Original Article

Heat transfer in visco-elastic fluid through rotating porous channel with Hall effect in presence of heat generation/absorption

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Abstract This paper presents the unsteady hydromagnetic oscillating viscous incompressible flow of visco-elastic fluid with heat transfer in a vertical channel saturated with a porous medium in a rotating system in the presence of strong uniform magnetic field normal to the plates has been studied on taking Hall currents into account. The entire system rotates with uniform angular velocity (Ω) about the axis perpendicular to the plates. The governing equations are solved by perturbation technique and the method of ordinary differential equation to obtain an analytical result for velocity and temperature profile, and results are presented graphically for various values of visco elastic parameter (K_2), Prandtl number (Pr), radiation parameter (N), heat generation/absorption parameter (Q_H) and Hall parameter (m). The Skin friction coefficient and the Nusselt numbers are presented in Table (1) and Table (2) respectively.
Keywords: Visco-elastic fluid, MHD, Hall effect, Porous medium.

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INTRODUCTION

Hydromagnetic convection with heat transfer in a rotating medium through porous medium has important applications in geophysics, nuclear power reactors, in underground water sources and energy storage system. When the strength of the magnetic field is strong, one cannot neglect the effects of Hall currents, the Hall effects due to the gyration of the electrons become important. The current component created by the anisotropic conductivity is known as the Hall current. In the presence of a uniform magnetic field, this effect modifies the Ohm's law, describing the motion and yields a dispersive character to the flow. A comprehensive discussion of Hall current is given by Cowling[1], Soundalgekar[2], Soundalgekar and Uplekar[3]. Hossain and Rashid[4] analyzed Hall effect of MHD free convective flow along porous plate with mass transfer. Attia[5] studied Hall current on the velocity and temperature fields on unsteady Hartmann flow. Effects of Hall current on free convective flow past on accelerated vertical plate in a rotating system with heat source/sink is analyzed by Singh and Garg[6]. Saha *et al.*[7] perceived the Hall current effect on MHD natural convection from a vertical plate. Aboeldahad and Elbarbary[8] examined heat and mass transfer over a vertical plate in the presence of magnetic field and Hall effect. Abo-Eldahab and El Aziz[9] explored the Hall current and Joule heating effects on electrically conducting fluid past a semi-infinite plate with strong magnetic field and heat generation/absorption. Radiation effects on free convection flow have become very important due to its applications in space technology, processes having high temperature and design of pertinent equipments. Moreover, heat and mass transfer by thermal radiation on convective flows is very important due to its

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significant role in the surface heat transfer. Recent developments in gas cooled nuclear reactors, nuclear power plants, gas turbines, space vehicles, and hypersonic flights have attracted research in this field. The unsteady convective flow in a moving plate with thermal radiation was examined by Cogley *et al*.[10] and Mansour[11]. The combined effects of radiation and buoyancy force past a vertical plate were analyzed by Hossain and Takhar[12]. Hossain *et al*. [13] considered the influence of thermal radiation on convective flows over a porous vertical plate. Seddeek[14] explained the importance of thermal radiation and variable viscosity on unsteady forced convection with an align magnetic field. Muthucumaraswamy and Senthil[15] studied the effects of thermal radiation on heat and mass transfer over a moving vertical plate. Pal[16] investigated convective heat and mass transfer in a stagnation-point flow towards a stretching sheet with thermal radiation. Aydin and Kaya[17] justified the effects of thermal radiation on mixed convection flow over a permeable vertical plate with magnetic field. Mohamed[18] studied unsteady MHD flow over a vertical moving porous plate with heat generation and Soret effect. Chauhan and Rastogi[19] analyzed the effects of thermal radiation, porosity and suction on unsteady convective hydromagnetic vertical rotating channel. Ibrahim and Makinde[20] probed radiation effect on chemically reaction MHD boundary layer flow of heat and mass transfer past a porous vertical flat plate. Pal and Mondal[21] studied the effects of thermal radiation on MHD Darcy-Forchheimer convective flow past a stretching sheet in a porous medium. MHD flow in rotating system has received wide attention to many researchers due to is varied and wide applications in many areas of science of technology which are directly governed by the action of Coriolis force. The Coriolis force effect has great significance in subjects of oceanography, atmospheric science, meteorology, stellar dynamics, in which it is a controlling factor in the direction of rotation of sunspot, and is thus considerable practical and theoretical interest in engineering and science such as MHD boundary layer control of reentry vehicles, MHD pumps, MHD generators and many more. Several investigations have been carried out in hydrodynamic rotating system considering various variation and configurations. Greenspan[22], Walin[23], Siegmann[24], Hayat and Hutter[25], Singh *et al*.[26]. The same problem with magnetic field is explained by many researchers, namely Singh[27], Hossain *et al*.[28], Wang and Hayat[29], Hayat and Abelman[30], Hayat *et al*.[31], Abelman *et al*.[32], Seth *et al*[33] examined the transient hydromagnetic Couette flow of a viscous incompressible conducting fluid in a rotating system in the presence of a uniform transverse magnetic field. Ahmed *et al*.[34] discussed the Hartmann Newtonian radiating MHD flow for a rotating vertical porous channel immersed in a Darcian Porous Regime. Despite the above studies, attention has hardly been focused to study the effects of the Hall current on unsteady hydromagnetic Non-Newtonian fluid flows. Such work seems to be important and useful partly for gaining a basic understanding of such flows and partly possible applications of these fluids in chemical process industries, food and construction engineering, movement of biological fluids. Another important field of application is the electromagnetic propulsion. The study of such system, which is closely associated with magneto-chemistry, requires a complete understanding of the equation of state shear stress-shear rate relationship, thermal conductivity and radiation. Some of these properties undoubtedly be influenced by the presence of an external magnetic field. Aldos *et al*.[35] studied MHD mixed convection flow from a vertical plate embedded in porous medium. Rajgopal *et al*.[36] analyzed an oscillatory mixed convection flow of a viscoelastic electrically conducting fluid in an infinite vertical channel filed with porous medium considering the Hall effects Attia[37] conferred unsteady Hartmann flow of a visco-elastic fluid. Chaudhary and Jha[38] analyzed heat and mass transfer in elastic-viscous fluid past an impulsively started infinite vertical plate with Hall current. Singh[39] investigated MHD mixed convection visco-elastic slip-flow through a porous medium in a vertical porous channel with thermal radiation. Gaur and Jha[40] presented the heat and mass transfer in visco-elastic fluid with phase angle through rotating porous channel with Hall effect. The objective of the present study is to analyze the effects of Hall current, thermal radiation, and heat generation/absorption on the oscillatory convective flow of visco-elastic fluids with suction/injection in a rotating vertical porous channel.

MATERIAL AND METHODS

The constitutive equations for the rheological equation of state for an elastico-viscous fluid
(Walter's liquid B') are

$$p_{ik} = -p\, g_{ik} + p'_{ik} \qquad (1)$$

$$p'_{ik} = 2\int_{-\infty}^{t} \psi(t-t')\, e^{(1)}_{ik}(t')\, dt' \qquad (2)$$

in which $\psi(t-t') = \int_0^{\infty} \frac{N(\tau)}{\tau} e^{-(t-t')\tau}\, d\tau$ (3)

N (τ) is the distribution function of relaxation times. In the above equations p_{ik} is the stress tensor, p an arbitrary isotropic pressure, g_{ik} is the metric tensor of a fixed co-ordinate system x_i and $e^{(1)}_{ik}$, the rate of strain tensor. It was shown by Walter's [42] that equation (2) can be put in the following generalized form which is valid for all types of motion and stress

$$p'^{ik}(x,t) = 2\int_{-\infty}^{t} \psi(t-t')\frac{\partial x'^{i}}{\partial x'^{m}}\frac{\partial x^{k}}{\partial x'^{r}} e^{(1)mr}(x't')\, dt' \quad (4)$$

Where, x'^{i} is the position at time t' of the element that is instantaneously at the print x^{i} at time "t". The fluid with equation of state (1) and (4) has been designated as liquid B'. In the case of liquids with short memories, i.e. short relaxation times, the above equation of state can be written in the following simplified form

$$p'^{ik}(x,t) = 2\eta_0 e^{(1)ik} - 2k_0 \frac{\partial e^{(1)ik}}{\partial t}, \quad (5)$$

in which $\eta_0 = \int_0^{\infty} N(\tau)\, d\tau$ is limiting viscosity at small rates of shear, $k_0 = \int_0^{\infty} \tau N(\tau)\, d\tau$ and $\frac{\partial}{\partial t}$ denotes the convected time derivative. We consider Oscillatory free convective flow of a viscous incompressible and electrically conducting fluid between two insulating infinite vertical permeable plates. A strong transverse magnetic field of uniform strength H_0 is applied along the axis of rotation by neglecting induced electric and magnetic fields. The fluid is assumed to be a gray, emitting, and absorbing, but non scattering medium. The radiative heat flux term can be simplified by using the Cogley *et al.*[10].

The equations governing the flow of fluid together with Maxwell's electromagnetic equations are as follows:

Equation of Continuity:

$$\nabla . V = 0 \quad (6)$$

Momentum Equation:

$$\frac{\partial V}{\partial t} + (V\nabla)V + 2\Omega^* V = -\frac{1}{\rho}\nabla P + \nabla . p_{ij} + \frac{1}{\rho}(J \times B) + g\beta^* T \quad (7)$$

In equation (7) the last term on the left side is the Coriolis force and last term on the right hand side in same equation report for the force due to buoyancy. All other quantities have their as usual meaning.

Energy Equation:

$$\rho C_p \left[\frac{\partial T}{\partial t} + (V.\nabla)T\right] = k\nabla^2 T - \nabla q + Q_0 T \quad (8)$$

The generalized Ohm's law, in the absence of the electric field [41], is of the form.

$$\vec{J} + \frac{\omega_e \tau_e}{H_0}(\vec{J} \times \vec{H}) = \sigma\left(\mu_e \vec{V}1 \times \vec{H} + \frac{1}{e n_e}\nabla p_e\right) \quad (9)$$

where $\vec{V}1, \omega_e, \tau_e, \mu_e, n_e, e, \sigma$ and p_e are electron velocity, the electrical conductivity, the magnetic permeability, the cyclotron frequency, the electron collision time, the electric charge, the number density of the electron, and the electron pressure, respectively. Under the usual assumption, the electron pressure (for a weakly ionized gas), the thermoelectric pressure, and ion slip are negligible, so we have from the Ohm's law

$$J_x^* + \omega_e \tau_e J_y^* = \sigma \mu_e H_0 v^* \quad (10)$$

$$J_y^* - \omega_e \tau_e J_x^* = -\sigma \mu_e H_0 u^* \quad (11)$$

From which we obtain that

$$J_x^* = \frac{\sigma\mu_e H_0(mu^* + v^*)}{1+m^2}, J_y^* = \frac{\sigma\mu H_0(mv^* - u^*)}{1+m^2} \quad (12)$$

The Solenoidal relation for the magnetic field $\nabla.\vec{H} = 0$ where $\vec{H} = (\vec{H}_x, \vec{H}_y, \vec{H}_z)$ gives $\vec{H}_z = \vec{H}_0$ (constant) everywhere in the flow, which gives $\vec{H} = (0,0,\vec{H}_0)$. If $(\vec{J}_x, \vec{J}_y, \vec{J}_z)$ are the component of electric current density $\vec{J}$, then the equation of conservation of electric charge $\nabla.\vec{J} = 0$ gives $\vec{J}_z =$ constant. We consider the unsteady MHD free convective flow in a vertical parallel porous plate channel filled with a porous substrate with a visco-elastic electrically conducting fluid. The entire system rotates about the axis normal to the plates with uniform angular velocity Ω^*. The channel is of width d. A cartesian coordinate system is assumed and z^*-axis is taken normal to the plates, while x^*-axis and y^*-axis are in the upward and perpendicular directions on the plate $z^* = 0$ (origin), respectively. Two vertical plates are situated at $z^* = 0$ and $z^* = d$. These plates are subjected to a constant injection/suction velocity w_o at one plate $z^* = 0$ and the same constant injection/ suction velocity at $z^* = d$. A uniform magnetic field H_0 is applied along an axis (z^* axis) normal to the plates and the entire system rotates about this axis. Since the plates are infinite in extent, all the physical quantities except the pressure depend only on z^* and t^*. The velocity components u^*, v^*, w_0 are in the x^*, y^*, z^* directions respectively. The governing equations in the rotating system in presence of Hall current, thermal radiation and heat generation/absorption are given by the following equations.

$$\frac{\partial u^*}{\partial t^*}+w_0\frac{\partial u^*}{\partial z^*}-2\Omega^* v^*=-\frac{1}{\rho}\frac{\partial p^*}{\partial x^*}+\nu\frac{\partial^2 u^*}{\partial z^{*2}}-\nu\frac{u^*}{K_p^*}+\frac{\mu_e H_0}{\rho}J_y^*-K_0\frac{\partial^3 u^*}{\partial z^{*2}\partial t^*}+gBT^*. \quad (13)$$

$$\frac{\partial v^*}{\partial t^*}+w_0\frac{\partial v^*}{\partial z^*}+2\Omega^* u^*=-\frac{1}{\rho}\frac{\partial p^*}{\partial y^*}+\nu\frac{\partial^2 v^*}{\partial z^{*2}}-\nu\frac{v^*}{K^*_p}-\frac{\mu_e H_0}{\rho}J_x^*-K_0\frac{\partial^3 v^*}{\partial z^{*2}\partial t^*} \quad (14)$$

$$-\frac{1}{\rho}\frac{\partial p^*}{\partial z^*}=0 \quad (15)$$

$$\frac{\partial T^*}{\partial t^*}+w_0\frac{\partial T^*}{\partial z^*}=\frac{k}{\rho c_p}\frac{\partial^2 T^*}{\partial z^{*2}}+\frac{Q_0}{\rho c_p}T^*-\frac{1}{\rho c_p}\frac{\partial q_r^*}{\partial z^*} \quad (16)$$

The last term of equation (16) accounts for radiative heat flux.

Following Cogley *et al.*[10], it is assumed that the fluid is optically thin with a low density and the heat flux due to radiation in Equation (16) is given by

$$\frac{\partial q_r^*}{\partial z^*}=4\propto^2 T^* \quad (17)$$

Where, α is the radiation absorption coefficient. After the substitution of Equation (17) into the equation (16):

$$\frac{\partial T^*}{\partial t^*}+w_0\frac{\partial T^*}{\partial z^*}=\frac{k}{\rho c_p}\frac{\partial^2 T^*}{\partial z^{*2}}+\frac{Q_0}{\rho c_p}T^*-\frac{1}{\rho c_p}4\alpha^2 T^* \quad (18)$$

Equation (15) illustrates the constancy of the hydrodynamic pressure along the axis of rotation. We shall assume now that the fluid flow under the influence of pressure gradient varying periodically with time in the x^* axis is of the form:

$$-\frac{1}{\rho}\frac{\partial p^*}{\partial x^*}=\mathrm{P}\cos\omega^* t^* \quad (19)$$

Where m ($=\omega_e\ \tau_e$) is the Hall parameter, β is coefficients of thermal expansion, c_p is the specific heat at constant pressure, ρ is the density of the fluid, ν is the kinematics viscosity, k is the fluid thermal conductivity, g_0 is the acceleration of gravity, Q_0 is the additional heat source, q_r^* is the radiative heat flux

The initial and boundary conditions as suggested by the physics of the problem are:

$$u^*=v^*=0 \quad T^*=T_0\cos\omega^* t^*, \text{ at } z^*=d$$

$$u^*=0, v^*=0, T^*=0 \text{ at } z^*=0 \quad (20)$$

we, now introduce the dimensionless variables and parameters as follows:

$$\eta=\frac{z^*}{d},\ u=\frac{u^*}{w_0},\ v=\frac{v^*}{w_0},\quad t=t^*\omega^*,\ \omega=\frac{\omega^* d^2}{\nu},$$

$$\Omega=\frac{\Omega^* d^2}{\nu}\ \lambda=\frac{w_0 d}{\nu},\ \theta=\frac{T^*}{T_0},\ .\mathrm{p}=\frac{p^*}{\rho w_0^2} \quad (21)$$

After combining (13) and (14) and taking $q=u+iv$, then (13) – (16) reduce to:

$$\omega\frac{\partial q}{\partial t}+\lambda\frac{\partial q}{\partial\eta}=\frac{\partial^2 q}{\partial\eta^2}+\lambda p\cos t-(2i\Omega+1/kp)q-\frac{M^2(1+im)}{1+m^2}q+Gr\,\theta-K_2 w\frac{\partial^3 q}{\partial\eta^2\partial t} \quad (22)$$

$$\omega\frac{\partial\theta}{\partial t}+\lambda\frac{\partial\theta}{\partial\eta}=\frac{1}{\Pr}\frac{\partial^2\theta}{\partial\eta^2}-\frac{N^2\theta}{\Pr}+\frac{Q_H}{\Pr}\theta \quad (23)$$

Where, $Gr=g_0\beta\,d^2T_0/w_0\,\nu$ is the modified Grashofff number, $\Pr=\nu\rho c_p/k$ is the Prandtl number,, $M=H_0\mu_e d\sqrt{\sigma/\mu}$ is the Hartmann number, $N^2=4\frac{\alpha^2 d^2}{k}$ is the radiation parameter. $k_2=k_0/d^2$ is the visco-elastic parameter and, $K_p=\frac{K^*_p}{d^2}$ is the permeability of porous medium., Ω is the rotation parameter, λ is the injection/suction parameter and $Q_H=\frac{Q_0 d^2}{k}$ is the heat generation/absorption parameter.

For the oscillatory internal flow we shall assume that the fluid flow under the influence of non-dimension pressure gradient such as $-\frac{\partial p}{\partial x}=p\cos t$ and $-\frac{\partial p}{\partial y}=0$ (24)

The boundary conditions in the dimensionless form can be expressed in form as:

u=0, v=0, $\theta=\cos t$ at $\eta=1$

u=0, v=0, $\theta=0$ at $\eta=0$ (25)

Method of Solution:

The set of partial differential equations (22) and (23) cannot be solved in closed form. Now, it is solved analytically after these equations are reduced to a set of ordinary differential equations in dimensionless form.

We assume that

$R(\eta,t)=R_0(\eta)\,e^{it}$ and

$$-\frac{\partial p}{\partial x}=p\cos t=pe^{it},\ -\frac{\partial p}{\partial y}=0 \quad (26)$$

Where R stands for q or θ.

Substituting (26) into (22)-(23) and comparing the likewise term, we obtain the following ordinary differential equations:

$$(1-K_2wi)q''_{0'}-\lambda q'_0-Sq_0=-(Gr\theta_0+\lambda P) \quad (27)$$

$$\theta''_{0'}-\lambda \Pr\theta'_0-(N^2-Q_H+w\Pr i)\theta_0=0 \quad (28)$$

Where,

$S=(1/Kp)+wi+M^2(1+im)/(1+m^2)+2i\Omega$ and dashes denote the derivatives w.r.t η.

The Transformed boundary conditions are:

$q_0=0,\ \theta_0=0$ at $\eta=0$

$q_0=0,\ \theta_0=1$ at $\eta=1$ (29)

The solution of equations (27)-(28) under the boundary conditions (29):

$$q_0=C_3e^{m_3\eta}+C_4e^{m_4\eta}-(A_1e^{m_1\eta}-A_2e^{m_2\eta})+A_3 \quad (30)$$

$$\theta_0=\frac{e^{m_1\eta}-e^{m_2\eta}}{e^{m_1}-e^{m_2}} \quad (31)$$

From equation (26), we have:

$$q(\eta,t)=q_0(\eta)e^{it} \quad (32)$$

$$\theta(\eta,t)=\theta_0(\eta)e^{it} \quad (33)$$

Where,

$$m_1=\frac{\Pr\lambda+\sqrt{(\Pr\lambda)^2+4(N^2-Q_H+w\Pr i)}}{2}$$

$$m_2=\frac{\Pr\lambda-\sqrt{(\Pr\lambda)^2+4(N^2-Q_H+w\Pr i)}}{2}$$

$$m_3=\frac{\lambda+\sqrt{(\lambda)^2+4S(1-K_2wi)}}{2(1-K_2wi)}$$

$$m_4=\frac{\lambda-\sqrt{(\lambda)^2+4S(1-K_2wi)}}{2(1-K_2wi)}$$

$$C_3=\frac{1}{(e^{m_4}-e^{m_3})}\times\left[A_1(e^{m_4}-e^{m_1})+A_2(e^{m_2}-e^{m_4})-A_3(1-e^{m_4})\right]$$

$$C_4=\frac{1}{(e^{m_4}-e^{m_3})}\times\left[A_1(e^{m_1}-e^{m_3})+A_2(e^{m_2}-e^{m_3})-A_3(1-e^{m_3})\right]$$

$$A_1=\frac{Gr}{(e^{m_1}-e^{m_2})\left[m_1^{\,2}(1-iK_2w)-\lambda m_1-S\right]}$$

$$A_2=\frac{Gr}{(e^{m_1}-e^{m_2})\left[m_2^{\,2}(1-iK_2w)-\lambda m_2-S\right]}$$

$$A_3=\frac{\lambda p}{S}$$

Now, it is convenient to write the primary velocity (u) and secondary velocity (v) in terms of fluctuating parts, separating the real and imaginary part from the equation (32) and taking only the real parts as they have physical significance, the velocity distributions of the flow field can be expressed in fluctuating parts as given below:

$$u(\eta,t)=u_0(\eta)\cos t-v_0(\eta)\sin t.,$$

$$v(\eta,t)=u_0(\eta)\sin t-v_0(\eta)\cos t.,$$

$$q_0(\eta)=u_0(\eta)+iv_0(\eta). \quad (34)$$

The Shear stress or skin-friction at the left plate can be found from velocity field such as:

$$\tau=\mathrm{Real}\left(\frac{\partial q}{\partial\eta}\right)_{\eta=0}$$

Table1: Skin friction coefficient for the fix values of P=7, Pr=0.7, w=5, t=pi/4

N	Q_H	λ	Ω	K_2	m	M	K_p	Gr	τ
2	2	1	5	.01	1	2	1	5	1.7240
5	2	1	5	.01	1	2	1	5	1.6222
2	4	1	5	.01	1	2	1	5	1.7533
2	2	1.2	5	.01	1	2	1	5	1.9810
2	2	1	15	.01	1	2	1	5	1.1095
2	2	1	5	.05	1	2	1	5	1.7302
2	2	1	5	.01	2	2	1	5	1.7832
2	2	1	5	.01	1	3	1	5	1.5458
2	2	1	5	.01	1	2	10	5	1.7514
2	2	1	5	.01	1	2	1	7	1.7784

The rate of heat transfer in terms of Nusselt number is given as:

$$Nu=-\mathrm{Real}\left(\frac{\partial\theta}{\partial\eta}\right)_{\eta=0}$$

Table 2: Nusselt number or rate of heat transfer

N	Q_H	λ	Pr	t	Nu
2	2	1	.7	t=$\pi/4$	-0.4646
5	2	1	.7	t=$\pi/4$	-0.0482
2	4	1	.7	t=$\pi/4$	-0.6327
2	2	1.2	.7	t=$\pi/4$	-0.4297
2	2	1	1	t=$\pi/4$	-0.3901
2	-2	1	.7	t=$\pi/4$	-0.2696

RESULT AND DISCUSSION:

This study considers the configuration of unsteady MHD oscillating visco-elastic fluid in rotating vertical parallel plate channel filled with porous substrate in the presence of radiation, heat generation/ absorption, Hall current and

hydro magnetic behaviors of buoyancy-induced flow is studied. The analytically results are presented graphically and discussed in detail. Further, it is assumed that the temperature difference is small enough so that the density changes of the fluid in the system will be small. When the injection/suction parameter λ is positive, fluid is injected through the cold wall into the channel and sucked out through the hot wall and reversal phenomenon occurs in case of λ negative. The influence of various physical parameters of the flow field, temperature distribution, is shown graphically in figures (1)-(14) and skin friction, Nusselt number is evaluated for various parameters from table (1) and table (2) respectively. Interest in such type of work in rotating frame is motivated by its importance in several practical situations and such study has some relevance in metallurgy where the process of solidification is characterized by the presence of a liquid, a mushy zone and a solid zone, in geothermal systems, cooling of electronic components and many more fluid engineering applications. Further, the system of ordinary differential equations (27) - (28) with boundary condition (29) is solved analytically using the perturbation technique.
To have healthier insight of the physical problem the variations of the velocity , temperature , skin friction and Nusselt number are evaluated numerically for different sets of values of suction /injection parameter λ, rotation parameter Ω, visco-elastic parameter K_2, porosity parameter Kp, Magnetic parameter M, Hall current m, Grashoff number Gr, radiation parameter N and frequency of oscillation w, heat generation/absorption parameter Q_H, pressure P, Thermal conductivity parameter Pr (Prandtl number) .We fix the value of parameters namely P=7, N=4, w=5, Q_H =2, Pr=.71, t=$\frac{\pi}{4}$, Gr,=5, Ω=5, K_2=.01, m=1, M=2, Kp=1, λ=2 for primary velocity profile and λ=2, N=4, w=10, Q_H =2, Pr=.71, t=0 for temperature distribution profile and these values are then shown graphically to assess the effect of changing each parameter one by one. We have investigated the physical behavior of the primary velocity (u) for two cases of rotation Ω=5(small) and Ω=25 (large).

From figure (1), an increase in the magnitude of injection parameter is found to accelerate the velocity of the flow. This is because the injection accelerates the velocity of a fluid. Figure (2) illustrates the variation of the primary velocity u. It is quite obvious from this figure that velocity goes on decreasing with increasing rotation Ω of the entire system. The velocity profiles initially remain parabolic with maximum almost centerline of the channel for small values of rotation parameter Ω and then as rotation increases the velocity profile reduces and become more or less flatten. This is because so increasing Coriolis force retards the forward flow which is due to the pressure gradient. To further increase in Ω, the maximum velocity profiles no longer takes place at the centre but shift towards the plates of the channel. It means that for large rotation there arise boundary layers on the plates of the channel (This physical behavior is also applicable in all figures for high rotation). From figure (3), it inferred that primary velocity u goes on increasing with increasing value of visco-elastic parameter K_2. Figure (4) exhibits the effect of frequency of oscillations w on the primary velocity u. It is noticed that the velocity decreases with increasing frequency w. The influence of Hall parameter m is depicted in figure (5). It is observed that the velocity increases slightly with increasing hall parameter m as the effectual conducting $\frac{\sigma}{1+m^2}$ reduces with increasing m which decreases the magnetic damping force on primary flow u, and the reduction in the magnetic damping force is coupled with the fact that magnetic field has a pushing effect on flow u. It is observed from figure (6) that increase in Hartmann number M, causes a decrease in the magnitude of the velocity profile. When a transverse magnetic field is applied to an electrically conducting fluid, it gives rise to a force called the Lorentz force which acts against the flow if the magnetic field is applied in the normal direction as in the present study. This resistive force has a tendency to slow down the motion of the fluid in the boundary layer. Figure (7) represents the variation of velocity profile in the boundary layer for various values of porous parameter Kp. In this study, permeability parameter Kp is directly proportional to the actual permeability K^*_p of the porous medium. Hence, increasing Kp decreases the resistance of the porous medium since permeability physically becomes more with an increase in Kp, hence accelerating the flow velocity, whereas opposite behavior is viewed in case of high rotation Ω=25. This is because so high rotation Coriolis force dominates the decreasing permeability drag force resulting in retards the forward flow. It is interesting to note that permeability of porous medium does not show the much influence on variation of velocity profile for small as well as high rotation.

Again, similar consequence is observed for all above assessed figures whenever we take the high rotation parameter Ω=25.

Figure (8) and figure (12) depict the effects of heat generation/absorption parameter Q_H on velocity profile and temperature distribution in the fluid respectively. In general, the presence of a heat generation (Q_H >0) in the flow field causes the fluid temperature to enhance. This leads to raise the activity of motion due to increase in the induced buoyancy induced flow. On the contrary, the presence of heat absorption (Q_H <0) causes a reduction in the fluid temperature and therefore, the thermal buoyancy effects. This leads to fall of the fluid motion. Figure (9) shows that the fluid velocity increases with increase the Grashoff number . It is due to the reason that favorable buoyancy force accelerates the fluid flow resulting in more induced flow. The effect of radiation parameters (N) on the velocity profile and the temperature distribution is demonstrated in fig. (10) and fig. (11) respectively. It is viewed that as the value of N decreases the velocity profile and the temperature profile increases. This result can be explained by the fact that a decrease in the Radiation parameter N decrease in the Rosseland radiation absorptive (α) at the plate (R is proportional to α) . It is concluded that the divergence of radiative heat flux $\frac{\partial q_r}{\partial z}$ increases as a_r decreases and this means that the rate of radiative heat transferred to the fluid increases and consequently the fluid temperature increases and hence, the velocity of its particles increases. From figure (13), it is observed that decreasing the Prandtl number (Pr) increases the thermal conductivity and therefore, heat is able to diffuse away from plate more swiftly than the higher value of Pr. Here, it is very interesting to state that for the higher value of Pr (for water), temperature of fluid flow decreases, but a certain point of the temperature field, diffusion of heat become almost invariable, hence tends to negative which is away from the heated plate and towards the cold plate resulting in a very low temperature profile. Therefore, the negative bulk temperature occurring in figure (13) for high Pr may be associated with flow reversal due to the rotation of the vertical channel. Figure (14) reveals injection/suction parameter λ is positive; fluid is sucked out right hot plate and injected into the channel through the left cold plate. It is observed that by increase in the positive value of λ makes the temperature of the flow to decrease. Injecting cold fluid particles destabilizes the temperature at the boundary layer. Also, when injection/suction parameter λ is negative, fluid is sucked out left cold plate and injected into the channel through the right hot plate and, is to enhance the temperature of the fluid. It is due to injecting hot fluid particles. It is concluded above discussed outcome that by increasing λ from negative value to a positive value the temperature in the channel falls.

The Skin friction coefficient and the Nusselt numbers are presented in Table (1) and Table (2) respectively. The Skin friction coefficient for constant values of P=7, w=5, Pr=.71 and t=π/4 at the left plate (η=0) for different values of the Radiation parameter (N), heat generation/absorption parameter (Q_H) suction/injection parameter (λ) and magnetic parameter (M), rotation parameter (Ω), visco-elastic parameter K_2, Hall parameter (m), porous parameter Kp, Grashoff number (Gr) have been perceived in the table (1). The growing values of λ, Q_H, K_2, m, Kp, Gr is to enhance the skin friction coefficient accelerating the velocity of flow at the boundary (y =0) and reversal phenomenon occurs for M, Ω, and N. Nusselt numbers increase with increasing values of Pr, λ, N, - Q_H (heat absorption). The reason is that the thermal layer for high Pr, λ, N, - Q_H (heat absorption) is thin and consequently the rate of heat transfer increases, while Q_H (heat generation) reverses the consequence.

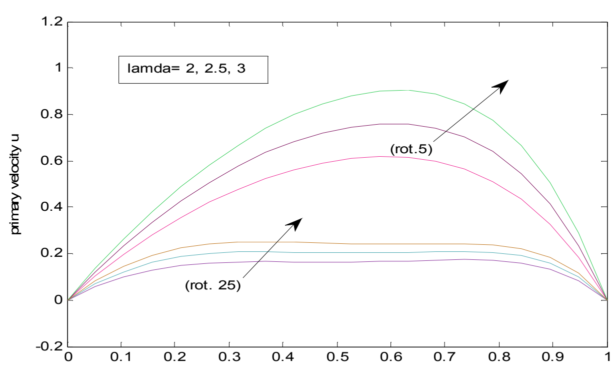


figure 1

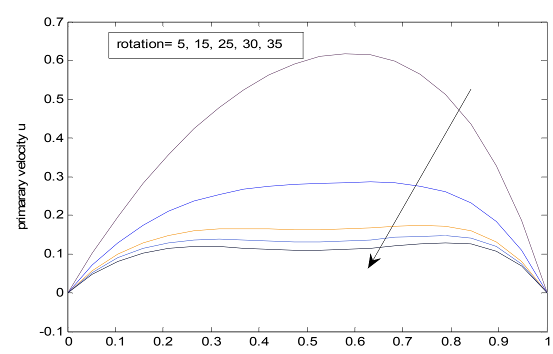


figure 2

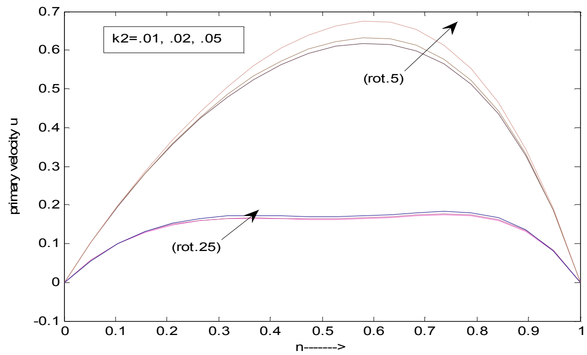


figure 3

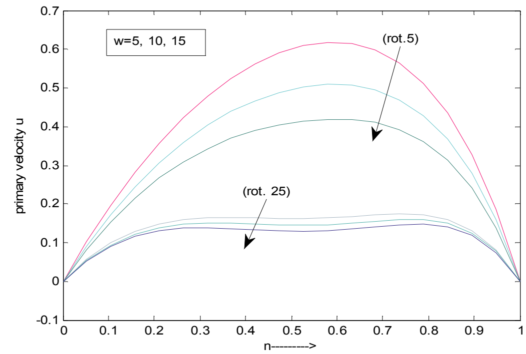


figure 4

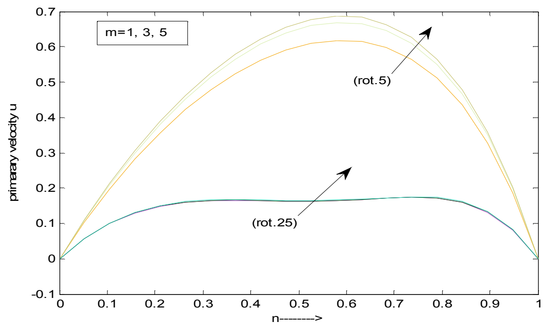


figure:5

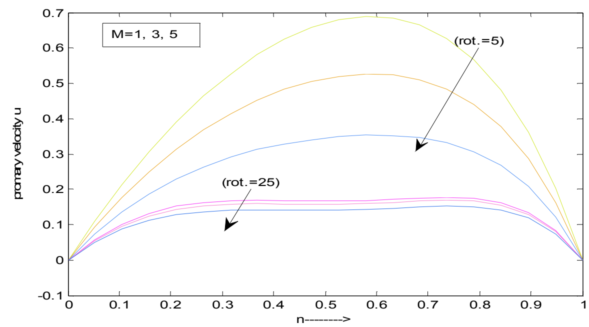


figure 6

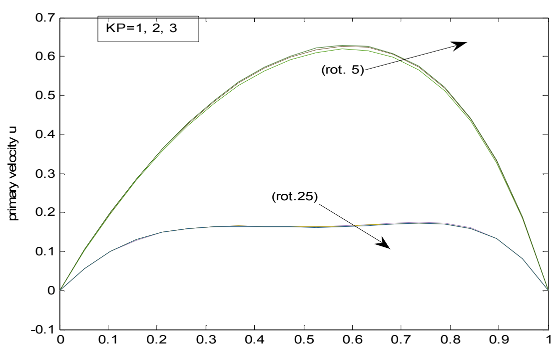


figure 7

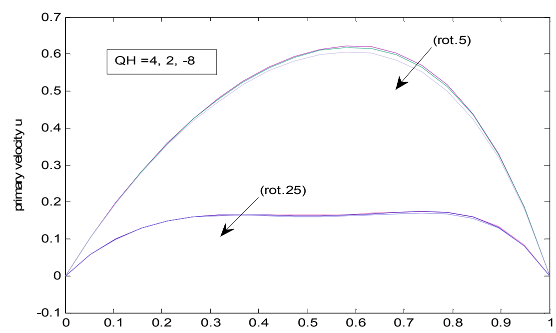


figure 8

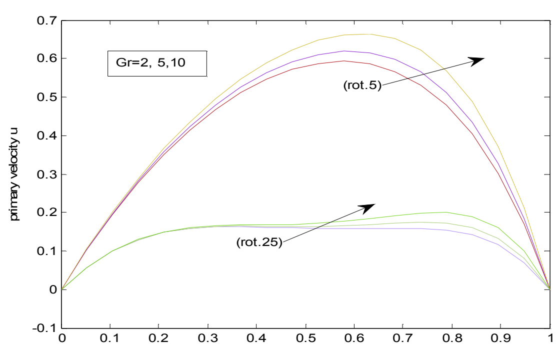


figure 9

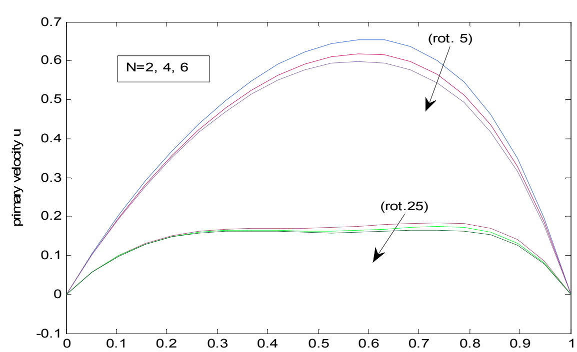
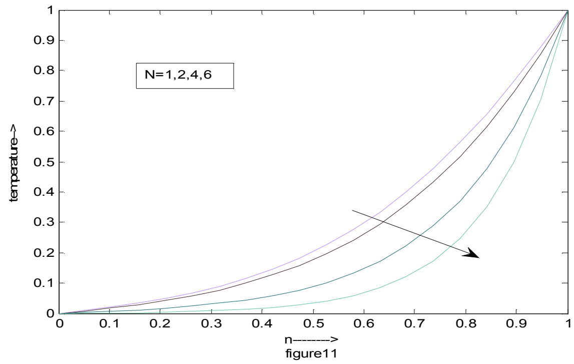
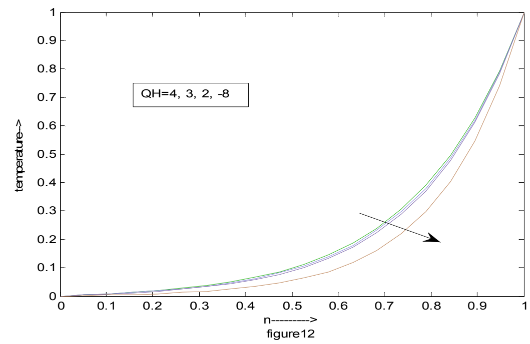
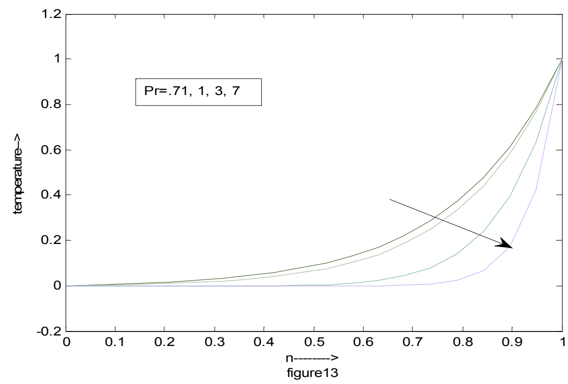


figure10







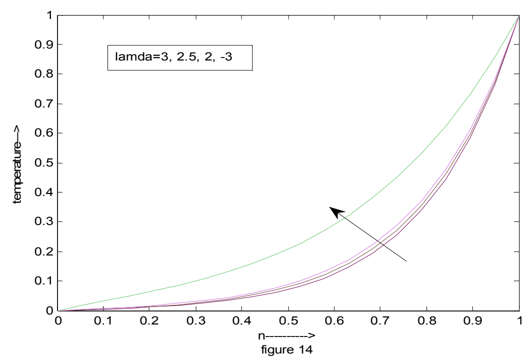


Figure 1: Primary velocity u versus η for different values of λ at $t = \frac{\pi}{4}$.
Figure 2: Primary velocity u versus η for different values of Ω at $t = \frac{\pi}{4}$
Figure 3: Primary velocity u versus η for different values of K_2 at $t = \frac{\pi}{4}$
Figure 4: Primary velocity u versus η for different values of w at $t = \frac{\pi}{4}$
Figure 5: Primary velocity u versus η for different values of m at $t = \frac{\pi}{4}$
Figure 6: Primary velocity u versus η for different values of M at $t = \frac{\pi}{4}$
Figure 7: Primary velocity u versus η for different values of K_p at $t = \frac{\pi}{4}$
Figure 8: Primary velocity u versus η for different values of K_p at $t = \frac{\pi}{4}$
Figure 9: Primary velocity u versus η for different values of G_r at $t = \frac{\pi}{4}$
Figure 10: Primary velocity u versus η for different values of N at $t = \frac{\pi}{4}$
Figure 11: Temperature distribution θ versus η for different values of N at t = 0
Figure 12: Temperature distribution θ versus η for different values of Q_H at t = 0
Figure 13: Temperature distribution θ versus η for different values of Pr at t = 0
Figure 14: Temperature distribution θ versus η for different values of λ at t = 0

Nomenclature:

J_x : x -component of current density
C_p : Specific heat at constant pressure
m : Hall parameter
d : Distance of the plate
M : Hartmann number
Nu : Nusselt number
e : Electric charge
n_e : Number density of the electron
g_0 : Acceleration due to gravity
P^* : Dimensional pressure
P_e : Electron pressure
G_r : Modified Grashofff number for heat transfer
P_r : Prandtl number
$\vec{H}$: Magnetic field
q_r^* : Radiative heat flux
H_0 : Magnetic field of uniform strength
Q_0 : Dimensional heat source

H_x : x -Component of magnetic field
Q_H : Heat source parameter
$\vec{J}$: Current density
N : Radition parameter
T_o : The mean temperature
u : Primary velocity
t^* : Dimensional time
T^* : Dimensional temperature
$\vec{V}1$: Electron velocity
K_2 : Visco-elastic parameter
w_0 : Dimensional injection /suction velocity
v : Secondary velocity component
u^*, v^*, w_0 : Velocity components are in the x^*, y^*, z^* directions respectively

Greek Symbols:
λ : Injection/suction parameter
μ : Dynamic viscosity
μ_e : Magnetic permeability
ν : Kinematic viscosity
ω : Oscillation parameter
η : Dimensionless distance
Ω^* : Dimensional parameter
Ω : Angular velocity
K : Fluid thermal conductivity
ω_e : Cyclotron frequency
σ : Electric conductivity
ρ : Density
θ : Non-dimensional temperature
σ : Coefficient of thermal expansion
β : Coefficient of thermal expansion
τ : shear stresses for the flow

CONCLUSIONS:

The problem of unsteady magnetohydrodynamic oscillatory convective and radiative visco-elastic fluid flow in a vertical porous channel filled with a porous substrate is examined. The fluid is injected through one of the porous plates and simultaneously sucked out through the other porous plate with the same constant velocity. The entire system rotates about an axis perpendicular to the plates. The closed form solutions for the velocity and temperature fields are obtained analytically and then assessed numerically for different physical parameters come into view in the equations. The above analysis brings out the following outcomes of physical interest on the velocity, temperature and Nusselt number and shear stress profiles of the flow fields.

1. It is explained that with the increasing rotation of the channel the velocity decreases and maximum of the parabolic velocity profiles in the almost center of the channel shifts towards the plate of the channel. The velocity profiles are looking more or less Parabolic, but for large rotation the profiles are perceived to be flattened.
2. The velocity increases with the increase of the injection/suction parameter, heat generation parameter, Grashoff number, Hall parameter, visco-elastic parameter and porous parameter.
3. The velocity decreases with the increase of the injection/suction parameter, heat absorption parameter, radiation parameter, magnetic parameter and frequency parameter.
4. It is found that with the increasing radiation parameter, Prandtl number, heat absorption parameter the temperature profile decreases, but reversal phenomenon occurs for growing values of heat generation, heat injection from the hot plate and suck at the cold plate.
5. It is very interesting to note that for the higher value of Pr (for water), the very low temperature profile is observed. Therefore, the negative bulk temperature occurring for high Pr (low conductivity) may be associated with flow reversal due to the rotation of the vertical channel.
6. The growing values of λ, Q_H, K_2, m, Kp, Gr is to enhance the skin friction coefficient (shear stress) accelerating the velocity of flow at the boundary (y =0) and reversal phenomenon occurs for M, Ω, and N.
7. Nusselt numbers (rate of heat transfer) increase with increasing values of Pr, λ, N, $-Q_H$ (heat absorption). The reason is that the thermal layer for high Pr, λ, N, $-Q_H$ (heat absorption) is thin and consequently the rate of heat transfer increases, while Q_H (heat generation) reverses the consequence.

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