# Wave Propagation at a Loosely Bonded Interface between Two Fluids Saturated Incompressible Porous Solids

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# Research Article

Abstract: Reflection and transmission phenomenon at a loosely bonded interface between two different fluid saturated porous half spaces is studied in the present investigation. P-wave or SV-wave incidents on the interface. The amplitude ratios for various reflected and transmitted waves to that of incident wave are obtained. After finding the amplitude ratios, they have been computed numerically for a specific model and plotted for different degree of bonding parameter. It is found that these amplitude ratios depend on angle of incidence of the incident wave and material properties of the considered medium and also these are affected by the bonding parameter. Also a special case is obtained and discussed from the present study accordingly.

*Keywords:* Porous solid, reflection, transmission, longitudinal wave, transverse wave, amplitude ratios, empty porous solid.

## Introduction

Elastic waves propagation in fluid saturated porous medium has been studied for a long time due to its importance in various fields such as soil dynamics, hydrology, seismology, earthquake engineering and geophysics. Layers of porous solids, such as sandstone or limestone, saturated with oil or groundwater are often present in the earth's crust. They are of great interest in geophysical exploration. Therefore the study of incompressible fluid saturated poroelastic solid in contact with another incompressible fluid saturated poroelastic solid is of great interest. Due to the different motions of the solid and liquid phases and different material properties and the complicated structures of pores, the mechanical behaviour of a fluid saturated porous medium is very complex. So many researchers tried to overcome this difficulty from time to time. Bowen (1980) and de Boer and Ehlers (1990a, 1990b) developed a theory for incompressible fluid saturated porous medium based on the work of Fillunger model (1913). For example, in the composition of soil both the solid constituents and liquid

constituents are incompressible. Based on this theory, many researchers like de Boer and Liu(1994,1995), Kumar and Hundal (2003), de Boer and Didwania (2004), Tajuddin and Hussaini (2006), Kumar et.al. (2011) etc. studied some problems of wave propagation in fluid saturated porous media. In the problems of wave propagation at the interface between two elastic half spaces, the contact between them is normally assumed to be welded. However, in certain situations, there are reasons for expecting that bonding is not complete. Murty in 1975 discussed a theoretical model for reflection, transmission, and attenuation of elastic waves through a loosely bonded interface between two elastic solid half spaces by assuming that the interface behaves like a dislocation which preserves the continuity of stresses allowing a finite amount of slip. A similar situation occurs at the two different poroelastic solids, as the liquid present in the porous skeleton may cause the two media to be loosely bonded. Vashisth and Gogna (1993), Kumar and Miglani (1996), Kumar and Singh (1997) etc. discussed the problems of reflection and transmission at the loosely bonded interface between two half spaces. Using de Boer and Ehlers (1990) theory for fluid saturated porous medium, the reflection and transmission of longitudinal wave (P-wave) or transverse wave (SVwave) at a loosely bonded interface between two different fluid saturated porous half spaces is investigated. A special case when fluid saturated porous half spaces reduce to empty porous solid half spaces has been deduced and discussed accordingly. Amplitudes ratios for various reflected and transmitted waves are computed for a particular model and depicted graphically.

# **Basic equations**

In 1990, de Boer and Ehlers described the governing equations for the deformation of an incompressible porous medium saturated with non-viscous fluid in the absence of body forces as

$$\nabla \cdot (\eta^{\mathbf{S}} \dot{\mathbf{u}}^{\mathbf{S}} + \eta^{\mathbf{F}} \dot{\mathbf{u}}^{\mathbf{F}}) = 0, (1)$$

$$\begin{split} & \left( \boldsymbol{\lambda}^S + \boldsymbol{\mu}^S \right) \! \nabla (\boldsymbol{\nabla}.\, \boldsymbol{u^S}) + \boldsymbol{\mu}^S \boldsymbol{\nabla}^2 \boldsymbol{u^S} - \boldsymbol{\eta}^S \boldsymbol{\nabla} \boldsymbol{p} - \boldsymbol{\rho}^S \boldsymbol{\ddot{u}}^S + S_v (\boldsymbol{\dot{u}}^F - \boldsymbol{\dot{u}}^S) = 0, (2) \\ & \boldsymbol{\eta}^F \boldsymbol{\nabla} \boldsymbol{p} + \boldsymbol{\rho}^F \boldsymbol{\ddot{u}}^F + S_v (\boldsymbol{\dot{u}}^F - \boldsymbol{\dot{u}}^S) = 0, (3) \\ & \boldsymbol{T_E^S} = 2 \boldsymbol{\mu}^S \boldsymbol{E^S} + \boldsymbol{\lambda}^S (E^S.\, \boldsymbol{I}) \boldsymbol{I}, (4) \\ & \boldsymbol{E^S} = \frac{1}{2} (grad \, \boldsymbol{u^S} + grad^T \boldsymbol{u^S}), (5) \end{split}$$

where  $\mathbf{u}^i, \dot{\mathbf{u}}^i, \ddot{\mathbf{u}}^i, \ddot{\mathbf{i}} = F, S$  denote the displacement, velocity and acceleration of fluid and solid phases, respectively and p is the effective pore pressure of the incompressible pore fluid.  $\rho^{S}$  and  $\rho^{F}$  are the densities of the solid and fluid constituents, respectively.  $T_E^S$  is the effective stress in the solid phase and  $E^S$  is the linearized langrangian strain tensor.  $\lambda^S$  and  $\mu^S$  are the macroscopic Lame's parameters of the porous solid and  $\eta^S$  and  $\eta^F$  are the volume fractions satisfying

$$\eta^{S} + \eta^{F} = 1.(6)$$

In the case of isotropic permeability, to describe the coupled interaction between the solid and fluid, de Boer and Ehlers (1990) gave the tensor  $S_v$  as

$$\mathbf{S}_{\mathrm{v}} = \frac{(\eta^{\mathrm{F}})^2 \gamma^{\mathrm{FR}}}{\mathrm{K}} \mathbf{I}, (7)$$

where  $\gamma^{FR}$  is the specific weight of the fluid and K is the Darcy's permeability coefficient of the porous medium and I stands for unit vector.

The displacement vector  $\mathbf{u}^{i}$  (i = F, S) can be assumed as

$$\mathbf{u}^{i} = (\mathbf{u}^{i}, 0, \mathbf{w}^{i}), \text{ where } i = F, S, (8)$$

and therefore equations (1)- (3) describing the equations of motion for fluid saturated incompressible porous medium in the component form can be written as

$$\begin{split} \left(\lambda^S + \mu^S\right) \frac{\partial \theta^S}{\partial x} + \mu^S \nabla^2 u^S - \eta^S \frac{\partial p}{\partial x} - \rho^S \frac{\partial^2 u^S}{\partial t^2} + S_v \left[ \frac{\partial u^F}{\partial t} - \frac{\partial u^S}{\partial t} \right] &= 0, (9) \\ \left(\lambda^S + \mu^S\right) \frac{\partial \theta^S}{\partial z} + \mu^S \nabla^2 w^S - \eta^S \frac{\partial p}{\partial z} - \rho^S \frac{\partial^2 w^S}{\partial t^2} + S_v \left[ \frac{\partial w^F}{\partial t} - \frac{\partial w^S}{\partial t} \right] &= 0, (10) \\ \eta^F \frac{\partial p}{\partial x} + \rho^F \frac{\partial^2 u^F}{\partial t^2} + S_v \left[ \frac{\partial u^F}{\partial t} - \frac{\partial u^S}{\partial t} \right] &= 0, (11) \\ \eta^F \frac{\partial p}{\partial z} + \rho^F \frac{\partial^2 w^F}{\partial t^2} + S_v \left[ \frac{\partial w^F}{\partial t} - \frac{\partial w^S}{\partial t} \right] &= 0, (12) \\ \eta^S \left[ \frac{\partial^2 u^S}{\partial x \partial t} + \frac{\partial^2 w^S}{\partial z \partial t} \right] + \eta^F \left[ \frac{\partial^2 u^F}{\partial x \partial t} + \frac{\partial^2 w^F}{\partial z \partial t} \right] &= 0, (13) \\ \text{where} \\ \theta^S &= \frac{\partial (u^S)}{\partial x} + \frac{\partial (w^S)}{\partial z} . (14) \\ \text{and} \\ \nabla^2 &= \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial z^2} . (15) \end{split}$$

With the help of Helmholtz decomposition of displacement vector, the displacement components ui and wi are related to the potential functions  $\phi^i$  and  $\psi^i$  as given below

$$u^{i} = \frac{\partial \phi^{i}}{\partial x} + \frac{\partial \psi^{i}}{\partial z}, w^{i} = \frac{\partial \phi^{i}}{\partial z} - \frac{\partial \psi^{i}}{\partial x}, i = F, S. (16)$$
Using, (16) in eqs. (9)- (13), we obtain the following equations:

$$\nabla^2 \varphi^S - \frac{1}{C^2} \frac{\partial^2 \varphi^S}{\partial t^2} - \frac{S_v}{\left(\lambda^S + 2\mu^S\right) (\eta^F)^2} \frac{\partial \varphi^S}{\partial t} = 0, (17)$$

$$\phi^{F} = -\frac{\eta^{S}}{\eta^{F}}\phi^{S}, (18)$$

$$\mu^{S}\nabla^{2}\psi^{S} - \rho^{S}\frac{\partial^{2}\psi^{S}}{\partial t^{2}} + S_{v}\left[\frac{\partial\psi^{F}}{\partial t} - \frac{\partial\psi^{S}}{\partial t}\right] = 0, (19)$$

$$\begin{split} & \rho^F \frac{\partial^2 \psi^F}{\partial t^2} + S_v \left[ \frac{\partial \psi^F}{\partial t} - \frac{\partial \psi^S}{\partial t} \right] = 0, (20) \\ & (\eta^F)^2 p - \eta^S \rho^F \frac{\partial^2 \varphi^S}{\partial t^2} - S_v \frac{\partial \varphi^S}{\partial t} = 0, (21) \\ & \text{where} \\ & C = \sqrt{\frac{(\eta^F)^2 \left(\lambda^S + 2\mu^S\right)}{(\eta^F)^2 \rho^S + (\eta^S)^2 \rho^F}}. (22) \end{split}$$

The normal and tangential stresses in the solid phase take the form,

$$t_{zz}^{S} = \lambda^{S} \left( \frac{\partial^{2} \phi^{S}}{\partial x^{2}} + \frac{\partial^{2} \phi^{S}}{\partial z^{2}} \right) + 2\mu^{S} \left( \frac{\partial^{2} \phi^{S}}{\partial z^{2}} - \frac{\partial^{2} \psi^{S}}{\partial x \partial z} \right), (23)$$

$$t_{zx}^{S} = \mu^{S} \left( 2 \frac{\partial^{2} \phi^{S}}{\partial x \partial z} + \frac{\partial^{2} \psi^{S}}{\partial z^{2}} - \frac{\partial^{2} \psi^{S}}{\partial x^{2}} \right). (24)$$

Taking the time harmonic solution of the system of equations (17) - (21) as  $(\varphi^S, \varphi^F, \psi^S, \psi^F, p) = \left(\varphi_1^{\ S}, \varphi_1^{\ F}, \psi_1^{\ S}, \psi_1^{\ F}, p_1\right) \exp(i\omega t) \, , (25)$ 

$$(\phi^{S}, \phi^{F}, \psi^{S}, \psi^{F}, p) = (\phi_{1}^{S}, \phi_{1}^{F}, \psi_{1}^{S}, \psi_{1}^{F}, p_{1}) \exp(i\omega t), (25)$$

where  $\omega$  is the complex circular frequency.

Using equation (25) in equations (17)-(21), we get 
$$\begin{bmatrix} \nabla^2 + \frac{\omega^2}{C^2} - \frac{i\omega S_v}{\left(\lambda^S + 2\mu^S\right)(\eta^F)^2} \end{bmatrix} \phi_1^S = 0, (26)$$
 
$$[\mu^S \nabla^2 + \rho^S \omega^2 - i\omega S_v] \psi_1^S = -i\omega S_v \psi_1^F, (27)$$
 
$$[-\omega^2 \rho^F + i\omega S_v] \psi_1^F - i\omega S_v \psi_1^S = 0, (28)$$
 
$$(\eta^F)^2 p_1 + \eta^S \rho^F \omega^2 \phi_1^S - i\omega S_v \phi_1^S = 0, (29)$$
 
$$\phi_1^F = -\frac{\eta^S}{\eta^F} \phi_1^S. (30)$$

Equation (26) represents the propagation of a longitudinal wave with velocity V<sub>1</sub>, where

$$V_1^2 = \frac{1}{G_1}, (31)$$
and  $G_1 = \left[\frac{1}{C^2} - \frac{iS_v}{\omega(\lambda^S + 2\mu^S)(\eta^F)^2}\right]. (32)$ 

Using equations (27) and (28), we get

$$\left[\nabla^2 + \frac{\omega^2}{{V_2}^2}\right] \psi_1^S = 0. (33)$$

Equation (33) describes the propagation of transverse wave with velocity V<sub>2</sub>, which is given by

$$V_2^2 = \frac{1}{G_2}$$

where

$$G_2 = \left\{ \frac{\rho^S}{u^S} - \frac{iS_v}{u^S_{\omega}} - \frac{S_v^2}{u^S(-\rho^S_{\omega^2} + i\omega S_v)} \right\}, (34)$$

# Formulation of the Problem and its Solution.

Consider a fluid saturated porous half space medium M<sub>2</sub> [z < 0] lying over another fluid saturated porous medium  $M_1$  [z > 0] (see figure 1). The interface between two half spaces is considered an imperfect boundary and taking the zaxis pointing into lower half-space. A longitudinal wave (P-wave) or transverse wave (SV-wave) propagating through the medium  $M_1$  and incident at the plane z=0 and making an angle  $\theta_0$  with normal to the surface. Corresponding to each incident wave (P-wave or SV-wave), we get two reflected waves P-wave and SV-wave in the medium M1 and two transmitted waves P-wave and SV-wave in medium M2. The Geometry of the problem conforms the two dimensional problem.

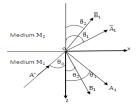


Figure 1: Geometry of the problem

## In medium M<sub>1</sub>

The potential functions satisfying the equations (17)-(21) can be taken as

$$\{\varphi^S, \varphi^F, p\} = \{1, m_1, m_2\}[A_{01} \exp\{ik_1(x \sin\theta_0 - z \cos\theta_0) + i\omega_1 t\} \\ + A_1 \exp\{ik_1(x \sin\theta_1 + z \cos\theta_1) + i\omega_1 t\}], (35) \\ \{\psi^S, \psi^F\} = \{1, m_3\}[B_{01} \exp\{ik_2(x \sin\theta_0 - z \cos\theta_0) + i\omega_2 t\} \\ + B_1 \exp\{ik_2(x \sin\theta_2 + z \cos\theta_2) + i\omega_2 t\}], (36) \\ \text{where}$$

$$m_1 = -\frac{\eta^S}{\eta^F}, m_2 = -\left[\frac{\eta^S \omega_1^2 \rho^F - i\omega_1 S_v}{(\eta^F)^2}\right], m_3 = \frac{i\omega_2 S_v}{i\omega_2 S_v - \omega_2^2 \rho^F}, (37)$$

and A<sub>01</sub> and B<sub>01</sub> are amplitudes of the incident P-wave and SV-wave, respectively. A<sub>1</sub>, B<sub>1</sub> are amplitudes of the reflected P-wave and SV-wave respectively.

# In medium M<sub>2</sub>

The potential functions satisfying the equations (17)-(21) can be taken as follow

$$\begin{split} &\left\{\overline{\varphi}^S, \overline{\varphi}^F, \overline{p}\right\} = \{1, \overline{m}_1, \overline{m}_2\} \left[ \ \overline{A}_1 exp \left\{ i \overline{k}_1 \left( x \sin \overline{\theta}_1 + z \cos \overline{\theta}_1 \right) + i \overline{\omega}_1 t \right\} \right], (38) \\ &\left\{\overline{\psi}^S, \overline{\psi}^F\right\} = \{1, \overline{m}_3\} \left[ \overline{B}_1 exp \left\{ i \overline{k}_2 \left( x \sin \overline{\theta}_2 + z \cos \overline{\theta}_2 \right) + i \overline{\omega}_2 t \right\} \right], (39) \end{split}$$

where  $\bar{k}_1$  and  $\bar{k}_2$  are wave numbers of transmitted P-wave and SV-wave, respectively.  $\bar{A}_1$  and  $\bar{B}_1$  are amplitudes of transmitted P-wave and transmitted SV-wave.

$$\overline{m}_{1} = -\frac{\overline{\eta}^{S}}{\overline{\eta}^{F}}, \overline{m}_{2} = -\left[\frac{\overline{\eta}^{S}\overline{\omega}_{1}^{2}\overline{\rho}^{F} - i\overline{\omega}_{1}\overline{S}_{v}}{\left(\overline{\eta}^{F}\right)^{2}}\right], \overline{m}_{3} = \frac{i\overline{\omega}_{2}\overline{S}_{v}}{i\overline{\omega}_{2}\overline{S}_{v} - \overline{\omega}_{2}^{2}\overline{\rho}^{F}}, (40)$$

# **Boundary Conditions**

The appropriate boundary conditions at the interface z=0 are the continuity of displacement and stresses. These boundary conditions can be expressed in the mathematical form as:

$$t_{zz}^{S} - p = \overline{t_{zz}}^{S} - \overline{p}, t_{zx}^{S} = \overline{t_{zx}}^{S},$$

$$w^{S} = \overline{w}^{S}, \overline{t_{zx}}^{S} = K_{t}(u^{S} - \overline{u}^{S}), (41)$$

where  $K_t = ik\mu\tau$  and  $\tau = \gamma/(1 - \gamma)\sin\theta_0$  (42)

 $\gamma$  is bonding constant.  $0 \le \gamma \le 1$ ,  $\gamma = 0$  corresponds to smooth surface and  $\gamma = 1$  corresponds to a welded interface In order to satisfy the boundary conditions, the extension of the Snell's law gives

$$\frac{\sin\theta_0}{V_0} = \frac{\sin\theta_1}{V_1} = \frac{\sin\theta_2}{V_2} = \frac{\sin\overline{\theta}_1}{\overline{V}_1} = \frac{\sin\overline{\theta}_2}{\overline{V}_2}, (43)$$

$$k_1V_1 = k_2V_2 = \overline{k}_1\overline{V}_1 = \overline{k}_2\overline{V}_2 = \omega$$
, at  $z=0$ , (44)

where  $\overline{V}_1$  and  $\overline{V}_2$  are the velocities of the transmitted P-wave and transmitted SV –wave respectively and can be obtained in the similar way as  $V_1$  and  $V_2$  are obtained.

For P-wave.

$$V_0 = V_1, \theta_0 = \theta_1, (45)$$
  
For SV-wave,

$$V_0 = V_2, \theta_0 = \theta_2, (46)$$

For incident longitudinal wave at the interface z=0, putting  $B_{01}=0$  in equation (36) and for incident transverse wave putting  $A_{01} = 0$  in equation (35). Substituting the expressions of potentials given by (35)-(36) and (38)-(39) in equations (16),(23)-(24) and the use of equations (41)-(46), gives a system of four non homogeneous which can be written as

$$\sum_{j=0}^{4} a_{ij} Z_j = Y_i, (i = 1,2,3,4) (47)$$

where

$$Z_1 = \frac{A_1}{A^*}$$
,  $Z_2 = \frac{A_2}{A^*}$ ,  $Z_3 = \frac{\overline{A}_1}{A^*}$ ,  $Z_4 = \frac{\overline{B}_1}{A^*}$  (48)

Also aii in non dimensional form can be written as

$$\begin{split} a_{11} &= \frac{\lambda^S}{\mu^S} + 2\text{cos}^2\theta_1 + \frac{m_2}{\mu^S k_1^{\ 2}}, a_{12} = -2\text{sin}\theta_2\text{cos}\theta_2 \frac{k_2^{\ 2}}{k_1^{\ 2}}, \ a_{13} = \frac{-\bar{k}_1^{\ 2}}{k_1^2 \mu^S} \Bigg[ \overline{\lambda}^S + 2\overline{\mu}^S \text{cos}^2 \overline{\theta}_1 + \frac{\overline{m}_2}{\bar{k}_1^{\ 2}} \Big], \\ a_{14} &= -\frac{\overline{\mu}^S \bar{k}_2^{\ 2}}{k_1^2 \mu^S} \text{sin} 2\overline{\theta}_2 \ , \ a_{21} = 2\text{sin}\theta_1 \text{cos}\theta_1 \ , \ a_{22} = \frac{k_2^{\ 2}}{k_1^2} (\text{cos}^2\theta_2 - \text{sin}^2\theta_2), a_{23} = \frac{\overline{\mu}^S \bar{k}_1^{\ 2}}{k_1^2 \mu^S} \text{sin} 2\overline{\theta}_1 \ , \\ a_{24} &= -\frac{\overline{\mu}^S \bar{k}_2^{\ 2}}{k_1^2 \mu^S} \text{cos} 2\overline{\theta}_2 \ , \ a_{31} = K_t \ i \ \text{sin}\theta_1, \ a_{32} = \frac{K_t \ i \ k_2 \text{cos}\theta_2}{k_1} \ , \ a_{33} = -\frac{K_t \ i \ \bar{k}_1}{k_1} \text{sin}\overline{\theta}_1 - \frac{\overline{\mu}^S \bar{k}_1^2 \text{sin} 2\overline{\theta}_1}{k_1} \\ a_{34} &= \frac{K_t \ i \ \bar{k}_2 \text{cos} \overline{\theta}_2}{k_1} + \frac{\overline{\mu}^S \bar{k}_2^2 \text{cos} 2\overline{\theta}_2}{k_1} \ , a_{41} = i \ \text{cos}\theta_1 \ , a_{42} = -\frac{i \ k_2 \text{sin}\theta_2}{k_1} \ , \\ a_{43} &= \frac{i \ \bar{k}_1 \text{cos} \overline{\theta}_1}{k_1} \ , \ a_{44} = \frac{i \ \bar{k}_2 \text{sin} \overline{\theta}_2}{k_1} \ , (49) \end{split}$$

For incident longitudinal wave (P-wave)

$$A^* = A_{01}$$
,  $Y_1 = -a_{11}$ ,  $Y_2 = a_{21}$ ,  $Y_3 = -a_{31}$ ,  $Y_4 = a_{41}$ , (50)

For incident transverse wave:

$$A^* = B_{01}, Y_1 = a_{12}, Y_2 = -a_{22}, Y_3 = a_{32}, Y_4 = -a_{42}$$
 (51)

# Special case:

If pores are absent or gas is filled in the pores then  $\rho^F$  is very small as compared to  $\rho^S$  and can be neglected, so the relation (22) gives us

$$C = \sqrt{\frac{\lambda^S + 2\mu^S}{\rho^S}}. (52)$$

and the coefficients  $a_{11}$ ,  $a_{13}$  and  $a_{43}$  in (49) changes to

$$a_{11} = \frac{\lambda^{S}}{\mu^{S}} + 2\cos^{2}\theta_{1}, \ a_{13} = \frac{-\overline{k}_{1}^{2}}{k_{1}^{2}\mu^{S}} \left[\overline{\lambda}^{S} + 2\overline{\mu}^{S}\cos^{2}\overline{\theta}_{1}\right], (53)$$

and the remaining coefficients in (49) remain same. In this situation the problem reduces to the problem of empty porous solid half space over empty porous solid half space.

## Numerical results and discussion

In medium M<sub>1</sub>, the physical parameters for fluid saturated incompressible porous medium are taken from de Boer, Ehlers and Liu (1993) as

$$\begin{split} \eta^s &= 0.67, \eta^F = 0.33, \rho^s = 1.34 \text{ Mg/m}^3, \rho^F = 0.33 \text{ Mg/m}^3, \lambda^s = 5.5833 \text{ MN/m}^2, \\ K^F &= 0.01 \text{m/s}, \gamma^{FR} = 10.00 \text{KN/m}^3, \mu^s = 8.3750 \text{N/m}^2, \omega^* = 10/\text{s}, (54) \end{split}$$

In medium  $M_2$ , the physical parameters are

$$\overline{\eta}^{s} = 0.6, \overline{\eta}^{F} = 0.4, \overline{\rho}^{s} = 2.0 \text{ Mg/m}^{3}, \overline{\rho}^{F} = 0.01 \text{ Mg/m}^{3}, \overline{\lambda}^{s} = 4.2368 \text{ MN/m}^{2}, \overline{K}^{F} = 0.02 \text{ m/s}, \overline{\gamma}^{FR} = 9.00 \text{ KN/m}^{3}, \overline{\mu}^{s} = 3.3272 \text{N/m}^{2}, \omega^{*} = 10/\text{s}, (55)$$

Using MATLAB, a computer programme has been developed and modulus of amplitude ratios  $|Z_i|$ , (i = 1,2,3,4,) for various reflected and transmitted waves have been computed.  $|Z_1|$  and  $|Z_2|$  represent the modulus of amplitude ratios for reflected P and reflected SV-wave respectively. Also,  $|Z_3|$  and  $|Z_4|$  represent the modulus of amplitude ratios for transmitted P and transmitted SV-wave respectively.]

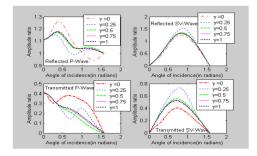
#### **Incident P-wave**

Figures (2)-(5) show the variations of the amplitude ratios of reflected P-wave, reflected SV-wave, transmitted P-wave and transmitted SV-wave with angle of incidence of incident P-wave in general case of fluid saturated half spaces. The

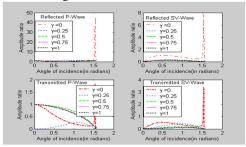
behaviour of all these distribution curves for reflected P-wave and for transmitted P-wave is similar i.e. they oscillate. For reflected SV-wave and transmitted SV-wave, the behaviour of all curves is also same i.e. increasing from normal incidence to maximum value and then decreasing from maximum value to grazing incidence. Figures (6)-(9) show the variations of the amplitude ratios of reflected P-wave, reflected SV-wave, transmitted P-wave and transmitted SV-wave with angle of incidence of incident P-wave in special case of empty porous solid half spaces. The effect of fluid filled in the pores of fluid saturated porous medium is clear by comparing the maximum values of corresponding amplitude ratio in figures (2)-(5) and (6)-(9). Also in these figures, the values corresponding to bonding parameter  $\gamma = 0$ , i.e., for smooth interface are large in comparison to other interface parameters.

#### **Incident SV-wave**

Figures (10)-(13) show the variations of the amplitude ratios for reflected P-wave, reflected SV-wave, transmitted P-wave and transmitted SV-wave with angle of incidence of the incident SV-wave whereas figures (14)-(17) represent the case of empty porous solid. The behaviour of all these curves in figures (6)-(9) and (14)-(17) is same i.e. they oscillates. In all the figures (10)-(17), the amplitude ratios for the bonding parameter  $\gamma = 0.25$  are maximum. The effect of fluid filled in the pores of fluid saturated porous medium is clear by comparing the maximum values of corresponding amplitude ratio in figures (10)-(13) and (14)-(17).



*Figure 2-5:* Variation of the amplitude ratios of reflected P-wave, reflected SV-wave, transmitted P-wave and transmitted SV-wave with angle of incidence of P-wave.



*Figure 6-9:* Variation of the amplitude ratios of reflected P-wave, reflected SV-wave, transmitted P-wave and transmitted SV-wave with angle of incidence of P-wave in case of empty porous solids.

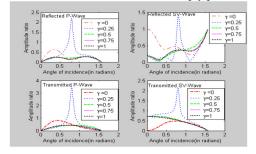
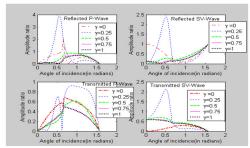


Figure 10-13: Variation of the amplitude ratios of reflected P-wave, reflected SV-wave, transmitted P-wave and transmitted SV-wave with angle of incidence of SV-wave.



*Figure 14-17:* Variation of the amplitude ratios of reflected P-wave reflected SV-wave, transmitted P-wave and transmitted SV-wave with angle of incidence of P-wave in case of empty porous solids.

# **Conclusion**

Numerical calculations are presented in detail for P-wave and SV-wave incident at the loosely bonded interface of considered model. For both the cases of incidence, it is observed the amplitudes ratios of various reflected and transmitted waves depend on the angle of incidence of the incident wave and material properties. The effect of fluid filled in the pores of incompressible fluid saturated porous medium is significant on amplitudes ratios. Effect of bonding parameter is significant on amplitude ratios. The research work is supposed to be useful in further studies; both theoretical and observational of wave propagation in more realistic models of fluid saturated porous solid present in the earth's interior.

# References

- 1. Bowen, R.M., *Incompressible porous media models by use of the theory of mixtures*, International Journal of Engineering Science, 18(9), 1129-1148, 1980.
- de Boer, and Didwania, A. K., Two phase flow and capillarity phenomenon in porous solid- A Continuum Thermomechanical Approach, Transport in Porous Media (TIPM), 56, 137-170, 2004.
- 3. de Boer, R. and Ehlers, W., *The development of the concept of effective stress*, Acta Mechanica A 83, 77-92,1990.
- 4. de Boer, R. and. Ehlers, W , Uplift, friction and capillarity-three fundamental effects for liquid- saturated porous solids, Int. J, Solid Structures B, 26,43-57,1990.
- 5. de Boer, R., Ehlers, W. and Liu, Z., One dimensional transient wave propagation in fluid saturated

- incompressible porous media, Arch. App. Mech. 63, 59-72, 1993.
- de Boer, R. and Liu, Z., Plane waves in a semi-infinite fluid saturated porous medium, Transport in Porous Media, 16 (2), 147-173, 1994.
- 7. de Boer, R. and Liu, Z., *Propagation of acceleration waves in incompressible liquid saturated porous solid*, Transport in porous Media (TIPM), 21, 163-173, 1995.
- Fillunger, P.; Der Auftrieb in Talsperren. Osterr., Wochenschrift fur den offentl. Baudienst, 19, 532 - 556, 1913.
- 9. Kumar, R. and Hundal, B.S., *Wave propagation in a fluid saturated incompressible porous medium*, Indian J. Pure and Applied Math.4, 51-65, 2003.
- 10. Kumar,R.,Miglani,A. and Kumar,S., Reflection and Transmission of plane waves between two different fluid saturated porous half spaces, Bull. Pol. Ac., Tech., 227-234, 59(2), 2011.
- 11. Kumar,R. and Singh,B., Reflection and transmission of elastic waves at a loosely bonded interface between an elastic and micropolar elastic solid, Indian Journal of Pure and Applied Mathematics, 28, 1133-1153, 1997.
- Murty, G.S.,Reflection, transmission and attenuation of elastic waves at a loosely bonded interface of two half spaces, Geophys. J.R. Astrom. Soc., 44, 389-404, 1976.
- 13. Tajuddin, M. and Hussaini, S.J., Reflection of plane waves at boundaries of a liquid filled poroelastic half-space, J. Applied Geophysics 58,59-86,2006.
- Vashisth, A.K., and Gogna, M.L., The effect of loose boundaries on wave propagation in a porous solid :reflection and refraction of seismic waves across a plane interface, international Journal of Solid Structure, 30,2485-2499,1993.