Original Research Article

Approach to Solve Multi Objective Linear Fractional Problem

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Abstract

In this paper I have suggested an approach to solve multi objective fractional programming problem. Initially each of the fractional programming problem is solved considering single objective function and then each of the linear fractional programming problems is converted into linear programming problem using Taylor's approximation. To solve single problem there are many analytical methods as well as software's. In this paper single objective fractional linear programming problem is solved by using LINGO software. Then each of the fractional objective function is expanded about optimal solution vector by Taylor's Series method and converted it into approximate linear programming problem, using partial differentiation. Finally we have solved this problem as Multi Objective Linear Programming Problem using Fuzzy compromise programming approach.
Key Words: *Fractional, Multi objective, Linear Programming.*

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INTRODUCTION

In decision making process often the objective function is ratio of two linear function and objective function is to be optimised. When there are several fractional objective functions then the problem is called multi objective fractional linear programming problem (MOFLPP). Fractional programming problem can be converted into linear programming problem (LPP) by using variable transformation given by Charnes and Cooper [1]. MOFLPP can be converted into MOLPP using Taylor's series method [2].With variable transformation method Chakraborty and Gupta [3] converted MOLFPP to MOLPP using Fuzzy set theoretic approach. SurapatiPramanik and Durga Banerjee [4] gave solution to chance constrained multi objective linear plus fractional programming problem. Singh et.al.[5]developed an algorithm for solving MOLPFPP with the help of Taylor series.Pitramsingh ,et.al. [6] gave approach for multi objective linear plus fractional programming problem

MOLPP is solved using fuzzy compromise approach by Lushu Li and K Lai [7]. Multi objective transportation problem is solved by Doke and Jadhav [8]using fuzzy compromise programming approach.

FORMULATION OF MOLFPP

i) Definition: A linear fractional programming problem (**LFPP**) may be defined as under.

Maximize (or Minimize) $Q(x)=\frac{P(x)}{D(x)}=\frac{\sum_{j=1}^{n} pjxj}{\sum_{j=1}^{n} djxj}$ .... (1)

Subject to constraint $\sum_{j=1}^{n} aijxj \geq or \leq or = bi$ ....(2)

For all i = 1 ,2,3,....,m

$x_{ij} \geq 0$ for all i = 1,2,.....m and j=1,2,...n ....(3)

Where $p_j$is profit associated with variable X_j and d_j is cost associated with variableX_j. P(x) is total profit and D(x) is total costand $D(x) \neq 0$ for every $x = (x_1, x_2, x_3, ..., x_n) \varepsilon S$. Where S denotes a feasible set.Above problem is said in

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canonical form if $\sum_{j=1}^{n} aijxj = bi$ For all i = 1 ,2,3,....,m.
Q(x) is called objective function which is fractional. Note that P(x) and D(x) are linear and (2) and (3) are also linear. Hence the name Linear fractional Programming problem (LFPP)

ii) Definition: Suppose $Q_1(x) = \frac{P1(x)}{Q1(x)}$, $Q_2(x) = \frac{P2(x)}{Q2(x)}$,........,$Q_k(x) = \frac{Pk(x)}{Qk(x)}$ are k fractional objective functions to be optimised simultaneously subject to constraint (2) and (3) and $D_k(x) \geq 0$ for every k then the problem is called Multi Objective Linear Fractional Programming Problem (**MOLFPP**).
Note that if the problem is not in standard form then convert it into standard form by adding appropriate slack, surplus and artificial variables.

iii) Definition

i) The linear fractional programming problem is solvable if a) feasible set S is non empty and there exists at least one vector X such that it satisfies constraint (2) and (3) and b) the objective function Q(x) has a finite upper bound over set S.

ii) The System B = { As1,As2,...Asm} of vectors Aj is basis of vectors if As1,As2,...Asm are linearly independent.

iii) The given vector X = $(x_1,x_2,...,x_n)'$ is a basic solution of the system AX = b , if vector X satisfies system $\sum_{j=1}^{n} aijxj = bi$ for all j=1,2,...,n.

iv) The basic solution is degenerate if at least one of its basic variable equal to zero.

v) A basic solution X of system AX = b is said to be basic feasible solution (BFS) of LFP (1-3) if all elements of X satisfy non negativity constraint.

iv) Theorems

i) A point X of set corresponding to system Ax = b, is its extreme point if and only if it is its basic solution.

ii) Objective function Q (x) is monotonic on any segment of a straight line in feasible set S.

iii) If feasible set S in LFP (1-3) is bounded then the objective function Q(x) attains its maximal value over S in an extreme point of S.

SOLUTION PROCEDURE FOR MOLFPP

i) Consider MOLFPP ,Maximize { $Q_1(x),Q_2(x),...,Q_k(x)$ } subject to constraint (2-3)

ii) Find optimal solution of each of the fractional objective function subject to constraint. To solve fractional programming problem use LINGO .

iii) Suppose X_l* is optimal solution of of $Q_l(x)$ for l =1,2,...,k

iv) Expanding objective function $Q_l(x)$ about X^*_lusing Taylor's theorem and ignoring second and higher order terms convert $Q_l(X)$ into linear function.
Consider $Q_l(x) = \frac{Pl(x)}{Ql(x)}$ and X^*_l = ($x_{l1},x_{l2},x_{l3},...,x_{ln}$) be the optimal solution of $Q_l(x)$,then using Taylor Series approach we have

$$Q_l(x) \sim \cong Q_l(x^*_l) + (x_1-x_{1l}) \frac{\partial Ql(Xl)}{\partial x1} + (x_2-x_{2l}) \frac{\partial Ql(Xl)}{\partial x2} + ... + (x_k-x_{kl}) \frac{\partial Ql(Xl)}{\partial xk} + O(h^2).$$

Using this expansion each of the objective function becomes linear function. To avoid complexity of notations write
$Q_l(x) \sim= \check{Z}_l(x)$, l =1,2,3,....,k.
The problem is Multi Objective Linear Programming problem. (MOLPP)
It can be solved by using fuzzy compromise approach as stated in next section.

FUZZY COMPROMISE APPROACH FOR MOLPP.

Using Fuzzy Compromise approach for MOLPP.
Consider MOLPP
Maximize Z (x) =[Z_1 (x), Z_2 , …, Z_k (x)] ----------------(5)
Subject to $x \in S$ where S set of feasible solutions.
Solution to (5) is often conflicting as several objectives cannot be optimized simultaneously. To find compromise solutions first solve each of the objective function as marginal or single objective function. In this paper we have

converted Linear Fractional Programming problem to Linear Programming problem using Taylor series approach. Note that basic feasible optimal solution of LFPP is same as basic feasible optimal solution of LPP converted using Taylor Series approach.

Suppose x_k* is optimal solution of k^{th} objective function. Find values of each objective at optimal solution of k^{th} objective for all k= 1,2,…,K. Thus we have matrix of evaluation of objectives.

To find compromise solution use different membership functions. These membership functions and process to find optimal solution is stated below.

1. **Membership Functions: Linear Membership function**:

Consider MOLPP as stated below,

$$Maximize\ [\ Z_1(x), Z_2(x), \dots, Z_k(x)]$$

$$Subject\ to \sum_{j=1}^{n} a_{ij}x_j\ \geq\ or\ \leq\ or\ =\ b_i\ for\ i = 1,2,\dots,m$$

$$x_{ij}\ \geq 0\ for\ i = 1,2,\dots,m\ and\ j = 1,2\dots,n$$

For each of the objective function find best and worst values.

$$U_k = Max\ \{Z_k(x)\}\ , L_k = Min\ \{Z_k(x)\}$$

For each particular objective we define marginal evaluation function

$$\varphi_k(x) : X \to [0,1]$$

$$\varphi_k(x) = \begin{cases} 0\ if\ L_k\ \leq Z_k(x) \\ \frac{Z_k - U_k}{L_k - U_k}\ if\ L_k < Z_k(x) < U_k \\ 1\ if\ Z_k(x) \geq U_k \end{cases}$$

According to fuzzy sets, $\varphi_k(x)$ is fuzzy subset describing fuzzy concept of optimum for objective$Z_k(x)$ on feasible solution space S. To find compromise solution the aggregation operator is given by,

$$\mu\ (x) = \varphi_w\ [\varphi_1(x), \varphi_2(x), \dots, \varphi_k(x)]$$

$$Maximize\ \mu\ (x) = M_w^{(\alpha)}[\ \varphi_1(x), \varphi_2(x), \dots, \varphi_k(x)]$$

$$Maximize\ \mu\ (x) = \sum_{i=1}^{k} [w_i \varphi_i^{\alpha}(x)]^{1/\alpha}$$

i) If α = 1 then,

$$Maximize\ \mu\ (x) = \sum_{i=1}^{k}\ [w_i \varphi_i(x)]\ subject\ to\ constraints$$

This is nothing but weighted Arithmetic Mean of marginal evaluation of individual objective.

ii) If α = 2 then

$$Maximize\ \mu\ (x) = \sum_{i=1}^{k} [w_i \varphi_i^2(x)]^{1/2}\ subject\ to\ constraints$$

This is nothing but weighted Quadratic Mean of marginal evaluation of individual objective. These procedures gives weighted arithmetic mean and weighted quadratic mean to find global evaluation of multiple objectives.

Example:

Consider followingfour objective functions,

$$Maximize\ Q_1(x) = \frac{6x_1 + 12x_2 + 15x_3 + 20x_4}{2x_1 + 3x_2 + 6x_3 + 10x_4}$$

$$Maximize\ Q_2(x) = \frac{7x_1 + 9x_2 + 12x_3 + 16x_4}{4x_1 + 8x_2 + 10x_3 + 14x_4}$$

$$Maximize\ Q_3(x) = \frac{6x_1 + 9x_2 + 10x_3 + 12x_4}{8x_1 + 10x_2 + 11x_3 + 12x_4}$$

$$Maximize\ Q_4(x) = \frac{3x_1 + 7x_2 + 8x_3 + 16x_4}{5x_1 + 8x_2 + 10x_3 + 12x_4}$$

Subject to

$$3x_1 + 2x_2 + x_3 + 4x_4 \leq 100$$
$$3x_1 + x_2 + 2x_3 + 2x_4 \leq 120$$
$$63 + 4x_2 + x_3 + x_4 \leq 150$$
$$2x_1 + 3x_2 + 2x_3 + x_4 \leq 130$$
$$x_1 \geq 0, x_2 \geq 0, x_3 \geq 0, x_4 \geq 0$$

Solving above objective functions the optimal solution are as under,

$$X_1 = [x_1 = 0, x_2 = 37.5, x_3 = 0, x_4 = 0]$$
$$X_2 = [x_1 = 100/3, x_2 = 0, x_3 = 0, x_4 = 0]$$
$$X_3 = [x_1 = 0, x_2 = 0, x_3 = 0, x_4 = 25]$$
$$X_4 = [x_1 = 0, x_2 = 0, x_3 = 0, x_4 = 25]$$

Expanding each of the objective functions about their optimal solution using Taylor series and approximating then to linear functions we have following four linear objectives.

$$Z_1 = 4 - 0.0177x_1 - 0.08x_3 - 0.1774x_4$$
$$Z_2 = \frac{7}{4} - 0.0375x_2 - 0.04125x_3 - 0.6375x_4$$
$$Z_3 = \frac{6}{8} - 0.00666x_1 - 0.0033333x_2 - 0.003333x_3 + 0x_4$$
$$Z_4 = \frac{16}{12} - 0.012222x_1 - 0.012222x_2 - 0.0177778x_3 + 0x_4$$

To find marginal evaluation of linear objective functions we have

$U_1 = 4\ and\ L_1 = -1.9259$

$U_2 = \frac{7}{4}\ and\ L_2 = 0.34375$

$$U_3 = \frac{6}{8}\ and\ L_3 = 0.7778$$

$U_4 = \frac{16}{12}\ and\ L_4 = 0.875$

Marginal evaluation functions are

$\varphi_k(x) = \frac{Z_k - U_k}{L_k - U_k}\ for\ k = 1,2,3,4.$

$\varphi_1(x) = 0.0300005x_1 + 0.x_2 + 0.0135001\ x_3 + 0.0300005x_4$

$\varphi_2(x) = 0.\ x_1 + 0.0266667\ x_2 + 0.0293333x_3 + 0.0453333x_4$

$\varphi_3(x) = 0.0300315x_1 + 0.0150149\ x_2 + 0.0150135\ x_3 + 0x_4$

$\varphi_4(x) = 0.0266668\ x_1 + 0.0266664\ x_2 + 0.0387886\ x_3 + 0x_4$.

To find compromise solution, maximize aggregation operator,

$\mu\ (x) = \varphi_w\ [\varphi_1(x), \varphi_2(x), \varphi_3(x), \varphi_4(x)]$

$$Maximize\ \mu(x) = \varphi_w(x)[\varphi_1(x), \varphi_2(x), \varphi_3(x), \varphi_4(x)]$$
$$Maximize\ \mu(x) = M_w^\alpha(\varphi_1(x), \varphi_2(x), \varphi_3(x), \varphi_4(x))$$
$$\mu(x) = [\sum w_i\, \varphi_i^\alpha(x)]^{\frac{1}{\alpha}}$$

i) If α = 1 then $Maximize\ \mu(x) = \sum[w_i \varphi_i(x)]$

$$Maximize\ \mu(x) = [w_1\varphi_1(x) + w_2\varphi_2(x) + w_3\varphi_3(x) + w_4\varphi_4(x)]$$

$subject\ given\ set\ of\ constraints$

$$3x_1 + 2x_2 + x_3 + 4x_4 \leq 100$$
$$3x_1 + x_2 + 2x_3 + 2x_4 \leq 120$$
$$63 + 4x_2 + x_3 + x_4 \leq 150$$
$$2x_1 + 3x_2 + 2x_3 + x_4 \leq 130$$

$$x_1 \geq 0, x_2 \geq 0, x_3 \geq 0, x_4 \geq 0$$
$$w_1 + w_2 + w_3 + w_4 = 1$$

Solutions of above problem for different combinations weight using LONGO are as under

Sr.No.	Weights (w_1, w_2, w_3, w_4)	Solution (x_1, x_2, x_3, x_4)	$Q_1(x)$	$Q_2(x)$	$Q_3(x)$	$Q_4(x)$
1	(0.2,0.3,0.4,0.1)	(0,9.33,46.66,8.66)	2.4966	1.18109	0.89927	0.8711961
2	(0.1,0.4,0.3,0.2)	(0,5,57.5,0)	2.5625	1.195122	0.9084	0.804878
3	(0.5,0.3,0.1,0.1)	(0,0,46.66,13.33)	2.3387	1.18367	0.8952	0.8979

iii) ii)If α = 2 then

$$Maximize\ \mu(x) = \sum_{i=1}^{k} [w_i \varphi_i^2(x)]^{1/2}$$

$subject\ given\ set\ of\ constraints$

Solutions of above problem for different combinations weight using LINGO are as under

Sr.No.	Weights (w_1, w_2, w_3, w_4)	Solution (x_1, x_2, x_3, x_4)	$Q_1(x)$	$Q_2(x)$	$Q_3(x)$	$Q_4(x)$
1	(0.2,0.3,0.4,0.1)	(0,9.33,46.66,8.66)	2.4966	1.18109	0.89927	0.8711961
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3	(0.5,0.3,0.1,0.1)	(0,0,46.66,13.33)	2.3387	1.18367	0.8952	0.8979

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