On Contra -πgb -Continuous Functions and Approximately -πgb-Continuous Functions in Topological Spaces

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Abstract: In this paper a new class of sets called contra-πgb-continuous functions is introduced and its properties are studied. Further the notion of approximately -πgb-continuous functions and almost contra- πgb-continuous functions are introduced.

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1.Introduction

Andrijevic [3] introduced a new class of generalized open sets in a topological space, the so-called b-open sets. This type of sets was discussed by Ekici and Caldas [11] under the name of γ-open sets. The class of b-open sets is contained in the class of semi-pre-open sets and contains all semi-open sets and preopen sets. The class of b-open sets generates the same topology as the class of preopen sets. Since the advent of these notions, several research paper with interesting results in different respects came to existence [1-3-6-11-13-18-19-20]). Levine [15] introduced the concept of generalized closed sets in topological space and a class of topological spaces called $T_{½}$ spaces. Extensive research on generalizing closedness was done in recent years as the notions of a generalized closed, generalized semi-closed, α-generalized closed, generalized semi-pre-open closed sets were investigated in [2-7-15-16-17].The finite union of regular open sets is said to be π-open. The complement of a π-open set is said to be π-closed.

The aim of this paper is to study the notion of contra-πgb-continuous functions, and its various characterizations are given in this paper. In Section 3, we study basic properties of approximately-πgb-continuous functions. In Section 4, some properties of almost contra- πgb- continuous functions are discussed.

2. Preliminaries

Throughout this paper (X, τ) and (Y, τ) represent non-empty topological spaces on which no separation axioms are assumed unless otherwise mentioned. For a subset A of a space (X,τ) , cl(A) and int(A) denote the closure of A and the interior of A respectively.(X, τ) will be replaced by X if there is no chance of confusion.

Let us recall the following definitions which we shall require later.

Definition 2.1: A subset A of a space (X, τ) is called

(1) a regular open set if A= int (cl(A)) and a regular closed set if A= cl(int (A));

(2) b-open [3] or sp-open [8], γ –open [11] if $A\subset cl(int(A))\cup int(cl(A))$.

The complement of a b-open set is said to be b-closed [3]. The intersection of all b-closed sets of X containing A is called the b-closure of A and is denoted by bCl(A). The union of all b-open sets of X contained in A is called b-interior of A and is denoted by bInt(A). The family of all b-open (resp. α-open, semi-open, preopen, β-open, b-closed, preclosed) subsets of a space X is denoted by bO(X)(resp. αO(X), SO(X), PO(X), βO(X), bC(X), PC(X)) and the collection of all b-open subsets of X containing a fixed point x is denoted by bO(X,x). The sets SO(X, x), αO(X, x), PO(X, x), βO(X, x) are defined analogously.

Lemma 2.2 [3]: Let A be a subset of a space X. Then

(1) $bCl(A)=sCl(A) \cap pCl(A) = A\cup[Int(Cl(A)) \cap Cl(Int(A))]$;

(2) $bInt(A) = sInt(A) \cup pInt(A) = A\cap [Int(Cl(A)) \cup Cl(Int(A))]$;

Definition 2.3: A subset A of a space (X, τ) is called
1) a generalized b-closed (briefly gb-closed)[14] if bcl(A) ⊂ U whenever A ⊂ U and U is open.
2) πg-closed [10] if cl(A) ⊂ U whenever A⊂ U and U is π-open.
3) πgb –closed [22] if bcl(A) ⊂ U whenever A ⊂U and U is π-open in (X, τ).
By πGBC(τ) we mean the family of all πgb- closed subsets of the space(X, τ).

Definition 2.4: A function f: (X, τ) → (Y, σ) is called
1) π- irresolute [5] if $f^{-1}(V)$ is π- closed in (X, τ) for every π-closed set V of (Y,σ);
2) b-irresolute: [11] if for each b-open set V in Y,$f^{-1}(V)$ is b-open in X;
3) b-continuous: [11] if for each open set V in Y,$f^{-1}(V)$ is b-open in X.
4) πgb- continuous [22] if every $f^{-1}(V)$ is πgb- closed in (X, τ) for every closed set V of (Y,σ).
5) πgb- irresolute [22] if $f^{-1}(V)$ is πgb- closed in(X, τ) for every gbr-closed set V in (Y, σ).

Definition 2.5[22]**:** A topological space X is a πgb-space if every πgb- closed set is closed.

Definition2.6 [22]: A space (X, τ) is called a πgb-$T_{1/2}$ space if every πgb- closed set is b-closed.

Definition2.7: A function f:(X,τ) → (Y,σ) is called
(i) contra –continuous[9] if $f^{-1}(V)$ is closed in X for each open set V of Y.
(ii) contra-b-continuous [20] if $f^{-1}(V)$ is b-closed in X for each open set V of Y.
(iii) contra-πg-continuous [12] if $f^{-1}(V)$ is πg-closed in X for each open set V of Y.
(iv) contra-πgα-continuous [4] if $f^{-1}(V)$ is πgα-closed in X for each open set V of Y.

Definition 2.8: A space X is said to be
(i) strongly-S-closed [9] if every closed cover of X has a finite sub-cover.
(ii) mildly compact [21] if every clopen cover of X has a finite sub-cover.
(iii) strongly-S-Lindelof [9] if every closed cover of X has a countable sub-cover.

3. Contra-πgb-continuous functions:

Definition 3.1: A function f:(X, τ)→(Y,σ) is called contra-πgb-continuous if $f^{-1}(V)$ is πgb-closed in (X, τ) for each open set V of (Y, σ).

Theorem 3.2:(i) Every contra continuous function is contra- πgb- continuous.
(ii) Every contra-b-continuous function is contra-πgb- continuous.
(iii) Every contra-πg-continuous function is contra-πgb-continuous.
(iv) Every contra-πgα-continuous function is contra-πgb-continuous.

Remark 3.3: Converse of the above statements is not true as shown in the following example.

Example3.4 (i) Let X = {a,b,c}, τ = {Φ,X,{a}}, σ = {Φ,X,{b},{c},{b,c}}.Then the identity function f: (X, τ) → (X,σ) is contra- πgb- continuous but not contra –continuous.
(ii) Let X = {a,b,c,d}, τ = {Φ,X,{a},{b},{a,b}}, σ = {Φ,X,{b,c}}.Then the identity function f: (X,τ) → (X,σ) is contra- πgb- continuous but not contra -b-continuous.
(iii)Let X={a,b,c,d},τ= {Φ,X,{a},{d},{a,d},{c,d}{a,c,d}}, σ ={Φ,X,{c}}.Then the identity function f: (X,τ) → (X,σ) is contra- πgb- continuous but not contra πg-continuous.
(iv)) Let X = {a,b,c,d,e}, τ = {Φ,X,{a,b},{c,d},{a,b,c,d}}, σ ={Φ,X,{a}}.Then the identity function f: (X,τ) → (X,σ) is contra- πgb-continuous but not contra πgα- continuous.

Definition 3.5: A space (X,τ) is called
(i) πgb-locally indiscrete if every πgb-open set is closed.
(ii) a $T_{\pi gb}$-space if every πgb-closed set is πg-closed.

Theorem3.6 (i) If a function f :(X,τ) → (Y,σ) is πgb-continuous and (X,τ) is πgb-locally indiscrete, then f is contra-continuous.
(ii) If a function f :(X,τ) → (Y,σ) is contra-πgb-continuous and (X,τ) is πgb-$T_{1/2}$ space, then f is contra-b-continuous.
(iii) If a function f :(X, τ) → (Y, σ) is contra-πgb-continuous and (X,τ) is πgb- space, then f is contra-continuous.
(iv) If a function f :(X, τ) → (Y, σ) is contra-πgb-continuous and (X,τ) is $T_{\pi gb}$ - space, then f is contra-πg-continuous.

Proof :(i)Let V be open in (Y,σ).By assumption, f^{-1} V) is πgb-open in X. Since X is locally indiscrete, $f^{-1}(V)$ is closed in X. Hence f is contra-continuous.
(ii) Let V be open in (Y, σ). By assumption, $f^{-1}(V)$ is πgb-closed in X. Since X is πgb-$T_{1/2}$ space, $f^{-1}(V)$ is b-closed in X. Hence f is contra-b-continuous.
(iii) Let V be open in (Y, σ). By assumption,$f^{-1}(V)$ is πgb-closed in X. Since X is πgb-space, $f^{-1}(V)$ is closed in X. Hence f is contra-continuous.
(iv) Let V be open in (Y,σ). By assumption, $f^{-1}(V)$ is πgb-closed in X. Since X is $T_{\pi gb}$-space, $f^{-1}(V)$ is πg-closed in X. Hence f is contra-πg-continuous.

Theorem 3.7: Let $A \subset Y \subset X$. (i) If Y is open in X, then $A \in \pi GBC(X)$ implies $A \in \pi GBC(Y)$.
(ii)If Y is regular open and πgb-closed in X, then $A \in \pi GBC(Y)$ implies $A \in \pi GBC(X)$

Theorem 3.8: Suppose $\pi GBO(X, \tau)$ is closed under arbitrary union. Then the following are equivalent for a function f :
$(X, \tau) \to (Y, \sigma)$:
(i) f is contra-πgb-continuous.
(ii) For every closed subset F of Y, $f^{-1}(F) \in \pi GBO(X)$
(iii) For each $x \in X$ and each $F \in C(Y,f(x))$,there exists $U \in \pi GBO(X,x)$ such that $f(U) \subset F$.

Proof: (i) ⇔ (ii) and (ii) ⇒ (iii) is obvious.

(iii)⇒ (ii): Let F be any closed set of Y and $x \in f^{-1}(F)$.Then $f(x) \in F$ and there exists $U_x \in \pi GBO(X)$ such that $f(U_x) \subset F$. Therefore we obtain $f^{-1}(F) = \cup\{U_x : x \in f^{-1}(F)\}$ and $f^{-1}(F)$ is πgb-open.

Theorem 3.9: Suppose $\pi GBO(X,\tau)$ is closed under arbitrary intersections. If a function f: $(X,\tau) \to (Y,\sigma)$ is contra-πgb-continuous and U is open in X, then $f / U: (U,\tau) \to (Y,\sigma)$ is contra-πgb-continuous.

Proof: Let V be closed in Y. Since f: $(X,\tau) \to (Y,\sigma)$ is contra- πgb-continuous, $f^{-1}(V)$ is πgb-open in (X,τ).
$(f/ U)^{-1}(V) = f^{-1}(V) \cap U$ is πgb-open in X. By theorem 3.7 (i) $(f/U)^{-1}(V)$ is πgb-open in U.

Theorem 3.10: Suppose $\pi GBO(X,\tau)$ is closed under arbitrary unions. Let $f :(X, \tau) \to (Y,\sigma)$ be a function and $\{U_i : i \in I\}$ be a cover of X such that $U_i \in \pi GBC(X)$ and regular open for each $i \in I$. If $f / U_i : (U_i, \tau /U_i) \to (Y,\sigma)$ is contra-πgb-continuous for each $i \in I$, then f is contra-πgb-continuous.

Proof: Suppose that F is any closed set of Y. We have
$f^{-1}(F) = \cup \{f^{-1}(F) \cap U_i : i \in I\} = \cup\{(f/U_i)^{-1}(F): i \in I\}$. Since f/U_i is contra-πgb-continuous for each $i \in I$, it follows $(f/U_i)^{-1}(F) \in \pi GBO(U_i)$. By theorem 3.7 (ii), it follows that $f^{-1}(F) \in \pi GBO(X)$. Therefore f is contra- πgb-continuous.

Theorem 3.11: Suppose $\pi GBO(X, \tau)$ is closed under arbitrary unions. If $f :(X, \tau) \to (Y,\sigma)$ is contra-πgb-continuous and Y is regular open, then f is πgb-continuous.

Proof: Let x be an arbitrary point of X and V an open set of Y containing f(x).Y is regular implies that there exists an open set W in Y containing f(x) such that cl (W) ⊂V. Since f is contra -πgb-continuous, by theorem 3.8, there exists $U \in \pi GBO(X, x)$ such that $f(U) \subset cl(W) \subset V$. Hence f is πgb-continuous.

4. Approximately -πgb-continuous functions

Definition4.1: A map f: X→Y is said to be approximately- πgb-continuous (ap- πgb-continuous) if $bcl(F) \subset f^{-1}(U)$ whenever U is an open subset of Y and F is a πgb-closed subset of X such that $F \subset f^{-1}(U)$.

Definition4.2: A map f: X→Y is said to be approximately-πgb-closed (briefly ap- πgb-closed) if $f(F) \subset bint(V)$ whenever V is a πgb-open subset of Y, F is a closed subset of X and $f(F) \subset V$.

Definition4.3: A map f: X→Y is said to be approximately -πgb-open (briefly ap-πgb-open if $bcl(F) \subset f(U)$ whenever U is an open subset of X, F is a πgb-closed subset of Y and $F \subset f(U)$.

Definition4.4: A map f: X→Y is said to be contra-πgb-closed (resp.contra πgb-open) if f (U) is πgb-open (resp πgb-closed) in Y for each closed (resp. open) set U of X.

Theorem4.5: Let f: X→Y be a function, then
(1) If f is contra -b- continuous, then f is an ap- πgb-continuous.
(2) If f is contra- b-closed, then f is ap- πgb-closed.
(3) If f is contra -b-open, then f is ap- πgb -open.

Proof: (1) Let $F \subset f^{-1}(U)$ where U is a open subset in Y and F is a πgb-closed subset of X. Then $bcl(F) \subset bcl(f^{-1}(U))$.Since f is contra-b- continuous, $bcl(F) \subset bcl(f^{-1}(U)) = f^{-1}(U)$.This implies f is ap- πgb-continuous.

(2) Let $f(F) \subset V$, where F is a closed subset of X and V is a πgb-open subset of Y. Therefore f (F) = bint(f(F))bint (V).Thus f is ap- πgb-closed.

(3) Let $F \subset f(U)$ where F is πgb-closed subset of Y and U is an open subset of X. Since f is contra -b-open,f(U) is b-closed in Y for each open set U of X. $bcl(F) \subset bcl(f(U)) = f(U)$.Thus f is ap- πgb -open.

Theorem 4.6: Let f: $(X, \tau) \to (Y, \sigma)$ be a map.
(1) If the open and b-closed sets of (X, τ) coincide, then f is a ap- πgb-continuous if and only if f is contra -b-continuous.
(2) If the open and b-closed sets of (Y, σ) coincide, then f is ap- πgb-closed if and only if f is contra - b-closed.

(3) If the open and b-closed sets of (Y, σ) coincide, then f is ap- πgb-open if and only if f is contra-b-open.

Proof: (1) Assume f is ap- πgb-continuous. Let A be an arbitrary subset of (X, τ) such that A ⊂U, where U is π-open in X. Then bcl (A) ⊂ bcl (U) = U .Therefore all subsets of (X, τ) are πgb-closed (hence all are πgb-open). So for any open set V in (Y, σ), we have $f^{-1}(V)$ is πgb-closed in (X, τ). Since f is ap-πgb-continuous, bcl(f^{-1}(V)) ⊂f^{-1}(V).Therefore f^{-1}(V)) is b-closed in (X,τ) and f is contra-b-continuous.

Converse is obvious from theorem 4.5.

(2) Assume f is ap-πgb-closed.As in (1), we get that all subsets of (Y, σ) are πgb-open. Therefore for any closed subset F of (X,τ),f(F) is πgb-open in Y.Since f is ap- πgb-closed, f(F) ⊂ bint f(F).Hence f(F) is b-open and thus f is contra b-closed.

Converse is obvious from theorem 4.5.

(3) Assume f is ap-πgb-open. As in (1) all subsets of Y are πgb-closed. Therefore for any open subset F of (X, τ), f(F) is πgb-closed in (Y, σ).Since f is ap-πgb-open, bcl(F)⊂f(F).Hence f(F) is b-closed and thus f is contra b-open. Converse is obvious from theorem 4.5.

Theorem 4.7: If a map f: X→Y is ap-πgb-continuous and b-closed map, then the image of each πgb-closed set in X is πgb-closed set in Y.

Proof: Let F be a πgb-closed subset of X. Let f(F) ⊂V where V is an open subset of Y. Then F⊂ f^{-1} (V) holds. Since f is ap-πgb- continuous, bcl(F) ⊂ f^{-1}(V).Thus f(bcl(F) ⊂V.Therefore we have bcl(f(F))⊂ bcl(f(bcl(F))=f(bcl(V))⊂V.Hence f(F) is πgb-closed set in Y.

Theorem 4.8: If f: X→Y is a π-continuous and b-closed function, then f (A) is πgb-closed in Y for every πgb-closed set A of X.

Proof: Let A be πgb-closed set in X. Let f (A) ⊂ V, where V is a π-open set in Y. Since f is π-continuous, f^{-1}(V) is π-open in X and A ⊂ f^{-1} (V)**.**Then we have bcl(A) ⊂ f^{-1} (V) and so f(bcl(A)) ⊂V. Since f is b-closed, f (bcl (A)) is b-closed in Y. Hence bcl(f(A)) ⊂ bclf(bcl(A)) = f(bcl(A)) ⊂V.This shows that f(A) is πgb closed in Y.

Definition 4.9: A map f: X→Y is said to be contra-πgb-irresolute if f^{-1}(V) is πgb- closed in X for each V∈ πGBO(Y).

Definition4.10: A space X is said to be πgb-Lindelof if every cover of X by πgb-open sets has a countable sub cover.

Theorem 4.11: Let f: X→Y and g: Y→Z be two maps such that gof: X→Z.

(i) If g is πgb-continuous and f is contra -πgb-irresolute, then gof is contra πgb- continuous.

(ii) If g is πgb-irresolute and f is contra-πgb irresolute, then gof is contra-πgb- irresolute.

Proof :(i) Let V be closed set in Z. Then g^{-1}(V) is πgb-closed in Y. Since f is contra -πgb- irresolute,f^{-1}(g^{-1}(V)) is πgb-open in X. Hence gof is contra πgb-continuous.

(ii)Let V be πgb-closed in Z. Then g^{-1}(V) is πgb-closed in Y. Since f is contra -πgb- irresolute, f^{-1}(g^{-1}(V)) is πgb-open in X. Hence gof is contra πgb-irresolute.

Theorem 4.12: Let f: X→Y and g: Y→Z be two maps such that (gof): X→Z.

(i)If f is closed and g is ap-πgb- closed, then (gof) is ap-πgb- closed.

(ii) If f is ap-πgb- closed and g is πgb-open and g^{-1} preserves πgb-open sets, then

(gof) is ap-πgb- closed.

(iii) If f is ap-πgb-continuous and g is continuous, then gof is ap-πgb-continuous

Proof :(i) Suppose B is an arbitrary closed subset in X and A is a πgb- open subset of Z for which (gof) (B) ⊆A . Then f(B) is closed in Y because f is closed. Since g is ap-πgb -closed, g (f(B)) ⊂ bint (A) .This implies (gof) is ap- πgb-closed.

(ii) Suppose B is an arbitrary closed subset of X and A is a πgb-open subset of Z for which (gof) (B) ⊆ A.Hence f(B) ⊂ g^{-1} (A).Then f(B) ⊂ bint (g^{-1} (A)) because g^{-1} (A) is πgb-open and f is ap-πgb-closed. Hence (gof) (B) = g(f(B)) ⊆g[bint (g^{-1} (A)] ⊆ bint (g g^{-1} (A) ⊆ bint (A).This implies that (gof) is ap-πgb-closed.

(iii)Suppose F is an arbitrary πgb-closed subset of X and U is open in Z for which F ⊂ (gof) $^{-1}$ (U).Then g^{-1} (U) is open in Y ,because g is continuous. Since f is ap-πgb- continuous, then we have bcl (F) ⊆ f^{-1} (g^{-1} (U))= $(gof)^{-1}$(U).This shows that gof is ap- πgb-continuous.

5. Almost Contra- πgb-Continuous Functions

Definition 5.1: A function f :(X, τ) → (Y,σ) is said to be almost contra- πgb- continuous if f^{-1}(V)∈ πGBC(X,τ) for each V∈RO(Y,σ).

Theorem 5.2: Suppose πGBO(X,τ) is closed under arbitrary unions.Then the following statements are equivalent for a function f: (X, τ) → (Y,σ).

(i) f is almost contra- πgb- continuous.

(ii) f^{-1}(F) ∈πGBO(X,τ) for every F ∈RC(Y,σ).

(iii) For each x∈X and each regular closed set F in Y containing f(x), there exists a πgb-open set U in X containing x such that f(U)⊂F.

(iv)For each x∈X, and each regular open set V in Y not containing f(x),there exists a πgb-closed set K in X not containing x such that f^{-1}(V)⊂K.

(v) f^{-1}(int(cl(G)))∈πGBC(X,τ) for every open subset G of Y.

(vi) f^{-1}(int(cl(F)))∈πGBO(X,τ) for every closed subset F of Y.

Proof :(i)⇒(ii).Let F∈RC(Y,σ).Then Y-F∈RO(Y,σ) by assumption. Hence f^{-1}(Y-F)=X-f^{-1}(F)∈ πGBC(X,τ).This implies f^{-1}(F)∈ πGBO(X,τ).

(ii) ⇒ (i).Let V∈RO(Y,σ).Then by assumption (Y-V) ∈RC(Y,σ). Hence f^{-1}(Y-F) =X-f^{-1}(F) ∈ πGBO(X,τ).This implies f^{-1}(F)∈ πGBC(X,τ).

(ii)⇒(iii).Let F be any regular closed set in Y containing f(x).f^{-1}(F)∈πGBO(X,τ) and x∈f^{-1}(F) (by(ii)). Take U= f^{-1}(F).Then f(U)⊂F.

(iii)⇒ (ii) Let F∈RC(Y,σ) and x∈f^{-1}(F).From (iii), there exists a πgb-open set U_x in X containing x such that U_x ⊂ f^{-1}(F). We have f^{-1}(F) =∪ {U_x: x∈f^{-1}(F)}.Then f^{-1}(F) is πgb-open.

(iii)⇒(iv) Let V be any regular open set in Y containing f(x).Then Y-V is a regular closed set containing f(x).By (iii), there exists a πgb-open set U in X containing x such that f(U)⊂ Y-V. Hence U⊂ f^{-1}(Y-V) ⊂ X- f^{-1}(V).Then

f^{-1}(V) ⊂X-U.Take K=X-U. We obtain a πgb-closed set in X not containing x such that f^{-1}(V) ⊂K.

(iv)⇒ (iii). Let F be regular closed set in Y containing f(x).Then Y-F is regular open set in Y containing f(x).By (iv), there exists a πgb-closed set K in X not containing x such that f^{-1}(Y-F) ⊂K. Then X- f^{-1}(F) ⊂ K implies X-K ⊂f^{-1}(F).Hence f (X-K) ⊂F. Take U=X-K.Then U is a πgb-open set U in X containing x such that f(U)⊂F.

(i)⇒(v). Let G be a open subset of Y. Since int(cl(G)) is regular open, then by (i),f^{-1}(int(cl(G))∈πGBC(X,τ).

(v)⇒(i).Let V∈RO(Y,σ).Then V is open in Y. By (v), f^{-1}(int(cl(G))∈πGBC(X,τ).This implies f^{-1}(V)∈πGBC(X,τ)

(ii)⇔ (vi) is similar as (i)⇔(v).

Theorem 5.3: If f :(X , τ) → (Y,σ) is an almost contra- πgb- continuous function and A is a open subset of X, then the restriction f/A:A→Y is almost contra- πgb- continuous.

Proof: Let F ∈RC(Y). Since f is almost contra-πgb- continuous, f^{-1}(F)∈ πGBO(X).Since A is open, it follows that $(f/A)^{-1}$(F)= A∩ f^{-1}(F)∈ πGBO(A).Therefore f/A is an almost contra- πgb-continuous.

Theorem 5.4: Let f: (X,τ) →(Y, σ) be an almost contra-πgb-continuous surjection. Then the following statements hold.

(i)If X is πGB-closed, then Y is nearly compact.

(ii)If X is πGB-Lindelof,then Y is nearly Lindelof.

(iii)If X is countably-πGB-closed,then Y is nearly countably compact.

(iv)If X is πGBO-compact,then Y is S-closed.

(v)If X is πGB-Lindelof,then Y is S- Lindelof.

(vi)If X is countable πGB-compact, then Y is countably S-closed compact.

Proof: (i) Let {V_b :b∈I} be regular open cover of Y. Then f is almost contra-πgb-continuous implies {f^{-1}(V_b):b∈I} is a πgb-closed cover of X.
Since X is πGB-closed,there exists a finite subset I_0 of I such that X=∪ {f^{-1} (V_b):b∈I_0}.Then we have Y=∪ {V_b:b∈I_0}.Hence Y is nearly compact.
Proof of (ii) and (iii) is similar to that of (i).

(iv)Let {V_b :b∈I} be regular closed cover of Y. Then f is almost contra-πgb-conntinuous implies {f^{-1} (V_b):b∈I} is a πgb-open cover of X.By assumption, there exists a finite subset I_0 of I such that X=∪ {f^{-1} (V_b):b∈I_0}.Then we have Y=∪ {V_b :b∈I_0}.Hence Y is nearly compact. Proof of (v) and (vi) is similar to that of (iv).

Theorem 5.5: If f: (X,τ) →(Y, σ) is almost contra-πgb-continuous and almost πgb-continuous surjection.Then

(i)If X is mildly πgb-compact, then Y is nearly compact.

(ii)If X is mildly countably -πgb-compact, then Y is nearly countably compact.

(iii) If X is mildly πgb-Lindelof,, then Y is nearly Lindelof.

Proof: (i) Let V∈RO(Y).Since f is almost contra-πgb-continuous and almost πgb-continuous, f^{-1} (V) is πgb-closed and πgb-open in X respectively. Then f^{-1}

(V) is πgb-clopen in X. Let $\{V_b : b \in I\}$ be any regular open cover of Y. Then

$\{f^{-1}(V_b) : b \in I\}$ is πgb-clopen in X.Since X is mildly πgb-compact, there exists a finite subset I_0 of I such that $X = \cup$

$\{f^{-1}(V_b) : b \in I_0\}$.Since X is surjective, we obtain $Y = \cup\{V_b : b \in I_0\}$.Hence Y is nearly compact.

Proof of (ii) and (iii) is similar to that of (i).

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