Original Article

Bayesian nonlinear structural equation modeling

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Abstract Many researchers concentrate on non-linear approaches to build the model to bring out the relationship among the latent variables. In the past two decades, many researchers introduced a latent nonlinear approach in Structural Equation Modeling. In this Paper, we study various methods for the simultaneous analysis of multiple nonlinear relations namely latent interaction and latent quadratic effects through Bayesian approach. The Bayesian approach is used to explain the relationship among the exogenous latent variables and endogenous manifest variables.
Keywords: Bayesian Confirmatory Factor Analysis, Bayesian Nonlinear Structural Equation Modeling, Cardio Vascular Disease, Linear Measurement Model and Non-linear Measurement Model.

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1 INTRODUCTION

The factor analysis model and the LISREL type models are models in which the latent variables are related by linear functions. Busemeyer and Jones (1983), Jonsson (1998), Ping (1996), Kenny and Judd (1984), Bagozzi, Baumgartner (1992), Schumacker and Marcoulides (1998) are studied the importance of quadratic and interaction effects of latent variables in various applied research. Nonlinear factor analysis models with polynomial relationships were first explored by McDonald (1962) and later on Etezadi-Amoli and McDonald (1983) and Mooijaart and Bentler (1986). Kenny and Judd (1984), Ping (1996), Jaccard and Wan (1995), Jöreskog and Yang (1996) and Marsh, Wen and Hau (2004) developed models which are useful in LISREL. The asymptotically distribution-free (ADF) theory (Browne, 1984; Bentler, 1983, 1992) is an alternative for rigorous treatment for the non-normality induced by the nonlinear latent variables. However, it is well known that (Hu, Bentler and Kano, 1992; Bentler and Dudgeon, 1996) ADF theory requires very large sample sizes to attain its large sample properties.The Bayesian approach has also been developed for nonlinear Structural Equation Modeling (SEM) with general forms, see Zhu and Lee (1999) and Lee and Song (2003). Lee, Song and Poon (2004) conducted simulation studies to compare the Bayesian approach with the product indicator approach. Their simulation results indicate that the Bayesian approach is better to estimate the parameters. Recently, Lee and Zhu (2000) and Lee and Song (2004) developed Bayesian methods for analyzing nonlinear SEMs with mixed continuous and ordered categorical variables. The main objective of this Paper is to describe Bayesian estimation procedures for analyzing nonlinear Structural Equation Models. Bayesian estimates are obtained through a hybrid algorithm which combines the Gibbs sampler (Geman and Geman, 1984) and the MH algorithm (Metropolis *et al.*, 1953; Hastings, 1970). In this Paper, The basic ideology of Bayesian model is explained in Section 2. The influence of Linear, Interaction and Quadratic effects through Bayesian approach on Ejection Fraction and Survival of patients are discussed in Section 4. Also the model specification, Bayesian Confirmatory Factor Analysis and Bayesian non-linear Structural Equation Modeling are explained in

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Section 4.1, Section 4.2 and Section 4.3 respectively. Statistical Software R Language (Version 3.2.5) is used to estimate the Posterior parameters and results are explained in Section 4.4.

2 BASIC IDEOLOGY OF BAYESIAN ANALYSIS OF SEM

Under Bayesian approach in SEM, Let **X and Y** be data matrices and let $\boldsymbol{\Omega}$ be the matrix of latent variables and the structural parameter **θ**, all unknown parameters are considered as a vector. We apply Markov Chain Monte Carlo (MCMC) methods to obtain the Bayesian estimates and $\boldsymbol{\Omega}$. To achieve our goal, a sequence of random observations from the joint posterior distribution will be generated via the Gibbs sampler procedures by providing prior distribution of the parameters. The Gibbs sampler is a MCMC technique that generates a sequence of random observations from the full conditional posterior distribution of unknown model parameters. Under Gibbs sample procedures, the samples converge to the desired posterior distribution.

3 DATA CHARACTERISTICS

This study is conducted in Cardio Vascular Disease patient in Chennai City. A total of 405 samples are collected with help of Dr. Immanuel who is running private hospital in Chennai. There are 13 variables information of patient were observedin clinical laboratory namely, Name of the patient, Gender, Age, Body Mass Index (BMI), Place of residence (Urban / Rural), Smoking habits, Alcohol habits, Family History, Blood Glucose level (BGL), Blood Cholesterol level (BCL), Blood Pressure (BP), Ejection Fraction (EF) of the patients and their Survival status. The variables Age, BMI, BGL, BCL, BP and EF are continuous variables and Gender, Place of residence, smoking habits, alcohol habits and family history are categorical variables. The data were collected between the years 2010 and 2013.

4 BAYESIAN NON LINEAR STRUCTURAL EQUATION MODELING FOR EJECTION FRACTION AND SURIVIVAL OF PATIENSTS

The observed data is classified into three factors namely Blood factor, Life Style factor and Physical factor based on the nature of the independent variables. The three factors are considered as major factors which influence the Cardio Vascular Disease and Survival status of patients. The following Bayesian non-linear structural equation model is specified as follows.

4.1 Model Specification

Finally the two factors namely blood factor (ξ_{i1}) and life style factor (ξ_{i2}) are used to build the model in order to explain the relationship among the variables are shown in Figure 1. The Blood factor (ξ_{i1}) is measured by three variables namely Blood Glucose (x_{i1}), Blood Cholesterol (x_{i2}) and Blood Pressure (x_{i3}). The Life Style factor (ξ_{i2}) is measured by three variables namely Smoking habits (x_{i4}), Alcohol habits (x_{i5}) and Family History (x_{i6}) of the patients. The nonlinear structural equation model includes two linear latent exogenous variables $\xi_{i1}$and ξ_{i2}, a latent interaction term $\xi_{i3} = \xi_{i1} * \xi_{i2}$, a latent quadratic term $\xi_{i4} = \xi_{i1} * \xi_{i1}$, two manifest endogenous variables $Y_{i1}$and Y_{i2} and two disturbance term ζ_{i1} and ζ_{i2}. The parameters γ_{11} and γ_{12} are linear effect of ξ_{i1} and ξ_{i2} respectively. The parameters $\gamma_{13}$is the interaction effect of ξ_{i3} and γ_{14} is the quadratic effect of ξ_{i4}. The parameter β_{21} is the effect of Y_{i1} on the variable Y_{i2}.The measurement equation of the model is defined by twelve manifest variables in $X_i = (x_{i1}, x_{i2}, \dots, x_{i12})^T$, i=1,2,…,n and four latent variables in$\omega_{i1} = (\xi_{i1}, \xi_{i2}, \xi_{i3}, \xi_{i4})^T$, i = 1,2,…,n.

The measurement model for ξ_{i1} is as follows:

$$x_{i1} = \alpha_1 + \xi_{i1} + \varepsilon_{i1}$$

$$x_{ik} = \alpha_k + \lambda_{k1}\xi_{i1} + \varepsilon_{ik}\ , k = 2, 3 \qquad (1)$$

The general matrix expression for the above equation is given in the following format:

$$\begin{bmatrix} x_{i1} \\ x_{i2} \\ x_{i3} \end{bmatrix} = \begin{bmatrix} \alpha_1 \\ \alpha_2 \\ \alpha_3 \end{bmatrix} + \begin{bmatrix} 1 & 0 & 0 \\ 0 & \lambda_{21} & 0 \\ 0 & 0 & \lambda_{31} \end{bmatrix} \begin{bmatrix} \xi_{i1} \\ \xi_{i1} \\ \xi_{i1} \end{bmatrix} + \begin{bmatrix} \varepsilon_{i1} \\ \varepsilon_{i2} \\ \varepsilon_{i3} \end{bmatrix}$$

The measurement model for ξ_{i2} is as follows:

$$x_{i4} = \alpha_4 + \xi_{i2} + \varepsilon_{i4}$$

$$x_{ik} = \alpha_k + \lambda_{k2}\xi_{i2} + \varepsilon_{ik}\ , k = 5, 6 \qquad (2)$$

The general matrix expression for the above equation is given in the following

$$\begin{bmatrix} x_{i4} \\ x_{i5} \\ x_{i6} \end{bmatrix} = \begin{bmatrix} \alpha_4 \\ \alpha_5 \\ \alpha_6 \end{bmatrix} + \begin{bmatrix} 1 & 0 & 0 \\ 0 & \lambda_{52} & 0 \\ 0 & 0 & \lambda_{62} \end{bmatrix} \begin{bmatrix} \xi_{i2} \\ \xi_{i2} \\ \xi_{i2} \end{bmatrix} + \begin{bmatrix} \varepsilon_{i4} \\ \varepsilon_{i5} \\ \varepsilon_{i6} \end{bmatrix}$$

The measurement model for ξ_{i3} is as follows:

$$x_{i7} = \alpha_7 + \xi_{i3} + \varepsilon_{i7}$$

$$x_{ik} = \alpha_k + \lambda_{k3}\xi_{i3} + \varepsilon_{ik}\ , k = 8, 9 \qquad (3)$$

The general matrix expression for the above equation is given as follows:

$$\begin{bmatrix} x_{i7} \\ x_{i8} \\ x_{i9} \end{bmatrix} = \begin{bmatrix} \alpha_7 \\ \alpha_8 \\ \alpha_9 \end{bmatrix} + \begin{bmatrix} 1 & 0 & 0 \\ 0 & \lambda_{83} & 0 \\ 0 & 0 & \lambda_{93} \end{bmatrix} \begin{bmatrix} \xi_{i3} \\ \xi_{i3} \\ \xi_{i3} \end{bmatrix} + \begin{bmatrix} \varepsilon_{i7} \\ \varepsilon_{i8} \\ \varepsilon_{i9} \end{bmatrix}$$

The measurement model for ξ_{i4} is as follows:

$$x_{i10} = \alpha_{10} + \xi_{i4} + \varepsilon_{i10}$$
$$x_{ik} = \alpha_k + \lambda_{k4}\xi_{i4} + \varepsilon_{ik}\ , k = 11, 12 \quad (4)$$

The general matrix expression for the above equation is given in the following

$$\begin{bmatrix} x_{i10} \\ x_{i11} \\ x_{i12} \end{bmatrix} = \begin{bmatrix} \alpha_{10} \\ \alpha_{11} \\ \alpha_{12} \end{bmatrix} + \begin{bmatrix} 1 & 0 & 0 \\ 0 & \lambda_{114} & 0 \\ 0 & 0 & \lambda_{124} \end{bmatrix} \begin{bmatrix} \xi_{i4} \\ \xi_{i4} \\ \xi_{i4} \end{bmatrix} + \begin{bmatrix} \varepsilon_{i10} \\ \varepsilon_{i11} \\ \varepsilon_{i12} \end{bmatrix}$$

where$\varepsilon_{ik}, k = 1,2,\ldots,p$ is independently distributed as $N[0, \psi_{\varepsilon k}]$, and is also independent with ω_i.

4.2 Bayesian Estimation of the Confirmatory Factor Analysis Model

The factor model is defined in Section 4.1. As per the concept, two linear factors are extracted and then interaction factor and quadratic factors are derived. Each factor is measured by three manifest variables. The twelve manifest variables of four factors coeffients are estimated through Bayesian approach and the associative MCMC method using R Language Version 3.2.5 and the output are as follows:

Bayesian Confirmatory Factor Analysis of ξ_{i1}

The posterior mean, posterior standard deviation and posterior standard error of measurement model of ξ_{i1} are estimated using R Language Version 3.2.5. The posterior mean of $x_{i1}, x_{i2} and x_{i3}$ denoted as $\lambda_{11}, \lambda_{21} and \lambda_{31}$ respectively. The trace and density plots of these estimates are observed and it is shown in the following Figure 1.

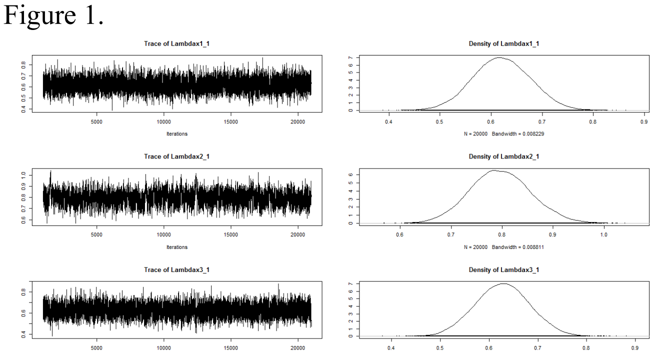

Figure 1

The trace plot of Lamdax1(λ_{11}), Lamdax2 (λ_{21}) and Lamdax3 (λ_{31}) are centered on its posterior means. The MCMC chain is converged at the sample size 20000. The density plots of $\lambda_{11}, \lambda_{21} and \lambda_{31}$ are follows perfect bell shaped curve which conforms that the estimates satisfies normal property.

Bayesian Confirmatory Factor Analysis of ξ_{i2}

The posterior mean, posterior standard deviation and posterior standard error of measurement model of ξ_{i2} are estimated using R Language Version 3.2.5. The posterior mean of $x_{i4}, x_{i5} and x_{i6}$ denoted as $\lambda_{42}, \lambda_{52} and \lambda_{62}$ respectively. The trace and density plots of these estimates are observed and it is shown in the following Figure 2.

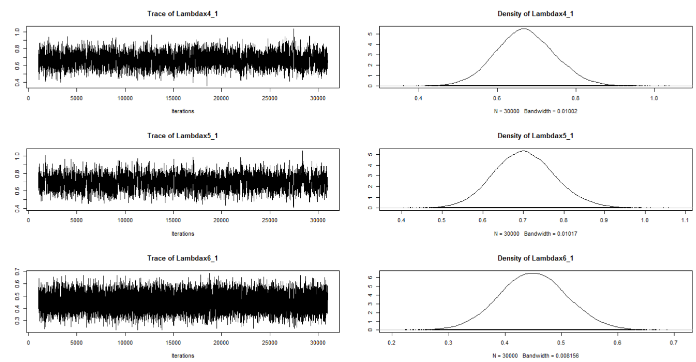

Figure 2

The trace plot of Lamdax4 (λ_{42}), Lamdax5 (λ_{52}) and Lamdax6 (λ_{62}) are centered on its posterior means. The MCMC chain is converged at the sample size 30000. The density plots of $\lambda_{42}, \lambda_{52} and \lambda_{62}$ are follows perfect bell shaped curve which conforms that the estimates satisfies normal property.

Bayesian Confirmatory Factor Analysis of ξ_{i3}

The posterior mean, posterior standard deviation and posterior standard error of measurement model of ξ_{i3} are estimated using R Language Version 3.2.5. The posterior mean of $x_{i7}, x_{i8} and x_{i9}$ denoted as $\lambda_{73}, \lambda_{83} and \lambda_{93}$ respectively. The trace and density plots of these estimates are observed and it is shown in the following Figure 3.

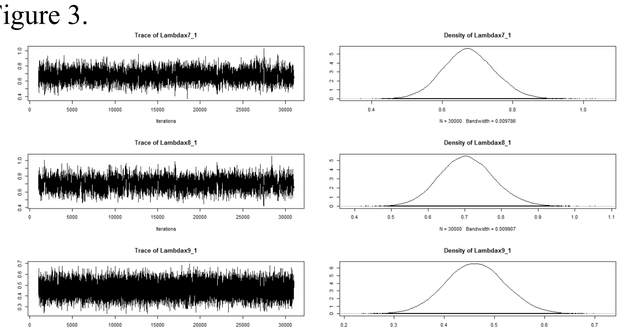

Figure 3

The trace plot of Lamdax7 (λ_{73}), Lamdax8 (λ_{83}) and Lamdax9 (λ_{93}) are centered on its posterior means. The MCMC chain is converged at the sample size 30000. The density plots of $\lambda_{73}, \lambda_{83} and \lambda_{93}$ are follows perfect bell shaped curve which conforms that the estimates satisfies normal property.

Bayesian Confirmatory Factor Analysis of ξ_{i4}

The posterior mean, posterior standard deviation and posterior standard error of measurement model of ξ_{i4} are estimated using R Language Version 3.2.5. The posterior mean of x_{i10}, $x_{i11} and\ x_{i12}$ denoted as $\lambda_{104}, \lambda_{114} and \lambda_{124}$ respectively. The trace and density plots of these estimates are observed and it is shown in the following Figure 4.

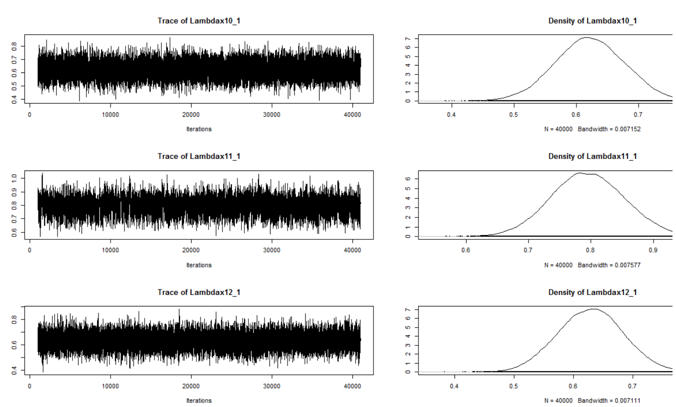

Figure 4

The trace plot of Lamdax10 (λ_{104}), Lamdax11 (λ_{114}) and Lamdax12 (λ_{124}) are centered on its posterior means. The MCMC chain is converged at the sample size 40000. The density plots of $\lambda_{104}, \lambda_{114} and \lambda_{124}$ are follows perfect bell shaped curve which conforms that the estimates satisfies normal property.

4.3 Bayesian Structural Equation Modeling

A nonlinear SEM is used with two linear terms, one interaction term, one quadratic term and two manifest endogenous terms. The effect of linear terms, interaction term and quadratic term is studied on Ejection Fraction (Y_{i1}) andSurvival Y_{i2}. Finally efficacy of methods will be studied. Also we studied how the Survival of the patients is influenced by linear and non linear terms and also studied the influence of Ejection Fraction on Survival.The linear measurement model and nonlinear measurement model is explained in Section 4.3.1 and Section 4.3.2 respectively in order to estimate the parameters.

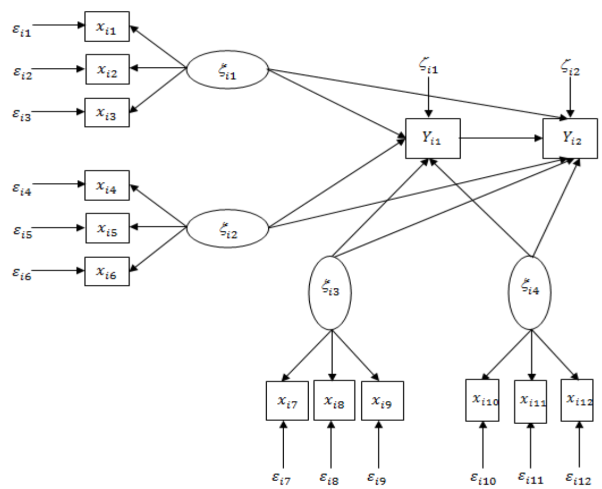

Figure 5

Bayesian Nonlinear structural equation model consists latent linear criterion $\xi_{i1}$and ξ_{i2}, a latent interaction termξ_{i3} and a latent quadratic term ξ_{i4} and a manifest endogenous variable Y_{i1}andY_{i2}The structural equation of the nonlinear model with an intercept term α_1 is given in the following structural equation model:

$$Y_{i2} = \alpha_1 + \beta_{21} Y_{i1} + \gamma_{21}\xi_{i1} + \gamma_{22}\xi_{i2} + \gamma_{23}\xi_{i3} + \gamma_{24}\xi_{i4} + \zeta_{i2} \qquad (5)$$

where

$$Y_{i1} = \alpha_2 + \gamma_{11}\xi_{i1} + \gamma_{12}\xi_{i2} + \gamma_{13}\xi_{i3} + \gamma_{14}\xi_{i4} + \zeta_{i1}$$

The general matrix expression is given in the following equation:

$$Y_i = \alpha + \beta\psi + \Gamma\xi' + \zeta_i \qquad (6)$$

where$\beta = (0 \quad \beta_{21})$, $\psi = \begin{pmatrix} 0 \\ Y_{i1} \end{pmatrix}$, $\Gamma = \begin{pmatrix} \gamma_{11} & \gamma_{12}\,\gamma_{13} & \gamma_{14} \\ \gamma_{21} & \gamma_{22}\gamma_{123} & \gamma_{24} \end{pmatrix}$,
$\xi = (\xi_{i1} \quad \xi_{i2}\xi_{i3}\xi_{i4})' and \zeta_i = (\zeta_{i2} \quad \zeta_{i1})'$.

In the above Equ. (6), Y_i is the manifest endogenous variable, $\alpha$is the latent intercept, $\Gamma$is the coefficient vector for the linear and non-linear effects and ζ_i is the latent disturbance.

4.3.1 Linear Measurement Model

The latent exogenous variable ξ_{i1} is measured by three indicators x_{i1}, x_{i2} and x_{i3} . Also ξ_{i2} is also measured by three indicators x_{i4}, x_{i5} and x_{i6}. The measurement model of ξ_{i1} and ξ_{i2} is given as follows:

$$\begin{bmatrix} x_{i1} \\ x_{i2} \\ x_{i3} \\ x_{i4} \\ x_{i5} \\ x_{i6} \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ \lambda_{21} & 0 \\ \lambda_{31} & 0 \\ 0 & 1 \\ 0 & \lambda_{52} \\ 0 & \lambda_{62} \end{bmatrix} \begin{bmatrix} \xi_{i1} \\ \xi_{i2} \end{bmatrix} + \begin{bmatrix} \varepsilon_{i1} \\ \varepsilon_{i2} \\ \varepsilon_{i3} \\ \varepsilon_{i4} \\ \varepsilon_{i5} \\ \varepsilon_{i6} \end{bmatrix}$$

The λ_{21} and λ_{31} are the factor loading of the linear measurement modelξ_{i1} and theλ_{25} and λ_{26} are the factor loading of the linear measurement model ξ_{i2}. The factor loadings ofthe indicatorsx_{i2}, x_{i3}, x_{i5} and x_{i6}areλ_{21}, λ_{31},λ_{52} and λ_{62} respectively which are freely estimated. The factor loading of the manifest variablesx_{i1} and x_{i4} are $\lambda_{11}$and λ_{42} respectively which are fixed parameters and it is denoted as$\lambda_{11} = \lambda_{42} = 1$. The error of the indicator variables x_{i1}, x_{i2}, x_{i3}, x_{i4}, x_{i5} and x_{i6} areε_{i1}, ε_{i2}, ε_{i3}, ε_{i4}, ε_{i5} and ε_{i6} respectively. The endogenous manifest variable is denoted as Y_{i1}.

4.3.2 Nonlinear latent Measurement Model

The latent exogenous variable ξ_{i3} is measured by three interaction indicators x_{i7}, x_{i8}andx_{i9} andξ_{i4} is also measured by three quadratic indicators x_{i10}, x_{i11} and x_{i12} and it is shown in the following nonlinear measurement model equation:

$$\begin{bmatrix} x_{i7} \\ x_{i8} \\ x_{i9} \\ x_{i10} \\ x_{i11} \\ x_{i12} \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ \lambda_{83} & 0 \\ \lambda_{93} & 0 \\ 0 & 1 \\ 0 & \lambda_{114} \\ 0 & \lambda_{124} \end{bmatrix} \begin{bmatrix} \xi_{i3} \\ \xi_{i4} \end{bmatrix} + \begin{bmatrix} \varepsilon_{i7} \\ \varepsilon_{i8} \\ \varepsilon_{i9} \\ \varepsilon_{i10} \\ \varepsilon_{i11} \\ \varepsilon_{i12} \end{bmatrix}$$

The factor loadings of the nonlinear measurement model ξ_{i3} are $\lambda_{73}, \lambda_{83}$ and λ_{93} and the factor loadings of the nonlinear measurement model ξ_{i4} are $\lambda_{114}$and λ_{124}. The parametersλ_{83} and $\lambda_{93}$are the factor loadings of the interaction indictors x_{i8} and x_{i9} respectively. The parameters λ_{114} and $\lambda_{124}$are the factor loadings of quadratic indicators x_{i11} andx_{i12} respectively. The factor loadings λ_{83}, λ_{93}, λ_{114} and λ_{124} are freely estimating parameters. The parametersλ_{73} and λ_{104} are the factor loadings of the indicators x_{i7} and x_{i10} respectively which are fixed parameters and it is denoted as $\lambda_{73} = \lambda_{104} = 1$. The parameters ε_{i7}, ε_{i8}, ε_{i9}, ε_{i10}, ε_{i11} and ε_{i12} are errors of the indicator variables x_{i7} , x_{i8}, x_{i9}, x_{i10}, $x_{i11}$and x_{i12} respectively are also estimated.

4.4 Findings of the Bayesian Nonlinear SEM for Survival of the Patients

The factor model is defined in Section 4.3.1.As per the concept, four factors are extracted through Bayesian approach out of which two linear forms, one interaction form and one quadratic form. Each factor is measured by three manifest variables. The four factor coefficients value are estimated through Bayesian approach and the associative MCMC method using R Language Version 3.2.5 and the output are as follows:

Table 1

Model	Post.Mean	Post.SD	PSRF	Prior
$\xi^*_{i1} \rightarrow Y^*_{i1}$	-0.509	0.026	1.010	dnorm(0,1e-2)
$\xi^*_{i2} \rightarrow Y^*_{i1}$	-0.421	0.018	1.002	dnorm(0,1e-2)
$\xi^*_{i3} \rightarrow Y^*_{i1}$	-0.608	0.025	1.011	dnorm(0,1e-2)
$\xi^*_{i4} \rightarrow Y^*_{i1}$	-0.432	0.052	1.006	dnorm(0,1e-2)
$Y^*_{i1} \rightarrow Y_{i2}$	0.781	0.106	1.004	dnorm(0,1e-2)
$\xi^*_{i1} \rightarrow Y_{i2}$	-0.789	0.074	1.019	dnorm(0,1e-2)
$\xi^*_{i2} \rightarrow Y_{i2}$	-0.557	0.173	1.014	dnorm(0,1e-2)
$\xi^*_{i3} \rightarrow Y_{i2}$	-0.794	0.022	1.012	dnorm(0,1e-2)
$\xi^*_{i4} \rightarrow Y_{i2}$	-0.501	0.097	1.015	dnorm(0,1e-2)

From the above Table 1, the Posterior Mean, Standard Deviation, Potential Scale Reduction Factor (PSFR) and Prior are given. Each factor PSFR value are less than 1.2 which shows the results are converged perfectly. The MCMC chain is converged at the sample 56000. The coefficients are also having negative impact on the dependent variables. Also the indirect effects of Factors are calculated and shown in the following Table 2.

Table 2

Model	Coefficients
$\xi_{i1} \rightarrow Y_{i2}$	-0.3975
$\xi_{i2} \rightarrow Y_{i2}$	-0.3288
$\xi_{i3} \rightarrow Y_{i2}$	-0.4748
$\xi_{i4} \rightarrow Y_{i2}$	-0.3373

The indirect effect of two latent exogenous variables (ξ_{i1}) and (ξ_{i2})are -0.3975 and -0.3288 respectively.The indirect effect of latent interaction term (ξ_{i3}) is -0.4748 andlatent quadratic (ξ_{i4}) term's indirect effect is – 0.3373 on the manifest endogenous Survival (Y_{i2}). Out of four factors of indirect effect, the Interaction factor indirect effect has more negative impact on the manifest endogenous Survival (Y_{i2}). The second leading factor is Blood factor, third factor is Quadratic factor and the least factor is Life Style factor.

SUMMARY

The non-linear structural equation modeling through constrained approach, unconstrained approach and latent moderated structural model was discussed by Xavier and Leo Alexander (2015). Also they proved that the constrained approach is the best method to explain the relationship between the latent exogenous variables and manifest endogenous variables. In this Paper, the Bayesian approach is used to explain the relationship among the exogenous latent variables and endogenous manifest variables. Generally theory says that the Bayesian approach is the best approach compared with all other methods to explain the relationship among the variables.In our study, we used the observed data set to find out the relationship among the variables. It is proved that the Bayesian approach is best method to explain the relationship among the variables as explained in Equ. 5.The indirect effects of four factors namely ξ_{i1}, ξ_{i2}, ξ_{i3} and ξ_{i4} are measured and their importance are also studied. The leading indirect contributing factor is Interaction Factor and least indirectly contributing factor is Life Style Factor. Thus it is concluded that the Bayesian approachhas a high explanatory power in terms of describing the interrelationship between the latent exogenous and manifest endogenous variable compared with latent moderated approach, unconstrained approach and constrained approach.

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