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Reliability and Cost-Effectiveness of a Redundant System with Three Types of Failures and Repairs

Dharmvir Singh Vashisth[1], A. S. Gousia Banu[2]

[1]Department of Mathematics, R.N. Engg. College, Rohtak (Haryana), INDIA.

[2]Department of Management, Narayana Engineering College (Nec) Nellore (A.P), INDIA

Correspondence addresses

dharmvir.vashisth@yahoo.in , gbanuzia@gmail.com

Research Article

Abstract: The goal of this paper is to carry out the reliability and cost effectiveness of a two unit cold standby system with allowed downtime and correlated failure and repair times. For this purpose, a stochastic Model is developed in which one unit is operative and other is kept as cold standby. Each unit has three modes of failure viz. normal, degraded and total failure. The time taken in replacement of a failed unit is negligible but it is a random variable. The failure rates are constant and system repair time are arbitrary distributed. The numerical results for reliability and cost-effectiveness are obtained using supplementary variable technique. The repair and failure times are correlated .

Introduction: In most of the reliability models of dissimilar units studied so far with three modes, viz, normal, partial and total failure have been considered. Gupta et al. [1] have studied two unit stand by system under partial failure and pre-emplive repair priority. They have all assumed that failure and repair are independent but it looks in practice that they have certain correspondence. Goel et al. [2] have considered that failure and repair time follow a behaviour exponential distribution. Sharma [4] has initiated the study of availability of the system which consist of two identical cold stand by units with constant failure rates. Initially one unit is operative while the other remains as stand by. Each of the units has three modes, i.e., normal, degraded and total failure. The system fails when both the units fail and may also fail due to common cause failure. The time taken in replacement of a failed unit by a stand by unit is not negligible but it is a random variable. Goel et al. [2] also presented a mathematical model for predicting a two identical active units redundant system with three types of failures namely mechanical, catastrophic and human failure. Any units may fail either partially or completely. And the system is only required when all the units fail including the helping unit. The failure rates are constant and system repair times are arbitrarily distributed. He obtained state probabilities by Laplace transforms.

In this paper, we are interested to analyse the two unit cold stand by system with allowed downtime and correlated failure and repair times. Using this concept of correlation into consideration the joint distribution of failure and repair times is taken to be bivariate exponential distributed. The numerical results for reliability and cost-effectiveness are obtained to improve the importance of the study.

Notations:

(a) $D/Dx/Dt$: $\frac{d}{dt}, \frac{\partial}{\partial x}, \frac{\partial}{\partial t}$

(b) λ_c : Constant failure rate due to catastrophic failure.

(c) λ_h : Constant failure rate due to human failure.

(d) $\lambda_1 / \lambda_2 / \lambda_{1p} / \lambda_{2p}$: Constant failure rate due to mechanical failure. (1)

(e) w_1 : Constant waiting due to mechanical failure of both the units.

(f) $P_{00}(t)$: P_r (at time t the system is in state S_0)

(g) $P_{0p}(x,t)\Delta$: P_r (the system is in S_1 state at time t due to partial mechanical failure of helping unit and elapsed repair time lies in the interval $(x, x+\Delta)$

(h) $P_{CF}(x,t)\Delta$: P_r (the system is in state S_2 at time t due to the complete mechanical failure of helping unit and elapsed repair time lies in the interval $(x, x+\Delta))$

(i) $P_{CC}(x,t)\Delta$: P_r (the system is in the state S_3 at time t due to catastrophic failure and elapsed repair time lies in the interval $(x, x+\Delta))$

(j) $P_w(x,t)\Delta$: P_r (the system is in the state S_4 at time t and elapsed repair time in the interval $(x, x+\Delta))$

(k) $P_{hh}(x,t)\Delta$: P_r (the system is in state S_5 at time t due to human error and elapsed repair time lies in the interval $(x, x+\Delta))$

(l) $P_{PP}(t)$: P_r (at time t, the system is in state S_6 due to partial failure of maintenance helping unit)

(m) $P_{PF}(t)$: P_r (at time t, the system is in state S_7 due to partial failure of maintenance and complete failure of helping unit).

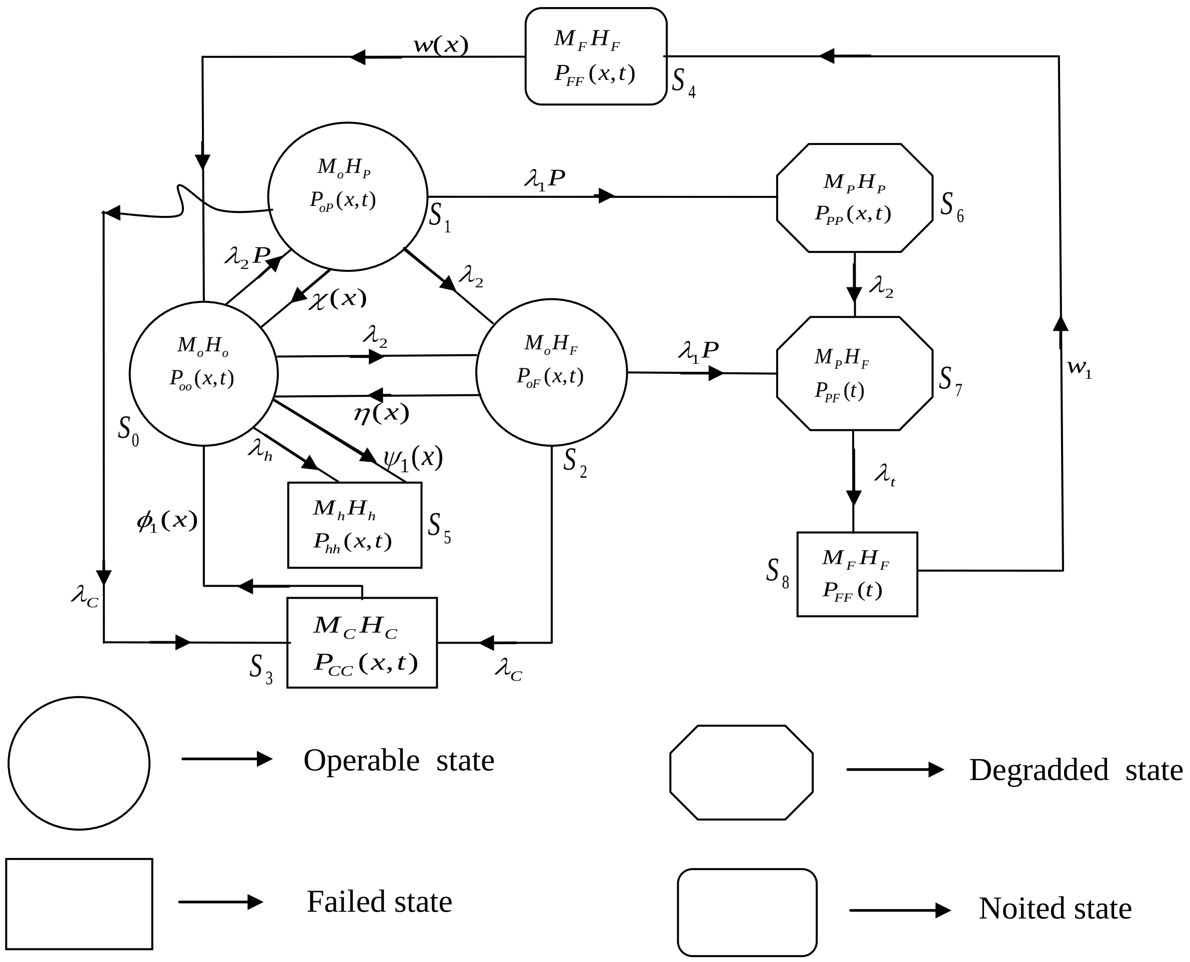
(n) $P_{FF}(t)$: P_r (the system is in state S_8 at time t due to complete failure of unit)

(o) $\phi_1(x)\Delta / \Psi_1(x)\Delta$: First order probabilities that failed / failed / operable / waited.

(p) $\eta(x)\Delta / \chi(x)\Delta$:system repaired in time $(x, x+\Delta) / (x, x+\Delta) / (x, x+\Delta) / (x, x+\Delta) / (x, x+\Delta)$

(q) $w(x)\Delta : \overline{S}_i^K(s) = k(x)e^{-\int_0^K K(x)dx}$, $\int = \int_0^\infty$ unless otherwise stated.

Transition of diagram



Formulation of Mathematical Models:

Probabilistic consideration and preceding stated procedures yield the following equations:

$$(D+\lambda_2+\lambda_{2P}+\lambda_h+\lambda_c)P_{00}(\tau) = \int P_{0F}(x,t)\eta(x)dx + \int P_{0P}(x,t)\chi(x)dx + \int P_{hh}(x,t)\Psi_1(x)dx + \int P_{cc}(x,t)\phi_1(x)dx + \int P_w(x,t)w(x)dx$$

$$[Dx + Dt + \lambda_{1P} + \lambda_2 + \lambda_c + \chi(x)]P_{0P}(x,t) = 0$$
$$[Dx + Dt + \lambda_{1P} + \lambda_c + \eta(x)]P_{0F}(x,t) = 0$$
$$[Dx + Dt + \phi_1(x)]P_{cc}(x,t) = 0$$
$$[Dx + Dt + \Psi_1(x)]P_{hh}(x,t) = 0$$
$$[Dx + Dt + w(x)]P_w(x,t) = 0$$
$$[D + \lambda_2]P_{PP}(t) = \lambda_1 P \int P_{0P}(x,t)dx$$
$$[D + \lambda_1]P_{PF}(x,t) = \lambda_2 P_{PP}(t) + \lambda_1 P \int P_{0F}(x,t)dx$$
$$[D + w_1]P_{FF}(t) = \lambda_1 P_{FF}(t)$$
$$P_{0P}(0,t) = \lambda_{2P} P_{00}(t) \quad ......(2)$$
$$P_{0F}(0,t) = \lambda_2 P_{00}(t) + \lambda_2 \lambda_{2P} P_{00}(t)$$
$$P_{cc}(0,t) = \lambda_c P_{00}(t) + \lambda_c \lambda_{2P} P_{00}(t) + \lambda_c \lambda_2 P_{00}(t) + \lambda_2 \lambda_c \lambda_{2P} P_{00}(t)$$
$$= (\lambda_c + \lambda_{2P}\lambda_c + \lambda_c \lambda_2 + \lambda_c \lambda_2 \lambda_{2P}) P_{00}(t)$$
$$P_{hh}(0,t) = \lambda_h P_{00}(t)$$
$$P_w(0,t) = w_1 P_{FF}(t)$$

If a final state, it is ……….So $P_{00}(0) = 1$, otherwise 0.

Mathematical Analysis:

Taking Laplace of the equation (2) we obtain

$$\overline{P}_{00}(s) = \frac{1}{D(s)}$$

$$\overline{P}_{0P}(s) = \lambda_{2P} \frac{1 - \overline{S}_\chi(s + \lambda, P + \lambda_c + \lambda_2)}{s + \lambda_{1P} + \lambda_c} \cdot \frac{1}{D(s)}$$

$$\overline{P}_{0F}(s) = \left[\lambda_2 \frac{1 - \overline{S}_\eta(s + \lambda, P + \lambda_c)}{s + \lambda_{1P} + \lambda_c} + \lambda_2 \lambda_{2P} \frac{1 - \overline{S}_\eta(s + \lambda, P + \lambda_c)}{s + \lambda_{1P} + \lambda_c} + \frac{1 - \overline{S}_\chi(s + \lambda, P + \lambda_c + \lambda_2)}{s + \lambda_{1P} + \lambda_c + \lambda_2} \right] \frac{1}{D(s)}$$

$$\overline{P}_{cc}(0) = \left[\lambda_c \frac{1 - \overline{S}_{\phi_1}(s)}{s} + \lambda_2 \lambda_{2P} \frac{1 - \overline{S}_\chi(s + \lambda_{1P} + \lambda_c + \lambda_2)}{s + \lambda_{1P} + \lambda_c + \lambda_2} \cdot \frac{1 - \overline{S}_{\phi_1}(s)}{s} + \frac{1 - \overline{S}_\eta(s + \lambda_{1P} + \lambda_c)}{s + \lambda_{1P} + \lambda_c} \frac{1 - \overline{S}_{\phi_1}(s)}{s} \right.$$
$$\left. + \lambda_c \lambda_2 \lambda_{2P} \frac{1 - \overline{S}_\chi(s + \lambda_{1P} + \lambda_c + \lambda_2)}{s + \lambda_{1P} + \lambda_c + \lambda_2} \cdot \frac{1 - \overline{S}_\eta(s + \lambda_{1P} + \lambda_c)}{s + \lambda_{1P} + \lambda_c} \cdot \frac{1 - \overline{S}_{\phi_1}(s)}{s} \right] \frac{1}{D(s)}$$

(3)

Initial conditions

$$\overline{P}_w(s) = \left[\frac{w_1 \lambda_1 \lambda_2 \lambda_{1P} \chi_{2P}}{(s + w_1)(s + \lambda_1)(s + \lambda_2)} \cdot \frac{1 - \overline{S}_w(s)}{s} \cdot \frac{1 - \overline{S}_\chi(s + \lambda_{1P} + \lambda_c + \lambda_2)}{s + \lambda_{1P} + \lambda_c + \lambda_2} \right.$$
$$+ \frac{w_1 \lambda_1 \lambda_2 \lambda_{1P}}{(s + w_1)(s + \lambda_1)} \cdot \frac{1 - \overline{S}_w(0)}{s} \cdot \frac{1 - \overline{S}_\eta(s + \lambda_{1P} + \lambda_c)}{s + \lambda_{1P} + \lambda_c}$$
$$\left. + \frac{w_1 \lambda_1 \lambda_2 \lambda_{2P} \chi_{1P}}{(s + w_1)(s + \lambda_1)} \cdot \frac{1 - \overline{S}_w(s)}{s} \frac{1 - \overline{S}_\eta(s + \lambda_{1P} + \lambda_c)}{s + \lambda_{1P} + \lambda_c} \cdot \frac{1 - \overline{S}_\chi(s + \lambda_{1P} + \lambda_c + \lambda_2)}{s + \lambda_{1P} + \lambda_c + \lambda_2} \right] \frac{1}{D(s)}$$

$$\overline{P}_{hh}(s) = \lambda_h \frac{1 - \overline{S}_{\Psi_1}(s)}{s} \cdot \frac{1}{D(s)}$$

$$\overline{P}_{PP}(s) = \frac{\lambda_{1P} \lambda_{2P}}{s + \lambda_2} \frac{1 - \overline{S}_\chi(s + \lambda_{1P} + \lambda_c + \lambda_2)}{s + \lambda_{1P} + \lambda_c + \lambda_2} \cdot \frac{1}{D(s)}$$

$$\overline{P}_{PF}(s) = \left[\frac{\lambda_3 \lambda_{1P} \lambda_{2P}}{(s + \lambda_1)(s + \lambda_2)} \frac{1 - \overline{S}_\chi(s + \lambda_{1P} + \lambda_c + \lambda_2)}{s + \lambda_{1P} + \lambda_c + \lambda_2} \right.$$
$$\left. + \frac{\lambda_2 \lambda_{1P}}{s + \lambda_1} \cdot \frac{1 - \overline{S}_\eta(s + \lambda_{1P} + \lambda_c)}{s + \lambda_{1P} + \lambda_c} \frac{\lambda_2 \lambda_{1P} \lambda_{2P}}{s + \lambda_1} \cdot \frac{1 - \overline{S}_\eta(s + \lambda_{1P} + \lambda_c)}{s + \lambda_{1P} + \lambda_c} \cdot \frac{1 - \overline{S}_\chi(s + \lambda_{1P} + \lambda_c + \lambda_2)}{s + \lambda_{1P} + \lambda_c + \lambda_2} \right] \frac{1}{D(s)}$$

$$\overline{P}_{FF}(s) = \left[\frac{\lambda_1 \lambda_2 \lambda_{1P} \lambda_{2P}}{(s + w_1)(s + \lambda_1)(s + \lambda_2)} \cdot \frac{1 - \overline{S}_\chi(s + \lambda_{1P} + \lambda_c + \lambda_2)}{s + \lambda_{1P} + \lambda_c + \lambda_2} + \frac{\lambda_1 \lambda_2 \lambda_{1P}}{(s + w_1)(s + \lambda_1)} \cdot \frac{1 - \overline{S}_\eta(s + \lambda_{1P} + \lambda_c)}{s + \lambda_{1P} + \lambda_c} \right.$$
$$\left. + \frac{\lambda_1 \lambda_2 \lambda_{1P} \lambda_{2P}}{(s + w_1)(s + \lambda_1)} \cdot \frac{1 - \overline{S}_\eta(s + \lambda_{1P} + \lambda_c)}{s + \lambda_{1P} + \lambda_c} \cdot \frac{1 - \overline{S}_\chi(s + \lambda_{1P} + \lambda_c + \lambda_2)}{s + \lambda_{1P} + \lambda_c + \lambda_2} \right] \frac{1}{D(s)}$$

Gupta, Singh and Kishore [2] also obtained the up and down state Probabilitie

$$\overline{P}_{up}(s) = \overline{P}_{00}(s) + \overline{P}_{0P}(s) + \overline{P}_{0F}(s) + \overline{P}_{PP}(s) + \overline{P}_{PF}(s) + \overline{P}_{w}(s)$$

$$= 1 + \lambda_{2P}\frac{1-\overline{S}_{\chi}(s+\lambda_{1P}+\lambda_c+\lambda_2)}{s+\lambda_{1P}+\lambda_c+\lambda_2} + \lambda_2\frac{1-\overline{S}_{\eta}(s+\lambda_{1P}+\lambda_c)}{s+\lambda_{1P}+\lambda_c}$$

$$+\lambda_2\lambda_{2P}\frac{1-\overline{S}_{\eta}(s+\lambda_{1P}+\lambda_c)}{s+\lambda_{1P}+\lambda_c}.\frac{1-\overline{S}_{\chi}(s+\lambda_{1P}+\lambda_c+\lambda_2)}{s+\lambda_{1P}+\lambda_c+\lambda_2} + \frac{\lambda_2\lambda_{1P}\lambda_{2P}}{(s+\lambda_1)(s+\lambda_2)}\frac{1-\overline{S}_{\chi}(s+\lambda_{1P}+\lambda_c+\lambda_2)}{s+\lambda_{1P}+\lambda_c+\lambda_2}$$

$$+\frac{\lambda_2\lambda_{1P}}{s+\lambda_1}\frac{1-\overline{S}_{\eta}(s+\lambda_{1P}+\lambda_c)}{s+\lambda_{1P}+\lambda_c} + \frac{\lambda_2\lambda_{1P}\lambda_{2P}}{s+\lambda_1}\frac{1-\overline{S}_{\eta}(s+\lambda_{1P}+\lambda_c)}{s+\lambda_{1P}+\lambda_c}.\frac{1-\overline{S}_{\chi}(s+\lambda_{1P}+\lambda_c+\lambda_2)}{s+\lambda_{1P}+\lambda_c+\lambda_2}$$

$$+\frac{w_1\lambda_1\lambda_2\lambda_{1P}\chi_{2P}}{(s+w_1)(s+\lambda_1)(s+\lambda_2)}.\frac{1-\overline{S}_{w}(s)}{s}.\frac{1-\overline{S}_{\chi}(s+\lambda_{1P}+\lambda_c+\lambda_2)}{s+\lambda_{1P}+\lambda_c+\lambda_2}$$

$$+\frac{w_1\lambda_1\lambda_2\lambda_{1P}}{(s+w_1)(s+\lambda_1)}.\frac{1-\overline{S}_{w}(0)}{s}.\frac{1-\overline{S}_{\eta}(s+\lambda_{1P}+\lambda_c)}{s+\lambda_{1P}+\lambda_c}$$

$$\left.+\frac{w_1\lambda_1\lambda_2\lambda_{2P}\chi_{1P}}{(s+w_1)(s+\lambda_1)}.\frac{1-\overline{S}_{w}(s)}{s}\frac{1-\overline{S}_{\eta}(s+\lambda_{1P}+\lambda_c)}{s+\lambda_{1P}+\lambda_c}\frac{1-\overline{S}_{\chi}(s+\lambda_{1P}+\lambda_c+\lambda_2)}{s+\lambda_{1P}+\lambda_c+\lambda_2}\right].\frac{1}{D(s)}$$

$$P_{down}(s) = \frac{1}{S} - \overline{P}_{up}(s) \text{ where,} \quad (4)$$

$$D(s) = s+\lambda_2+\lambda_{2P}+\lambda_h+\lambda_c-\lambda_2\overline{S}_{\eta}(s+\lambda_{1P}+\lambda_c)$$

$$-\lambda_2+\lambda_{2P}\frac{1-\overline{S}_{\chi}(s+\lambda_{1P}+\lambda_c+\lambda_2)}{s+\lambda_{1P}+\lambda_c+\lambda_2}.\overline{S}_{\eta}(s+\lambda_{1P}+\lambda_c)$$

$$-\lambda_{2P}\overline{S}_{\chi}(s+\lambda_{1P}+\lambda_c+\lambda_2)-\lambda_h\overline{S}_{\Psi_1}(s)$$

$$-\lambda_2\lambda_{2P}\frac{1-\overline{S}_{\chi}(s+\lambda_{1P}+\lambda_c+\lambda_2)}{s+\lambda_{1P}+\lambda_c+\lambda_2}.\overline{S}_{\phi}(s)$$

$$-\lambda_c\lambda_2.\frac{1-\overline{S}_{\eta}(s+\lambda_{1P}+\lambda_c)}{s+\lambda_{1P}+\lambda_c}.\overline{S}_{\phi_1}(s)-\lambda_c\lambda_2\lambda_{2P}.\frac{1-\overline{S}_{\eta}(s+\lambda_{1P}+\lambda_c)}{s+\lambda_{1P}+\lambda_c}.\frac{1-\overline{S}_{\chi}(s+\lambda_{1P}+\lambda_c+\lambda_2)}{s+\lambda_{1P}+\lambda_c+\lambda_2}\overline{S}_{\phi_1}(s)$$

$$-\frac{w_1\lambda_1\lambda_2\lambda_{1P}\chi_{2P}}{(s+w_1)(s+\lambda_1)(s+\lambda_2)}.\frac{1-\overline{S}_{\chi}(s+\lambda_{1P}+\lambda_c+\lambda_2)}{s+\lambda_{1P}+\lambda_c+\lambda_2}.\overline{S}_{w}(s)-\frac{w_1\lambda_1\lambda_2\lambda_{1P}}{(s+w_1)(s+\lambda_1)}.\frac{1-\overline{S}_{\eta}(s+\lambda_{1P}+\lambda_c)}{s+\lambda_{1P}+\lambda_c}.\overline{S}_{w}(s)$$

$$-\frac{w_1\lambda_1\lambda_{1P}\lambda_{2P}}{(s+w_1)(s+\lambda_1)}\frac{1-\overline{S}_{\eta}(s+\lambda_{1P}+\lambda_c)}{s+\lambda_{1P}+\lambda_c}\frac{1-\overline{S}_{\chi}(s+\lambda_{1P}+\lambda_c+\lambda_2)}{s+\lambda_{1P}+\lambda_c+\lambda_2}.\overline{S}_{w}(s)$$

Ergodic Behavior of the System:

Goel, Singh and Kishore [2] using base, we have

$$\underset{s\to 0}{s}\,\overline{f}(s) = \underset{t\to\infty}{f}(t) = f\,'(say)$$

has obtained values as follows:

$$P_{00} = \frac{1}{D'(0)}$$

$$P_{0F} = \left[\lambda_2.\frac{1-\overline{S}_{\eta}(\lambda_{1P}+\lambda_c)}{\lambda_{1P}+\lambda_c}+\lambda_2\lambda_{2P}\frac{1-\overline{S}_{\eta}(\lambda_{1P}+\lambda_c)}{\lambda_{1P}+\lambda_c}.\frac{1-\overline{S}_{\xi}(\lambda_{1P}+\lambda_c+\lambda_2)}{\lambda_{1P}+\lambda_c+\lambda_2}\right]\frac{1}{D'(0)}$$

$$P_{cc} = \left[\lambda_c M^{\phi_1} + \lambda_2 \lambda_{2P} . \frac{1-\overline{S}_\chi(\lambda_{1P}+\lambda_c+\lambda_2)}{\lambda_{1P}+\lambda_c+\lambda_2} M^{\phi_1} + \lambda_c \lambda_2 \frac{1-\overline{S}_\eta(\lambda_{1P}+\lambda_c)}{\lambda_{1P}+\lambda_c} . M^{\phi_1} \right. \quad (5)$$

$$\left. + \lambda_c \lambda_2 \lambda_{2P} \frac{1-\overline{S}_\chi(\lambda_{1P}+\lambda_c+\lambda_2)}{\lambda_{1P}+\lambda_c+\lambda_2} . \frac{1-\overline{S}_\eta(\lambda_{1P}+\lambda_c)}{\lambda_{1P}+\lambda_c} . M^{\phi_1} \right] \frac{1}{D'(0)}$$

$$P_w = \lambda_{1P} \lambda_{2P} \frac{1-\overline{S}_\chi(\lambda_{1P}+\lambda_c+\lambda_2)}{\lambda_{1P}+\lambda_c+\lambda_2} M^w + \lambda_2 \lambda_{1P} \frac{1-\overline{S}_\eta(\lambda_{1P}+\lambda_c)}{\lambda_{1P}+\lambda_c} . M^w$$

$$\left. + \lambda_2 \lambda_{1P} \chi_{2P} \frac{1-\overline{S}_\eta(\lambda_{1P}+\lambda_c)}{\lambda_{1P}+\lambda_c} . \frac{1-\overline{S}_\chi(\lambda_{1P}+\lambda_c+\lambda_2)}{\lambda_{1P}+\lambda_c+\lambda_2} . M^w \right] \frac{1}{D'(0)}$$

$$P_{hh} = \lambda_h M^{\Psi_1} \frac{1}{D'(0)}$$

$$P_{PP} = \frac{\lambda_{1P}\lambda_{2P}}{\lambda_2} . \frac{1-\overline{S}_\chi(\lambda_{1P}+\lambda_c+\lambda_2)}{\lambda_{1P}+\lambda_c+\lambda_2} . \frac{1}{D'(0)}$$

$$P_{PF} = \left[\frac{\lambda_{1P}\lambda_{2P}}{\lambda_1} . \frac{1-\overline{S}_\chi(\lambda_{1P}+\lambda_c+\lambda_2)}{\lambda_{1P}+\lambda_c+\lambda_2} + \frac{\lambda_{1P}\lambda_2}{\lambda_1} . \frac{1-\overline{S}_\chi(\lambda_{1P}+\lambda_c)}{\lambda_{1P}+\lambda_c} \right.$$

$$\left. + \frac{\lambda_{1P}\lambda_2\lambda_{2P}}{\lambda_1} . \frac{1-\overline{S}_\eta(\lambda_{1P}+\lambda_c)}{\lambda_{1P}+\lambda_c} . \frac{1-\overline{S}_\chi(\lambda_{1P}+\lambda_c+\lambda_2)}{\lambda_{1P}+\lambda_c+\lambda_2} \right] \frac{1}{D'(0)}$$

$$P_{FF}(s) = \left[\frac{\lambda_{1P}\lambda_{2P}}{w_1} . \frac{1-\overline{S}_\chi(\lambda_{1P}+\lambda_c+\lambda_2)}{\lambda_{1P}+\lambda_c+\lambda_2} + \frac{\lambda_2\lambda_{1P}}{w_1} . \frac{1-\overline{S}_\eta(\lambda_{1P}+\lambda_c)}{\lambda_{1P}+\lambda_c} \right.$$

$$\left. + \frac{\lambda_2\lambda_{1P}\lambda_{2P}}{w_1} . \frac{1-\overline{S}_\eta(\lambda_{1P}+\lambda_c)}{\lambda_{1P}+\lambda_c} . \frac{1-\overline{S}_\chi(\lambda_{1P}+\lambda_c+\lambda_2)}{\lambda_{1P}+\lambda_c+\lambda_2} \right] \frac{1}{D'(0)}$$

Also, $P_{AV} = P_{00} + P_{0P} + P_{0F} + P_w + P_{PP} + P_{PF}$ (6)

$$= \left[1 + \lambda_{2P} . \frac{1-\overline{S}_\eta(\lambda_{1P}+\lambda_c+\lambda_2)}{\lambda_{1P}+\lambda_c+\lambda_2} + \lambda_2 . \frac{1-\overline{S}_\eta(\lambda_{1P}+\lambda_c)}{\lambda_{1P}+\lambda_c} + \lambda_2 \lambda_{2P} . \frac{1-\overline{S}_\eta(\lambda_{1P}+\lambda_c)}{\lambda_{1P}+\lambda_c} \frac{1-\overline{S}_\chi(\lambda_{1P}+\lambda_c+\lambda_2)}{\lambda_{1P}+\lambda_c+\lambda_2} \right.$$

$$+ \lambda_{1P}\lambda_{2P} . \frac{1-\overline{S}_\chi(\lambda_{1P}+\lambda_c+\lambda_2)}{\lambda_{1P}+\lambda_c+\lambda_2} M^w + \lambda_2 \lambda_{1P} . \frac{1-\overline{S}_\eta(\lambda_{1P}+\lambda_c)}{\lambda_{1P}+\lambda_c} . M^w$$

$$+ \lambda_2 \lambda_{1P} \chi_{2P} . \frac{1-\overline{S}_\eta(\lambda_{1P}+\lambda_c)}{\lambda_{1P}+\lambda_c} \frac{1-\overline{S}_\chi(\lambda_{1P}+\lambda_c+\lambda_2)}{\lambda_{1P}+\lambda_c+\lambda_2} M^w + \frac{\lambda_{1P}\lambda_{2P}}{\lambda_1} . \frac{1-\overline{S}_\chi(\lambda_{1P}+\lambda_c)}{\lambda_{1P}+\lambda_c}$$

$$+ \frac{\lambda_{1P}\lambda_{2P}}{\lambda_1} . \frac{1-\overline{S}_\chi(\lambda_{1P}+\lambda_c+\lambda_2)}{\lambda_{1P}+\lambda_c+\lambda_2} + \frac{\lambda_{1P}\lambda_2}{\lambda_1} . \frac{1-\overline{S}_\chi(\lambda_{1P}+\lambda_c)}{\lambda_{1P}+\lambda_c}$$

$$\left. + \frac{\lambda_{1P}\lambda_2\lambda_{2P}}{\lambda_1} . \frac{1-\overline{S}_\eta(\lambda_{1P}+\lambda_c)}{\lambda_{1P}+\lambda_c} . \frac{1-\overline{S}_\chi(\lambda_{1P}+\lambda_c+\lambda_2)}{\lambda_{1P}+\lambda_c+\lambda_2} \right] \frac{1}{D'(0)}$$

$$P_{DN} = 1 - P_{AV} \quad (7)$$

where, $D'(0) = \frac{d}{ds}[D]_{s=0}$ *and* $M^\phi = -\dot{S}'^{\phi}(0)$

Particular Cases:

(1) Constant Rate Repair: When repair rate is an exponential setting by $\overline{S}(s) = \frac{j}{s+j}, where\ J = \phi_1, \Psi_1, \eta_1, \chi_1, w$ in relations (6) one may get Goel, Singh and Kishore [2] the following value

$$\overline{P}_{00}(s) = \frac{1}{E(s)}$$

$$\overline{P}_{0P}(s)=\frac{\lambda_{2P}}{s+\lambda_{1P}+\lambda_c+\lambda_2+\chi}\frac{1}{E(s)}$$

$$\overline{P}_{0F}(s)=\frac{1}{E(s)}\left[\frac{\lambda_2}{s+\lambda_{1P}+\lambda_c+\eta}+\frac{\lambda_{2P}}{(s+\lambda_{1P}+\lambda_c+\eta)(s+\lambda_{1P}+\lambda_c+\lambda_2+\chi)}\right] \quad (8)$$

$$\overline{P}_{cc}(s)=\left[\frac{\lambda_c}{(s+\phi_1)}+\frac{\lambda_2\lambda_{2P}}{(s+\lambda_{1P}+\lambda_c+\lambda_2+\chi)(s+\phi_1)}+\frac{\lambda_c\lambda_2}{(s+\lambda_{1P}+\lambda_c+\eta)}\frac{1}{(s+\phi_1)}\right.$$
$$\left.+\frac{\lambda_c\lambda_2\lambda_{2P}}{(s+\lambda_{1P}+\lambda_c+\lambda_2+\chi)}\cdot\frac{1}{(s+\lambda_{1P}+\lambda_c+\eta)}\frac{1}{(s+\phi_1)}\right]\frac{1}{E(s)}$$

$$\overline{P}_{w}(s)=\frac{w_1\lambda_1\lambda_2\lambda_{1P}\chi_{2P}}{(s+w_1)(s+\lambda_1)(s+\lambda_2)}\cdot\frac{1}{(s+w)}\cdot\frac{1}{(s+\lambda_{1P}+\lambda_c+\lambda_2+\chi)}$$
$$+\frac{w_1\lambda_1\lambda_2\lambda_{1P}}{(s+w_1)(s+\lambda_1)}\frac{1}{s+w}\cdot\frac{1}{(s+\lambda_{1P}+\lambda_c+\eta)}$$
$$+\frac{1}{s+\lambda_{1P}+\lambda_c+\lambda_2+\chi}\cdot\frac{w_1\lambda_1\lambda_2\lambda_{1P}\chi_{2P}}{(s+w_1)(s+\lambda_1)}\frac{1}{(s+w)}\frac{1}{(s+\lambda_{1P}+\lambda_c+\eta)}$$

$$\overline{P}_{hh}(s)=\frac{\lambda_h}{s+\Psi_1}\frac{1}{E(s)}$$

$$\overline{P}_{PP}(s)=\frac{\lambda_{1P}\lambda_{2P}}{s+\lambda_2}\frac{1}{s+\lambda_{1P}+\lambda_c+\lambda_2+\chi}\cdot\frac{1}{E(s)}$$

$$\overline{P}_{PF}(s)=\left[\frac{\lambda_2\lambda_{1P}\lambda_{2P}}{(s+\lambda_1)(s+\lambda_2)}\cdot\frac{1}{(s+\lambda_{1P}+\lambda_c+\lambda_2+\chi)}+\frac{\lambda_2\lambda_{1P}}{s+\lambda_1}\cdot\frac{1}{(s+\lambda_{1P}+\lambda_c+\eta)}\right.$$
$$\left.+\frac{\lambda_2\lambda_{1P}\lambda_{2P}}{s+\lambda_1}\cdot\frac{1}{s+\lambda_{1P}+\lambda_c+\eta}\cdot\frac{1}{s+\lambda_{1P}+\lambda_c+\lambda_2+\chi}\right]\frac{1}{E(s)}$$

$$\overline{P}_{FF}(s)=\left[\frac{\lambda_1\lambda_2\lambda_{1P}\lambda_{2P}}{(s+w_1)(s+\lambda_1)(s+\lambda_2)}\frac{1}{(s+\lambda_{1P}+\lambda_c+\lambda_2+\chi)}+\frac{\lambda_1\lambda_2\lambda_{1P}}{(s+w_1)(s+\lambda_1)}\cdot\frac{1}{s+\lambda_{1P}+\lambda_2+\eta}\right.$$
$$\left.+\frac{\lambda_1\lambda_2\lambda_{1P}\lambda_{2P}}{(s+w_1)(s+\lambda_1)}\cdot\frac{1}{s+\lambda_{1P}+\lambda_c+\lambda_2+\chi}\cdot\frac{1}{s+\lambda_{1P}+\lambda_c+\eta}\right]\frac{1}{E(s)}$$

$$\overline{P}_{up}(s)=\left[1+\frac{\lambda_{2P}}{s+\lambda_{1P}+\lambda_c+\lambda_2+\chi}+\frac{\lambda_2}{s+\lambda_{1P}+\lambda_c+\eta}+\frac{\lambda_2\lambda_{2P}}{s+\lambda_{1P}+\lambda_c+\eta}\cdot\frac{1}{s+\lambda_{1P}+\lambda_c+\lambda_2+\chi}\right.$$
$$+\frac{\lambda_{1P}\lambda_{2P}}{s+\lambda_2}\cdot\frac{1}{s+\lambda_{1P}+\lambda_c+\lambda_2+\chi}+\frac{\lambda_2\lambda_{1P}\lambda_{2P}}{(s+\lambda_1)(s+\lambda_2)}\cdot\frac{1}{(s+\lambda_{1P}+\lambda_c+\lambda_2+\chi)}$$
$$+\frac{\lambda_2\lambda_{1P}}{s+\lambda_1}\cdot\frac{1}{s+\lambda_{1P}+\lambda_c+\eta}+\frac{\lambda_2\lambda_{1P}\lambda_{2P}}{s+\lambda_1}\cdot\frac{1}{s+\lambda_{1P}+\lambda_c+\eta}\cdot\frac{1}{s+\lambda_{1P}+\lambda_c+\lambda_2+\chi}$$
$$+\frac{w_1\lambda_1\lambda_2\lambda_{1P}\chi_{2P}}{(s+w_1)(s+\lambda_1)(s+\lambda_2)}\frac{1}{s+w)}\cdot\frac{1}{s+\lambda_{1P}+\lambda_c+\lambda_2+\chi}\cdot\frac{1}{s+\lambda_{1P}+\lambda_c+\lambda_2+\chi}$$
$$\left.+\frac{w_1\lambda_1\lambda_2\lambda_{1P}}{(s+w_1)(s+\lambda_1)}\cdot\frac{1}{(s+w)(s+\lambda_{1P}+\lambda_c+\eta)}\right]$$

Non Repairable System:

The Laplace system of transform of reliability when all repairs of the system are zero, i.e., given by $\phi_1 = w_1 = \eta = \chi = w = 0$

$$\overline{R}(s) = \frac{A}{s+A_1} + \frac{B}{s+A_2} + \frac{C}{s+A_3} + \frac{D}{s+\lambda_1} + \frac{E}{s+\lambda_2} + \frac{F}{s+w_1} + \frac{G}{s+w_2}$$

where, $A_1 = \lambda_2 + \lambda_{2P} + \lambda_n + \lambda_c$, $A_2 = \lambda_{1P} + \lambda_c + \lambda_2$, $A_3 = \lambda_{1P} + \lambda_c$

$$A = 1 + \frac{\lambda_{2P}}{A_2 - A_1} + \frac{\lambda_2}{A_3 - A_1} + \frac{\lambda_2\lambda_{2P}}{(A_1 - A_2)(A_1 - A_3)} + \frac{\lambda_{1P}\lambda_{2P}}{(A_1 - A_2)(A_1 - \lambda_2)} + \frac{\lambda_2\lambda_{1P}}{(A_1 - A_3)(A_1 - \lambda_1)}$$
$$- \frac{\lambda_2\lambda_{1P}\lambda_{2P}}{(A_1 - \lambda_1)(A_1 - \lambda_2)(A_1 - A_2)} - \frac{\lambda_2\lambda_{1P}\lambda_{2P}}{(A_1 - \lambda_1)(A_1 - A_2)(A_1 - A_3)} + \frac{w_1\lambda_1\lambda_2\lambda_{1P}}{A_1(A_1 - w_1)(A_1 - \lambda_1)(A_1 - A_3)}$$
$$- \frac{w_1\lambda_1\lambda_2\lambda_{1P}\lambda_{2P}}{A_1(A_1 - w_1)(A_1 - \lambda_1)(A_1 - A_2)(A_1 - A_3)}$$

$$B = \frac{\lambda_{2P}}{A_1 - A_2} + \frac{\lambda_2\lambda_{2P}}{(A_2 - A_1)(A_2 - A_3)} + \frac{\lambda_{1P}\lambda_{2P}}{(A_1 - A_2)(A_2 - \lambda_2)} + \frac{\lambda_2\lambda_{1P}\lambda_{2P}}{(A_1 - A_2)(\lambda_1 - A_2)(\lambda_2 - A_2)}$$
$$- \frac{\lambda_2\lambda_{1P}\lambda_{2P}}{(A_1 - A_2)(\lambda_1 - A_2)(\lambda_2 - A_3)} + \frac{w_1\lambda_1\lambda_2\lambda_{1P}\lambda_{2P}}{A_2(A_2 - A_1)(A_2 - w_1)(A_2 - \lambda_1)(A_2 - \lambda_2)}$$
$$- \frac{w_1\lambda_1\lambda_2\lambda_{1P}\lambda_{2P}}{A_2(A_2 - A_1)(A_2 - w_1)(A_2 - \lambda_1)(A_2 - \lambda_2)}$$

$$C = \frac{\lambda_2}{A_1 - A_3} + \frac{\lambda_2\lambda_{2P}}{(A_3 - A_1)(A_3 - A_2)} + \frac{\lambda_2\lambda_{1P}}{(A_1 - A_3)(A_1 - A_3)} + \frac{\lambda_2\lambda_{1P}\lambda_{2P}}{(A_1 - A_3)(\lambda_1 - A_3)(A_2 - A_3)}$$
$$- \frac{w_1\lambda_1\lambda_{1P}\lambda_2}{A_3(A_3 - A_1)(A_3 - W_1)(A_3 - \lambda_1)} - \frac{w_1\lambda_1\lambda_2\lambda_{1P}\lambda_{2P}}{A_3(A_3 - A_1)(A_3 - w_1)(A_3 - \lambda_1)(A_3 - A_2)}$$

$$D = \frac{w_1\lambda_2\lambda_{1P}}{(\lambda_1 - A_1)(\lambda_1 - w_1)(\lambda_1 - A_3)} + \frac{\lambda_2\lambda_{2P}}{(A_1 - \lambda_1)(\lambda_1 - A_2)} + \frac{\lambda_2\lambda_{1P}\lambda_{2P}}{(A_1 - \lambda_1)(\lambda_1 - A_2)(\lambda_1 - A_3)}$$
$$+ \frac{\lambda_2\lambda_{1P}\lambda_{2P}}{(A_1 - \lambda_1)(\lambda_1 - \lambda_2)(\lambda_1 - A_2)} - \frac{w_1\lambda_2\lambda_{1P}\lambda_{2P}}{(\lambda_1 - A_1)(\lambda_1 - w)(\lambda_1 - \lambda_2)(\lambda_1 - A_2)} - \frac{w_1\lambda_2\lambda_{1P}\lambda_{2P}}{(\lambda_1 - A_1)(\lambda_1 - w_1)(\lambda_1 - A_2)(\lambda_1 - A_3)}$$

$$E = \frac{\lambda_{1P}\lambda_{2P}}{(\lambda_2 - A_1)(\lambda_2 - A_2)} - \frac{\lambda_2\lambda_{1P}\lambda_{2P}}{(\lambda_2 - A_1)(\lambda_2 - A_1)(\lambda_2 - A_2)} - \frac{w_1\lambda_1\lambda_{1P}\lambda_{2P}}{(A_2 - A_1)(\lambda_2 - w_1)(\lambda_2 - \lambda_1)(\lambda_2 - A_2)}$$

$$F = \frac{\lambda_2\lambda_{1P}}{A_1A_3} - \frac{\lambda_{1P}\lambda_{2P}}{A_1A_2} + \frac{\lambda_{1P}\lambda_{2P}}{A_1A_2A_3}$$

$$G = \frac{\lambda_1\lambda_2\lambda_{1P}\lambda_{2P}}{(A_1 - w_1)(w_1 - \lambda_1)(w_1 - \lambda_2(w - A_2)} + \frac{\lambda_1\lambda_2\lambda_{1P}\lambda_{2P}}{(w_1 - A_1)(w_1 - \lambda_1)(w_1 - A_2)(w_1 - A_3)} - \frac{\lambda_1\lambda_2\lambda_{1P}}{(w_1 - A_1)(w_1 - \lambda_1)(w_1 - A_3)}$$

The reliability of the system is

$$R(t) = Ae^{-A_1t} + Be^{-A_2t} + Ce^{-A_3t} + De^{-\lambda_1t} + Ee^{-\lambda_2t} + F + Ge^{-w_1t} \tag{9}$$

where, $E(s) = s + \lambda_2 + \lambda_{2P} + \lambda_h + \lambda_c - \dfrac{\eta\lambda_2}{s + \lambda_{1P} + \lambda_c + \eta} - \dfrac{\eta\lambda_2\lambda_{2P}}{(s + \lambda_{1P} + \lambda_c + \eta)(s + \lambda_{1P} + \lambda_c + \lambda_2 + \chi)}$

$$+\frac{\lambda\lambda_{2P}}{s+\lambda_{1P}+\lambda_c+\lambda_2+\chi}\cdot\frac{\lambda_h\Psi_1}{s+\Psi_1}+\frac{\lambda_c\phi_1}{s+\phi_1}-\frac{\lambda_c\lambda_{2P}\phi_1}{(s+\phi_1)(s+\lambda_{1P}+\lambda_c+\lambda_2+\chi)}+\frac{\phi_1\lambda_c\lambda_2}{(s+\phi_1)(s+\lambda_{1P}+\lambda_c+\eta)}$$

$$+\frac{\phi_1\lambda_c\lambda_2\lambda_{2P}}{(s+\phi_1)(s+\lambda_{1P}+\lambda_c+\eta)(s+\lambda_{1P}+\lambda_c+\lambda_2+\chi)}+\frac{ww_1\lambda_1\lambda_2\lambda_{1P}\chi_{2P}}{(s+w)(s+w_1)(s+\lambda_1)(s+\lambda_{1P}+\lambda_c+\eta)}$$

$$-\frac{ww_1\lambda_1\lambda_2\lambda_{1P}\chi_{2P}}{(s+w)(s+w_1)(s+\lambda_1)(s+\lambda_2)(s+\lambda_{1P}+\lambda_c+\lambda_2+\eta)}-\frac{ww_1\lambda_1\lambda_2\lambda_{1P}\chi_{2P}}{(s+w)(s+w_1)(s+\lambda_1)(s+\lambda_{1P}+\lambda_c+\lambda_2+\eta)(s+\lambda_{1P}+\lambda_c+\lambda_2+\chi)}$$

Discussion:

We look parameters as follows as Goel, Singh and Kishore [2].
Let

$$\phi_1=\Psi_1=\eta=\chi=w=1,\lambda_1=0.1,\lambda_2=0.01,\chi_n=0.2$$
$$\lambda_c=0.05,w_1=0.1,\lambda_{2P}=0.15,\lambda_{1P}=0.25 \quad (10)$$

The analyses for availability is obtained as follows:

$$P_{UP}(t)=2.7678+4.2501e^{-0.1088t}+0.9412e^{-1.1611t}-4.558e^{-0.0387t}-1.3354e^{-0.085t}+1.167e^{-1.5259t}-2.1917e^{-1.7105t} \quad (11)$$

Calculating values of $P_{0P}(t)=att=0,1,2,3,4,5, we\,get\ P_{ut}(t)$

T	0	1	2	3	4	5
P_{up}	0.99	0.94	0.83	0.77	0.67	0.61

The values obtained above show that P_{up} decreases as time increases. It can also be shown that increase of λ_c also causes decrease in P_{up}.

Let us consider

$$\lambda_1=0.1,\ \lambda_2=0.2,\ \lambda_{1p}=0.4,\ \lambda_{2p}=0.2,\ w_1=0.5\ \lambda_c=0.08,\ \lambda_n=0.4 \quad (12)$$

On account of (12), (8) and (9), we get

$A_1=0.88, A_2=0.68,\ and\ A_3=0.48$ and also

$$R(t)=0.755e^{-0.88t}+0.522e^{-0.68t}-0.0514e^{-0.48t}-0.177e^{-0.1t}-0.327e^{-0.2t}+0.3342+0.0975e^{-0.5t}$$

R(t)

T	0	1	2	3	4	5
R(t)	1.153	0.552	0.233	0.153	0.132	0.128

We draw a graph in R(t), i.e., reliability time. It is found that reliability decreases with time initially decreases is sharp but gradually it is a uniform decrease.

Cost Effectiveness of the system:

Assuming that the service facility is always available and remains busy for time t during interval] 0, t [. Let $C_1\,and\,C_2$ be revariance cost per unit time and service cost per unit times.

The profit function Profit(t)

$$=C_1\int_0^1 P_{up}(t)dt-C_2t$$

$$C_1(t)=C_1[2.7678t-39.0634e^{-0.1088t}-0.8106e^{-1.161t}+0.7318e^{-1.5259t}+1.6724e^{-1.3105t}-94.5567]-C_2(t)$$

Let,

$C_1=1,\ t=0,1,2,.......,10,15\ and\ C_2=1,0.5,0.10,$ the expected profit are shown in the following table, we get respectively

$$C_1(t) = 1.7678t + 177.7e^{-0.038t} - 15.71e^{-0.065t} + 1.6729e^{-1.3105t} - [39.06e^{-1.088t} + 8106e^{-1.1611t} + 0.7318e^{-1.5259t}]$$

Particular values of $C_1, C_2, ...etc.$ we calculate values of $C_1(t)$ a profit function, we give them in this table .

Table : 1

Sr.No	Time	If $C_2 = 1$ G(t)	If $C_2 = 0.5$ G(t)	If $C_2 = 0.1$ G(t)
1	0	-0.067	-0.067	-0.067
2	1	-0.0922	-0.9012	0.867
3	2	-0.148	0.733	1.533
4	3	-0.267	1.043	2.237
5	4	-0.7611	1.247	2.848
6	5	-1.125	1.378	3.374
7	10	-3.933	1.605	3.606

This table shows the expected profit during this interval (0, t) for the fixed value per unit of time. This shows that expected profit v/s time decreases rapidly where service cost $C_2 \geq 1$ and increases if $C_2 < 1$.

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