Original Research Article

Topological conjugacy of logistic map and its applications

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Abstract Topological conjugacy property has an important role to study chaotic behavior of a map. With the help of this property we can study the chaotic significance by comparing one map with another map. In this paper topological property of logistic map and its application has defined in comparisons with Tent map.
Key Words: Logistic map, Topological conjugacy, periodic bifurcation, topological transitive.

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INTRODUCTION

Topological conjugacy has so importance as it can protect many topological dynamical properties. Therefore, if we can find topolological conjugacy between a map f with another map, then we can investigate the map g to obtain in sequence about dynamical properties of the previous map f. The logistic map appears very frequently as models of real-life dynamical systems, for example in biology. Many chaotic properties of the logistic map,

$$f_m(x) = mx(1-x), 0 < m \leq 4, 0 < x \leq 1$$

for m = 4 can be studied through the topological-conjugacy with the tent map:

$$T(x) = \begin{cases} 2x \ if \ x\epsilon \left[0,\frac{1}{2}\right] \\ 2-2x \ if x\epsilon \left[\frac{1}{2},1\right] \end{cases}$$

The tent map turns out to be easier to analyse, since it can be studied using dual expansions similarly to the doubling map (in particular, we can find all periodic points and create dense orbits). We have already seen that conjugacy maps periodic points of period n to periodic points of period n[1].

Definition: 1.1. The dynamical system defined by $f: X \to X$ is the family of functions $\{f^n\}_{n\in Z_+,}$ with each f^n mapping $X \ to \ X$. For example let $f: R \to R$ given by $f(x) = \frac{x}{3}$. Then the dynamical system defined by $f$is the family of functions given by $f^n(x) = {}^{x}/{}_{3^n}$.

Definition 1. 2. The functions $f: X \to X$ and $g: Y \to Y$ (and the dynamical systems defined by them) are said to be topologically conjugate if there exists a homomorphism $h: X \to Y$ such that $goh = hog$.The function h is called a topological conjugacy between $f \ and \ g$. Topological conjugacy is defined by the following figure. It is with the aim of both routes from the upper-left X to lower-right Y –from corner to corner the top, then the right side, and down the left side, then across the bottom-give the same result. we can state that the diagram commutes.

$$\begin{tikzcd} X \arrow[r, "f"] \arrow[d, "h"'] & X \arrow[d, "h"] \\ Y \arrow[r, "g"] & Y \end{tikzcd}$$

Figure 1:

For example: We have the dynamics of the functions $f(x) = 2x \, and \, g(x) = 3x$ become visible the same. In both the cases 0 is a fixed point, and all additional orbits continue either on the positive or negative side of 0 and move about outer from 0.In fact, these two functions are topologically conjugate. The function $h: R \to R$, defined by $h(x) = x^{log_2(3)}$, is a homomorphism so as to satisfy $goh = hog$. Theorem: 1. Let us consider h be a topological conjugacy between $f: X \to X$ and $g: Y \to Y$.For all $x \in X and \, n \in Z_+$, we have $h(f^n(x)) = g^n(h(x))$. As a result h maps the orbit $x \, under \, f$ to the orbit of $h(x)$ under g. proof: It can be prove by induction on n. By the definition of topological conjugacy, for $n = 1$ the result holds. Let us assume that the result holds for $n - 1$. Then we have,

$h(f^n(x)) = h(f^{n-1}(f(x)))$
$= \; g^{n-1}(h(f(x))$
$= \; g^{n-1}(g(h(x)))$
$= \; g^n(h(x))$.

Where the second equality holds by the inductive supposition. Hence if the result holds for $n - 1$, then it holds for n,.therefore, by induction $h(f^n(x) = \; g^n(h(x))$ for all $n \in Z_+$.

Proposition: Let us consider h be a topological conjugacy between $f: X \to X and g: Y \to Y$ and suppose that $x \in X$ Then we have the following results hold:

1. If x is a fixed point of f,then $h(x)$ is a fixed point of g.
2. If x is a period-m point of $f, h(x)$ is a period –m point of g.
3. If x is an concluding fixed point of f,then $h(x)$ is an concluding fixed point of g.
4. If x is an concluding periodic point of f,then $h(x)$ is an concluding periodic point of g.

Definition: 1.3. Let us consider X be a topological space. A function $f: X \to X$ is said to be chaotic or to have chaos if

1. The set of periodic points of f is dense in X.
2. For every U, V open in X, there exists $x \epsilon U$ and $n \epsilon Z_+$ such that $f^n(x) \epsilon V$.

The first condition indicates that there is regular periodic 20ehaviour densely spread throughout the region. It does not matter what point we choose in the region, there are periodic points randomly close in. The second condition, referred to topological transitivity, indicates that every pair of regions in the domain is mixed together by the system. For any pair of open sets, there is at least one point in the first set that, on a number of iteration, is mapped into the set. Let us consider the tent function

$$T(x) = \{ \begin{matrix} 2x \; if \; x\epsilon \left[0, \frac{1}{2}\right] \\ 2 - 2x \; if x\epsilon \left[\frac{1}{2}, 1\right] \end{matrix} and$$

The following figure shows the graph of $T^3 and \, T^4$. The pattern is apparent. The graph of T^n results in a 'tent' over $[\frac{j-1}{2^{n-1}}, \frac{j}{2^{n-1}}]$ for each $j = 1,2, \dots \dots \dots 2^{n-1}$. It follows that each such interval contains two intersections of the graph of T^n with the line $y = x$. These intersection points are periodic points of T. Thus as n gets larger and larger, the intervals $[\frac{j-1}{2^{n-1}}, \frac{j}{2^{n-1}}]$ partition $[0,1]$ into smaller and smaller intervals, each of which contains periodic points.

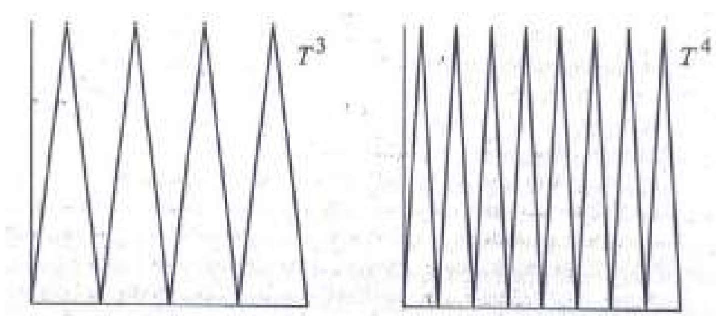

Figure 2: Graph of T^3 and T^4

Furthermore, T^n maps each interval $\left[\frac{j-1}{2^{n-1}}, \frac{j}{2^{n-1}}\right]$onto $[0,1]$.As a result as n gets larger and larger, smaller and smaller intervals are getting extend out "mixed" by T^n over the entire interval $[0,1]$. Thus the two major mechanism of chaos appear to be present in the tent function. In a set of accompanying implement at the end of this section, we work through the details in this regard to proving that T is chaotic. On the other hand, we will now take a different process to proving that T is chaotic; this process is based on binary expansions of the real numbers in $[0,1]$.In particular, we use the fact that every $x \, \epsilon[0,1]$ can be defined in form

$\frac{a_1}{2} + \frac{a_2}{2^2} + \; \dots \dots \dots \dots \dots \dots + \frac{a_m}{2^m} + \; \dots \dots \dots \dots$

Here each a_i equals either 0 or 1. For such x, we have the binary expansion

$x = .a_1 a_2 a_3 \dots . a_m \dots \dots$

For $a = 0 \, or \, 1$, we let $a^* = 1 - a$. It is to note that $(a^*)^* = a$. Again, if x has binary expansion.

$a_1a_2a_3 \ldots\ldots a_m \ldots\ldots\ldots\ldots\ldots$ than $1-x$ has binary expansion. $a_1{}^*a_2{}^* \ldots\ldots\ldots a_m{}^* \ldots\ldots\ldots$

Now, if $x \in \left[0,\frac{1}{2}\right]$ then x has a binary expansion in the form. $0a_2a_3 \ldots\ldots a_m \ldots\ldots$, and it follows that $2x = 2\left(\frac{0}{2}+\frac{a_2}{2^2}+ \ldots\ldots\ldots + \frac{a_m}{2^m} + \ldots\ldots\right) = .a_2a_3 \ldots\ldots a_m \ldots$

Furthermore, if $x \in \left[\frac{1}{2},1\right]$, than x has a binary expansion in the form. $a_2a_3 \ldots\ldots a_m \ldots\ldots$, so $1-x = .0a_2{}^* \ldots\ldots\ldots a_m{}^* \ldots\ldots\ldots$, and $2-2x = .a_2{}^*a_3{}^* \ldots\ldots\ldots a_m{}^* \ldots\ldots\ldots$,(Here it is to be noted that binary expansions are not unique. for example, $1/2$ can be expressed as both. $1000000\ldots$ and. $01111111\ldots\ldots$, and for that reason we include $1/2$ in each of the possibilities, $x \in [0,\frac{1}{2}]$ and $x \in [\frac{1}{2},1]$, obtainable here.)

Therefore, by using binary expansions, we can state the tent function $T:[0,1] \to [0,1]$. as follows :

$$T(.a_1a_2a_3 \ldots\ldots a_m \ldots\ldots) = \begin{cases} .a_2a_3 \ldots\ldots a_m \ldots\ldots if\, a_1 = 0 \\ .a_2{}^*a_3{}^* \ldots\ldots\ldots a_m{}^* \ldots if\, a_1 = 1 \end{cases}$$

For example $T(.00000\ldots\ldots) = .00000\ldots\ldots$ confirming the fixed point at 0. The binary expansion for 1 is. $111\ldots.$, and $T(.111\ldots\ldots) = .0000\ldots\ldots$, so $T(1) = 0$. For $2/3$, the binary expansion is. $1010\ldots\ldots$, and $T(.1010\ldots\ldots) = .1010\ldots$; thus $2/3$ is a fixed point. For $2/5$ we have binary expansion. $01100110\ldots$, and $T(.01100110\ldots\ldots) = .01100110\ldots\ldots = 4/5$, on the other hand, $T(.11001100\ldots\ldots) = .011011\ldots. = 2/5$. Therefore, as we saw in the previous portion, we have period-2 points at $2/5$ *and* $4/5$.

With the help of above explanation on T, we can also express the value of $T^n(.a_1a_2a_3 \ldots\ldots a_m \ldots\ldots)$, as the following lemma indicates:

Lemma 1: For. $a_1a_2a_3 \ldots\ldots a_m \ldots\ldots \in [0,1]$ *and* $n \in Z_+$,

$$T^n(.a_1a_2a_3 \ldots\ldots a_m \ldots\ldots) = \begin{cases} .a_{n+1}a_{n+2} \ldots\ldots\ldots\ldots if\, a_n = 0. \\ .a_{n+1}{}^*a_{n+2}{}^* \ldots\ldots\ldots\ldots if\, a_n = 1. \end{cases}$$

Proof: We can prove this by the induction process on n. By the definition the result holds for $n = 1$. Let us assume that the result holds for n−1. Then

$$T^n(.a_1a_2a_3 \ldots\ldots a_m \ldots\ldots) = T\left(T^{n-1}(.a_1a_2a_3 \ldots\ldots a_m \ldots)\right)$$
$$= \begin{cases} T(.a_na_{n+1} \ldots\ldots\ldots\ldots)\, if\, a_{n-1} = 0. \\ T(.a_n{}^*a_{n+1}{}^* \ldots\ldots\ldots)\, if\, a_{n-1} = 1. \end{cases}$$

Where the second equality holds by the inductive supposition. Now if $a_n = 0$; then $T^n(.a_1a_2a_3 \ldots\ldots a_m \ldots\ldots)$ equals either $T(.0a_{n+1}a_{n+2} \ldots\ldots\ldots\ldots)$ or $T(.1a_{n+1}{}^*a_{n+2}{}^* \ldots\ldots\ldots)$.

In either case, by the definition of T, the result is $.a_{n+1}a_{n+2} \ldots\ldots\ldots\ldots$, at the same time as preferred.

Now let us assume that $a_n = 1$. Then $T^n(.a_1a_2a_3 \ldots\ldots a_m \ldots\ldots)$ equals either $T(.1a_{n+1}a_{n+2} \ldots\ldots\ldots\ldots)$ or $T(.0a_{n+1}{}^*a_{n+2}{}^* \ldots\ldots\ldots)$, and in either case we get the preferred result $.a_{n+1}{}^*a_{n+2}{}^* \ldots\ldots$ Thus, if the result holds for $n-1$ it holds for n, and it implies by induction that the result holds for all $n \in Z_+$. In our proof that T is chaotic we have used the following lemma, which indicates that if two points in [0,1] have connected binary expansions that concur in their first n entries, then the distance between those two points is at most $\frac{1}{2^n}$. Lemma 2. Let x *and* y have binary expansions. $a_1a_2a_3 \ldots\ldots a_m \ldots\ldots$ and. $b_1b_2b_3 \ldots\ldots b_m \ldots.$, respectively. If $a_i = b_i for\ i = 1,2, \ldots\ldots n$, then $|x-y| \le \frac{1}{2^n}$.

Proof: We have

$$|x-y| = \left|\sum_{j=n+1}^{\infty} \frac{a_j - b_j}{2^j}\right|$$
$$\le \sum_{j=n+1}^{\infty} \left|\frac{a_j - b_j}{2^j}\right|$$
$$\le \sum_{j=n+1}^{\infty} \frac{1}{2^j} \left(since \left|a_j - b_j\right| \le 1\right).$$
$$= \frac{1}{2^n}\sum_{j=1}^{\infty} \frac{1}{2^j}$$
$$= \frac{1}{2^n} \left(since \sum_{j=1}^{\infty} \frac{1}{2^j} = 1\right).$$

Now we have established that T is chaotic.

Theorem: 2. The tent function is chaotic.

Proof **:** Let us start by establishing with periodic points of T are dense in [0,1]. It is sufficient to prove that if $x \in [0,1]$ and $x > 0$, then there is a periodic point p such that $|x-p| < \varepsilon$. Thus let $x \in [0,1]$ and $\varepsilon > 0$ be arbitrary. Let us consider that. $a_1a_2a_3 \ldots\ldots a_m \ldots\ldots$ is a binary expansion of x and $n \in Z_+$ is large enough that $\frac{1}{2^n} < \varepsilon$.

If we consider $p = .a_1a_2a_3 \ldots\ldots a_n0.a_1a_2a_3 \ldots\ldots a_n0.a_1a_2a_3 \ldots\ldots a_n0 \ldots\ldots\ldots\ldots\ldots$,

then by using Lemma 1. we have that p is a periodic point, and Lemma 2 implies that $|x-p| < \varepsilon$. Thus periodic points of T are dense in [0,1].

Now to show the topological transitivity, let us consider U *and* V be open in [0,1], and let $x = .a_1a_2a_3 \ldots a_m \ldots \in U$. be arbitrary. Since U is open, therefore there exists $\varepsilon > 0$ such that $(x-\varepsilon, x+\varepsilon) \cap [0,1] \in U$. Let $n \in Z_+$ be large enough as $\frac{1}{2^n} < \varepsilon$. Again let $y = .b_1b_2b_3 \ldots\ldots b_m \ldots \in V$, and consider the point p such that,

$p = .a_1a_2a_3 \ldots\ldots a_n0b_1b_2b_3 \ldots\ldots$

Now with the help of lemma 2. $|x - p| \le \frac{1}{2^n} < \varepsilon$, and therefore $p \in U$. Hence from lemma 1, we have $T^{n+1}(p) = y \in V$. It shows that T is topological transitive and consequently it is chaotic.

Transitivity:[2,4]

Occasionally for a given dynamical system $f: X \to X$, when we iterate $x_0 \in X$, the orbit $O(x_0) = \{x_0, f(x_0), \ldots\ldots\ldots\}$, spreads itself consistently over X, as a result that $O(x_0)$ is a dense set in X. Hence $f: X \to X$ is said to be topologically transitive if there exists $x_0 \in X$ such that $O(x_0)$ is dense subset of X. If f is transitive, then there is a dense set of transitive points, since each point in $O(x_0)$ will be a transitive point[2].

Proposition: 2. If $f: X \to X and g: Y \to Y$ are maps conjugate via a conjugacy $h: X \to Y :\ hof = goh$, then

1 $hof^n = g^n oh\, for all\, n \in Z_+$, $(f^n\ and g^n are\ also\ conjugative\ )$.

2. $If c\ is\ a\ point of period\ m\ for\ f, then\ h(c)\ is\ a\ point\ of\ period\ m\ for\ g, c\ is\ attracting\ iff\ h(c)\ is\ attracting.$

3. $f\ is\ transitive\ iff\ g\ is\ transitive.$

4. $f\ has\ a\ dense\ set\ of\ periodic\ points\ iff\ g\ has\ a\ dense\ set\ of\ periodic\ points.$

5. $f\ is\ chaotic\ iff\ g\ is\ chaotic.$

Proof: 1. $hof^2 = hofof = gohof = gogoh = g^2oh$, and in the same way $hof^3 = g^3oh$, and continuing in this way we have by induction process the result follows.

2. Suppose that $f^i(c) \ne c for\ 0 < i < m and\ f^m(c) = c$ then $hof^i(c) \ne h(c) for$ $0 < i < m$ since h is one -to–one, and as a result $g^i oh(c) \ne h(c)\ for\ 0 < i < m$.

In addition, $hof^m(c) = g^m oh(c)$, or $h(c) = g^m(h(c))$, so $h(c)$ is a period m point for g.

We shall only to show that if p is an attracting fixed point of f, (there exist an open sphere $B_\epsilon(p)$ such that $x \in B_\epsilon(p)\ then\ f^n(x) \to p\ as\ n \to \infty$), then $h(p)$ is an attracting fixed point of g.

Let $V = h\,(B_\epsilon(p))$, then since h is a homomorphism, V is open in Y and contains $h(p)$. Let $y \in V$, then $h^{-1}\,(y) \in B_\epsilon(p)$, so that $f^n(h^{-1}\,(y)) \to p$ as $n \to \infty$.

Since h is continuous, $h(f^n(h^{-1}\,(y))) \to h(p)$ as $n \to \infty$, i.e.

$g^n(y) = hof^n oh^{-1}(y) \to h(p), as\ n \to \infty$. So $h(p)$ is attracting.

3. Let us Suppose that $O(z) = \{z, f(z), f^2(z) \ldots\ldots\ldots\}$ is dense in X and consider $V \subset Y$ be a non-empty open set. Then since h is a homeomorphism, $h^{-1}(V)\ is\ open\ in\ X$, and hence there exists $k \in z^+$ with $fg^k(z) \in h^{-1}(V)$.

It follows that $h\left(f^k(z)\right) = g^k\left(h(z)\right) \in V$, so that $O(h(z)) = \{h(z), g\left(h(z)\right), g^2\left(h(z)\right), \ldots\ldots\ldots\}$ is dense in Y, i.e. g is transitive. Similarly, if g is transitive, then f is transitive.

4. Suppose that f has a dense set of periodic points and let $V \subset Y$ be non-empty and open. Then $h^{-1}(V)\ is\ open\ in\ X$, so contains periodic points of f. As in (3), we see that V contains periodic points of g. Similarly if g has a dense set of periodic points, so does.

5. (3) and (4) together implies this result.

3. $\boldsymbol{n - cycles}$

$\boldsymbol{3.1\ n - cycles\ of\ Tent\ map}$. [2,4].

1. The tent map f has two fixed points. One is at 0.. The slope of f at the fixed points is 2 so they are repellors.
2. fof has 2^2 fixed points. Two are fixed points of f. Two extra fixed points of fof form a 2-cycle of f. All are repellors.
3. $fofof$ has 2^3 fixed points. All repellors. Two are fixed points of f. The remaining six form two three cycles.
4. . f^4 has $2^4\ 4 - cycles$. 2^2 are fixed points of f^2. The remaining 12 form three four cycles.
5. The graph of f^n consists of 2^{n-1} tents of width 2^{1-n}. Each tent has two $n - cycles$ one on theleft side and one on the right.
6. The distance between adjacent $n - cycles$ is no larger than 2^{2-n}. Adjacent $n - cycles$ are connected by the graph rising to the top and bouncing back to
7. $y = x$ or sinking to the floor and bouncing back. The slopes of the segments are equal to 2^{n+1}. And it gives the lower bound.
8. For any compact subinterval $[a, b] \subset (0,1]$ there is a constant $c\ \in (a, b)$ independent of n to that the distance between adjacent $n - cycles$ in $[a, b]$ satisfies $c2^{-n} < distence < 2^{-n}$.

3.2 $\boldsymbol{n - cycles}$ of the Logistic Map.[2]

There are similar results for the logistic map $f(x) = 4x(1 - x)\ from\ [0,1]$ to itself.

1. The logistic map f has two fixed points. One is at 0. Both are repellors.
2. $.fof$ has 2^2 fixed points. Two are fixed points of f and the other two form a $2 - cycles$. all unstable.
3. f^3 has 2^3 fixed points. Two are fixed points. The remaining six form two three cycles.

4. f^4 has 2^4 fixed points. 2^2 are fixed points of g^2. The remaining 12 form three four cycles.
5. The graph of f^n consists of 2^{n-1} fingers with width $\leq {}^{1}/_{2^{n-1}}$. Each tent has two $n-cycles$.
6. The distance between adjacent n-cycles is no larger than ${}^{2}/_{2^{n-1}}$.
7. For any compact subinterval $[a,b] \subset (0,1]$ there is a constant $c \in (a,b)$independent of n to that the distance between adjacent $[a,b] \subset (0,1]$ there is a constant$c \in (a,b)$ independent of $n-cycles\ in\ [a,b]$is no smaller than $c2^{-n}$.
8. The proof is as for the tent map. The graph must reach the top or the bottom and return. It

covers a vertical distance that is bounded below. For the one tenth of the path that is nearest the top or bottom the slope is bounded above by $c2^n$. Mutually it gives the lower bound. For both logistic and tent maps the set of all cycles is dense in $[0,1]$.Which is one of the important characteristics of chaotic maps.

4. **Conjugations and $n-cycles$.**[2]

Definition: A homeomorphism h is a conjugation between maps $f\ and\ g$ when

$f = h^{-1} o\ g\ o\ h.$

1. Then $x\ is\ an\ n-cycle\ of\ f$ if and only if $h(x) is\ an\ n-cycles\ of\ g$.
2. h is a homeomorphism of $[0,1]$ when and only when h is a surjective strictly monotone map. It is increasing when $h(0) = 0$ and decreasing in case $h(0) = 1$. Since the tent map and the logistic map have only one endpoint that is a fixed point and that is 0, any conjugation must map 0 to itself. Therefore any conjugation is strictly increasing with $h(1) = 1$.
3. It follows that for each$n$ any conjugation of the tent map to the logistic map must map the $n-cycles$ of the tent map to the $n-cycles$ of the logistic map preserving the order. Thus any conjugation is uniquely determined on the set of all cycles.
4. Since the set of all cycles of the tent map are dense, and hence there is at most one conjugation.

Theorem 4.1 The tent map is topologically conjugate to the logistic map.

Proof. Step I. If there is a conjugacy then it values at the endpoints and all $n-cycles$ is uniquely determined. The conjugation h must take the nonzero fixed point of f to the non zero fixed point of g. The fixed points of fof must be mapped to the same number of fixed points of gog in increasing order. The mapping on the fixed points of f has already been assigned. It is not clear that the fixed points of fof can be mapped in increasing order while in respect of the previous obligation. For example, if there were two fixed points of fof between the fixed points of f and only one fixed point of gog between the fixed points of g. An essential and adequate condition for being able to allocate values constantly is as follows. For each $n\ and\ 1 \leq m \leq 2^n$ denote by $x_{m,n}$ the $m^{th} n-cycle$ of f^n counting from the left and $y_{m,n}$ the equivalent cycles for g^n. The eaac is that for each $k < n$ and $1 \leq j \leq 2^k . x_{j,k} \in]x_{m,n}, x_{m+1,n}]$ iff $y_{j,k} \in]y_{m,n}, y_{m+1,n}]$. For the convenience that this is satisfied, consider the j^{th} fixed point of f^k. It is in one of the intervals of length 2^k. Consider that interval J. All the intervals of size 2^n lying to the left of the interval contain fixed points of f^n that lie to the left. To make a decision for the ranking, we needs only to consider the intervals of width 2^n starting at the left endpoint of J. For intervals of length 2^n consider them as open on the left and closed on the right. Then the fixed point lies in exactly one of the intervals of size 2^n whose union in J.Let us consider that interval as K. Of the two fixed points of f^n in K, j^{th} one is either to the left or the right of centre. This can be reproduce for the logistic map and it shows that the ranking of the j^{th} fixed point of g^k among the fixed points of g is exactly the same. In this way the value of $h\ on\ all\ n-cycles$ is exclusively determined. This determines h on a dense subset. Step II. It suffices to show that the mapping defined on the cycles extends to a continuous Function on [0; 1]. Since the constraint of h to the set of cycles is strictly increasing it follows that any continuous extension is non decreasing. To show that it is strictly increasing suppose that $x_1 < x_2$, let us consider two cycles $\underset{x_1}{\sim} < \underset{x_2}{\sim}$ between $x_1\ and x_2$. Therefore we have $h(x_1) < h\left(\underset{x_1}{\sim}\right) < h\left(\underset{x_2}{\sim}\right) < h(x_2)$, which shows that h is strictly monotone.

Step III. Construction of a continuous extension.

To extend to the open subinterval $(0,1]$it sufficient to extend to each compact subinterval $[a,b] \subset (0,1]$. To extend to $[a,b]$ it is sufficient (and necessary) to show that if $z_n \neq w_n$ are sequences of cycles in $[a,b]$with $z_n - w_n \to 0$ then $h(z_n) - (w_n) \to 0$.

Now to verify the sufficient condition, let us denote $r(n)\ ands(n)$ be the degrees of$z_n and w_n$ respcetivly. Renaming $z_n and w_n$ we may suppose that $r(n) \geq s(n)$. Since $0 \neq z_n - w_n \to 0$, it follows that $r(n) \to \infty$.

In addition the nearest $r(n) cycle \underset{z_n}{\sim}$to w_n and further from $z_{\ n}$is at most 2^{-n} further from z_n. Hence $z_n - \underset{z_n}{\sim} \to$

0. Again consider the ranks of $z_n and \overset{\sim}{z_n}$ among fixed points of f^n differ by $j_n > 0$. Then we have $z_n - \overset{\sim}{z_n} \rightarrow 0$ iff $^{j_n}/_{2^n} \rightarrow 0$. In this case the images $h(z_n)$ and □$(\overset{\sim}{z_n})$ are fixed points of g_n and $h(\overset{\sim}{z_n})$ is the ${j_n}^{th}$ neighbour of $h(z_n)$ among such cycles. It shows that $\left|h(z_n) - h\left(\overset{\sim}{z_n}\right)\right| \leq c2^{-n} \rightarrow 0$. Since $h(w_n)$ lies between $z_n and \overset{\sim}{z_n}$. Hence $|h(z_n) - h(w_n)| \leq |h(z_n) - h\left(\overset{\sim}{z_n}\right)| \rightarrow 0.$ This satisfies the sufficient condition for a Continuous extension. Therefore □ extends to a continuous function on $(0, 1)$. Now for the entire proof it is sufficient to show that $lim_{x\rightarrow 0}^{h(x)} = 0\ and\ 1.$ and $lim_{x\rightarrow 1}^{h(1)} = 1.$ The largest $n - cycles\ x_n$ of the tent map and its image $y_n$both converge to 1. Again as □ is monotone with the values in $[0,1\ ]$ and it implies that as $x \rightarrow 1, h(x) \rightarrow 1.$ Let us Consider the least $n - cycles\ (0, 1)$ proves that $\lim_{x\rightarrow 0} h(x) = 0$. Therefore h is extended so that $h(0) = 0$ and $h(1) = 1$ is continuous on $[0,1]$.

CONCLUSION

From the above discussion we conclude that the logistic map and tent map are topological conjugate.
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