit failed due to strong winds is waiting for repair of Type-1

TRANSITION PROBABILITIES

Simple probabilistic considerations yield the following expressions:

$p_{01} = \lambda_1 / \lambda_1 + \lambda_2$, $p_{02} = \lambda_2 / \lambda_1 + \lambda_2$, $p_{10} = pF_1^*(\lambda_1)+q F_2^*(\lambda_2)$, $p_{14} = p- pF_1^*(\lambda_1) = p_{11}^{(4)}$,

$p_{15} = q- q F_1^*(\lambda_2) = p_{12}^{(5)}$, $p_{23} = pF_2^*(\lambda_1)+q F_2^*(\lambda_2)$, $p_{26} = p- pF_2^*(\lambda_1) = p_{29}^{(6)}$,

$$p_{27} = q- qF_2^*(\lambda_2) = p_{28}^{(7)},\ p_{30} = p_{82} = p_{91} = 1 \quad (1)$$

We can easily verify that

$$p_{01} + p_{02} = 1,\ p_{10} + p_{14}\ (=p_{11}^{(4)}) + p_{15}\ (=p_{12}^{(5)}) = 1,$$
$$p_{23} + p_{26}\ (=p_{29}^{(6)}) + p_{27}\ (=p_{28}^{(7)}) = 1 \quad (2)$$

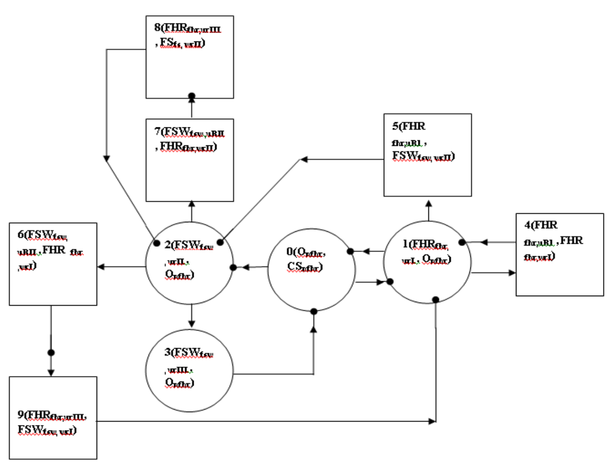

Figure 1: The State Transition Diagram

⬭ Up-State ☐ Down-State • Regeneration point

And mean sojourn time is

$\mu_0 = E(T) = \int_0^\infty P[T > t]dt$

MEAN TIME TO SYSTEM FAILURE

$Ø_0(t) = Q_{01}(t)[s]\, Ø_1(t) + Q_{02}(t)[s]\, Ø_2(t)$,

$Ø_1(t) = Q_{10}(t)[s]\, Ø_0(t) + Q_{14}(t) + Q_{15}(t)$

$Ø_2(t) = Q_{23}(t)[s]\, Ø_3(t) + Q_{26}(t) + Q_{27}(t)$,

$Ø_3(t) = Q_{30}(t)[s]\, Ø_0(t)$ (3-6)

We can regard the failed state as absorbing Taking Laplace-Stiljes transform of eq. (3-6) and solving for

$ø_0^*(s) = N_1(s) / D_1(s)$ (7)

where

$N_1(s) = Q_{01}^*[\, Q_{14}^*(s) + Q_{15}^*(s)\,] + Q_{02}^*[\, Q_{26}^*(s) + Q_{27}^*(s)\,]$

$D_1(s) = 1 - Q_{01}^* Q_{10}^* - Q_{02}^* Q_{23}^* Q_{30}^*$

Making use of relations (1) & (2) it can be shown that $ø_0^*(0) = 1$, which implies that $ø_0(t)$ is a proper distribution.

$MTSF = E[T] = \left.\frac{d}{ds} ø_0^*(s)\right|_{s=0}$

$= (D_1'(0) - N_1'(0)) / D_1(0)$

$= (\mu_0 + p_{01}\mu_1 + p_{02}\mu_2) / (1 - p_{01}p_{10} - p_{02}p_{23})$

where

$\mu_0 = \mu_{01} + \mu_{02}$, $\mu_1 = \mu_{10} + \mu_{11}^{(4)} + \mu_{12}^{(5)}$, $\mu_2 = \mu_{23} + \mu_{28}^{(7)} + \mu_{29}^{(6)}$

AVAILABILITY ANALYSIS

Let $M_i(t)$ be the probability of the system having started from state i is up at time t without making any other regenerative state. By probabilistic arguments, we have

$M_0(t) = e^{-\lambda_1 t} e^{-\lambda_2 t}$, $M_1(t) = p\, F_1(t)\, e^{-\lambda_1 t}$, $M_2(t) = q\, F_2(t)$, $M_3(t) = F_3(t)$

The point wise availability $A_i(t)$ have the following recursive relations

$A_0(t) = M_0(t) + q_{01}(t)[c]A_1(t) + q_{02}(t)[c]A_2(t)$

$A_1(t) = M_1(t) + q_{10}(t)[c]A_0(t) + q_{12}^{(5)}(t)[c]A_2(t) + q_{11}^{(4)}(t)[c]A_1(t)$,

$A_2(t) = M_2(t) + q_{23}(t)[c]A_3(t) + q_{28}^{(7)}(t)[c]\, A_8(t) + q_{29}^{(6)}(t)]\,[c]A_9(t)$

$A_3(t) = M_3(t) + q_{30}(t)[c]A_0(t)$, $A_8(t) = q_{82}(t)[c]A_2(t)$,

$A_9(t) = q_{91}(t)[c]A_1(t)$ (8-12)

Taking Laplace Transform of eq. (8-12)

and solving for $\hat{A}_0(s)$

$\hat{A}_0(s) = N_2(s) / D_2(s)$ (13)

where

$N_2(s) = \hat{M}_0 [\{1 - \hat{q}_{11}^{(4)}\}\{1 - \hat{q}_{28}^{(7} \hat{q}_{82}\} - \hat{q}_{12}^{(5)} \hat{q}_{29}^{(6)} \hat{q}_{91}] + \hat{q}_{01}[\hat{M}_1\{1 - \hat{q}_{28}^{(7)} \hat{q}_{82}\} + \hat{q}_{12}^{(5)} \hat{q}_{23} \hat{M}_3] + \hat{q}_{02}[\{\hat{q}_{23} \hat{M}_3 + \hat{M}_2\}\{1 - \hat{q}_{11}^{(4)}\} + \hat{q}_{29}^{(6)}) \hat{q}_{91} \hat{M}_1]$

$D_2(s) = \{1 - \hat{q}_{11}^{(4)}\}\{1 - \hat{q}_{28}^{(7} \hat{q}_{82}\} - \hat{q}_{12}^{(5)} \hat{q}_{29}^{(6)} \hat{q}_{91} - \hat{q}_{01}[\hat{q}_{10}\{1 - \hat{q}_{28}^{(7)} \hat{q}_{82}\} + \hat{q}_{12}^{(5)} \hat{q}_{23}] - \hat{q}_{02}[\{\hat{q}_{23} \hat{q}_{30}\{1 - \hat{q}_{11}^{(4)}\} + \hat{q}_{29}^{(6)} \hat{q}_{91} \hat{q}_{10}]$

(Omitting the arguments s for brevity)

The steady state availability

$A_0 = \lim_{t\to\infty}[A_0(t)] = \lim_{s\to 0}[s\,\hat{A}_0(s)] = \lim_{s\to 0} \frac{s\,N_2(s)}{D_2(s)}$

Using L' Hospitals rule, we get

$A_0 = \lim_{s\to 0} \frac{N_2(s) + s\, N_2'(s)}{D_2'(s)} = \frac{N_2(0)}{D_2'(0)}$ (14)

The expected up time of the system in (0,t] is

$\lambda_u(t) = \int_0^\infty A_0(z)dz$ So that $\overline{\lambda_u}(s) = \frac{\hat{A}_0(s)}{s} = \frac{N_2(S)}{SD_2(S)}$ (15)

The expected down time of the system in (0,t] is

$\lambda_d(t) = t - \lambda_u(t)$ So that $\overline{\lambda_d}(s) = \frac{1}{s^2} - \overline{\lambda_u}(s)$ (16)

Similarly, we can find out

1. The expected busy period of the server when there is failure due to heavy rainfall and strong winds in (0,t]-B_0
2. The expected number of visits by the repairman Type-1 or Type-2 for repairing the identical units in (0,t]-V
3. The expected number of visits by the disaster management repairman Type-3 for repairing the identical units in (0,t]-W_0

BENEFIT- FUNCTION ANALYSIS

The Benefit-Function analysis of the system considering mean up-time, expected busy period of the system under failure due to heavy rainfall and failure due to strong winds, expected number of visits by the repairman for unit failure.

$C = \lim_{t\to\infty}(C(t)/t) = \lim_{s\to 0}(s^2 C(s)) = S_1A_0 - S_2B_0 - S_3V_0 - S_4W_0$

where

S_1: Revenue per unit up-time, S_2 - cost per unit time for which the system is busy under repairing,

S_3: Cost per visit by the repairman type- 1 or type- 2 for units repair,

S_4: Cost per visit by the disaster management repairman Type- 3 for units repair

CONCLUSION

After studying the system, we have analyzed graphically that when the failure rate due to heavy rainfall and strong winds increases, the MTSF, steady state availability decreases and the Profit-function decreased as the failure increases.

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