**A Summation Formula of Half Argument Influence with Recurrence Relation**

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***Research Article***

***Abstract:*** The main aim of the present paper is to evaluate a summation formula of half argument associated with contiguous relation and recurrence relation of gamma function.

***Key words****:* Gaussian Hypergeometric function, Recurrence relation, Bailey summation theorem, Contiguous relation.

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**Introduction:**

 **Generalized Gaussian Hypergeometric function of one variable is defined by**

AFB($a\_{1}$,a2,…,aA;b1,b2,…bB;z ) =$\sum\_{k=0}^{\infty }\frac{\left(a\_{1}\right)\_{k}\left(a\_{2}\right)\_{k}…..\left(a\_{A}\right)\_{k} z^{k}}{\left(b\_{1}\right)\_{k}\left(b\_{2}\right)\_{k}…..\left(b\_{B}\right)\_{k} k!}$ …(1)

where the parameters b1 , b2 , ….,bB are neither zero nor negative integers and A , B are non negative integers.

**Contiguous Relation is defined by**

[Andrews p.363(9.16)]

(a-b) 2F1(a, b ; c; z) = a 2F1(a+1,b; c; z) - b 2F1(a,b+1; c; z) …(2)

**Recurrence relation is defined by**

Γ(ξ+1) **=** ξ Γ(ξ) …(3)

**Bailey summation theorem[Prud, p.491(7.3.7.3)**

2F1(a, 1-a ; $c$; $\frac{1}{2}$) = $\frac{Γ\left(\frac{c}{2}\right)Γ(\frac{c+1}{2}) }{Γ\left(\frac{c+a}{2}\right)Γ(\frac{c+1-a}{2}) } $ = $\frac{\sqrt{π}Γ(c) }{2^{c-1}Γ\left(\frac{c+a}{2}\right)Γ(\frac{c+1-a}{2}) }$ …(4)

**Main Summation Formula:**

2F1(a, -a-40 ;$ c$; $\frac{1}{2}$) =$\frac{\sqrt{π}Γ(c) }{2^{c+40}}$[ $ \frac{-66875101180623272603049984000 a71357280184447789909895884800a^{2} }{Γ\left(\frac{c+a+40}{2}\right)Γ(\frac{c-a+1}{2}) }$

+$\frac{-25714790950373338695805785600a^{3}+3649155977880410544364179456a^{4}}{Γ\left(\frac{c+a+40}{2}\right)Γ\left(\frac{c-a+1}{2}\right) }$

+$\frac{-104982886696471128317302560a^{5}-17081120544451777974736080a^{6}+731764160260783185841360a^{7}}{Γ\left(\frac{c+a+40}{2}\right)Γ\left(\frac{c-a+1}{2}\right) }$

+$\frac{49058956189087369506136a^{8}-1127898659648383594530a^{9}-89322308991894454125a^{10}}{Γ\left(\frac{c+a+40}{2}\right)Γ\left(\frac{c-a+1}{2}\right) }$

+$\frac{-267351987755153730a^{11}+63311743532688261a^{12}+1230183880910140a^{13}-3243688031810a^{14}}{Γ\left(\frac{c+a+40}{2}\right)Γ\left(\frac{c-a+1}{2}\right) }$

+$\frac{-341178590820a^{15}-3651511854a^{16}-5077050a^{17}+139215a^{18}+790a^{19}+a^{20}}{Γ\left(\frac{c+a+40}{2}\right)Γ\left(\frac{c-a+1}{2}\right) }$

+$\frac{66875101180744917703458816000c-189985004451228730616119296000 ac}{Γ\left(\frac{c+a+40}{2}\right)Γ\left(\frac{c-a+1}{2}\right) }$

+$\frac{127461722050600090647554949120a^{2}c-32698614748137402381734117376a^{3}c}{Γ\left(\frac{c+a+40}{2}\right)Γ\left(\frac{c-a+1}{2}\right) }$

+$\frac{3112488390013770587916985344a^{4}c+17435208969056442656858112a^{5}c}{Γ\left(\frac{c+a+40}{2}\right)Γ\left(\frac{c-a+1}{2}\right) }$

+$\frac{-15453151414910521210524992a^{6}c+97642175088798664593728a^{7}c+36361145615195178372192a^{8}c}{Γ\left(\frac{c+a+40}{2}\right)Γ\left(\frac{c-a+1}{2}\right) }$

+$\frac{207062572140949520736a^{9}c-39702606806587503636a^{10}c-801954460495433388a^{11}c}{Γ\left(\frac{c+a+40}{2}\right)Γ\left(\frac{c-a+1}{2}\right) }$

+$\frac{9766776111021904a^{12}c+461063762814512a^{13}c+3875318999688a^{14}c-28428184584a^{15}c}{Γ\left(\frac{c+a+40}{2}\right)Γ\left(\frac{c-a+1}{2}\right) }$

+$\frac{-645434640a^{16}c-3251760a^{17}c-980a^{18}c+20a^{19}c+118627724267212505853041049600c^{2}}{Γ\left(\frac{c+a+40}{2}\right)Γ\left(\frac{c-a+1}{2}\right) }$

+$\frac{-193639952718842374558148198400ac^{2}+93016963884979881869293559808a^{2}c^{2}}{Γ\left(\frac{c+a+40}{2}\right)Γ\left(\frac{c-a+1}{2}\right) }$

+$\frac{-17422819402724518077246311424a^{3}c^{2}+1031610380484178452170147328a^{4}c^{2}}{Γ\left(\frac{c+a+40}{2}\right)Γ\left(\frac{c-a+1}{2}\right) }$

+$\frac{46196414077865484231694272a^{5}c^{2}-4959237589489857015278272a^{6}c^{2}-100719136266782788084512a^{7}c^{2}}{Γ\left(\frac{c+a+40}{2}\right)Γ\left(\frac{c-a+1}{2}\right) }$

+$\frac{8587061429047147285824a^{8}c^{2}+226898390603909225916a^{9}c^{2}-4552047674626168860a^{10}c^{2}}{Γ\left(\frac{c+a+40}{2}\right)Γ\left(\frac{c-a+1}{2}\right) }$

+$\frac{-208525479763087344 a^{11}c^{2}-1124881537937872a^{12}c^{2}+42278077673832a^{13}c^{2}+684327983064a^{14}c^{2}}{Γ\left(\frac{c+a+40}{2}\right)Γ\left(\frac{c-a+1}{2}\right) }$

+$\frac{2194766640a^{15}c^{2}-21386640a^{16}c^{2}-156420a^{17}c^{2}-220a^{18}c^{2}+91893021619284232336740188160c^{3}}{Γ\left(\frac{c+a+40}{2}\right)Γ\left(\frac{c-a+1}{2}\right) }$

+$\frac{-105894362587221112530214453248ac^{3}+38184824036314771057292378112a^{2}c^{3}}{Γ\left(\frac{c+a+40}{2}\right)Γ\left(\frac{c-a+1}{2}\right) }$

+$\frac{-5217954699738689318667147264a^{3}c^{3}+154802720535224701053833216a^{4}c^{3}}{Γ\left(\frac{c+a+40}{2}\right)Γ\left(\frac{c-a+1}{2}\right) }$

+$\frac{18035890704304867606142336a^{5}c^{3}-720787628093566157429760a^{6}c^{3}-37777438252222068429120a^{7}c^{3}}{Γ\left(\frac{c+a+40}{2}\right)Γ\left(\frac{c-a+1}{2}\right) }$

+$\frac{748744915379634640896a^{8}c^{3}+46358887174170154584a^{9}c^{3}+116326659272412160a^{10}c^{3}}{Γ\left(\frac{c+a+40}{2}\right)Γ\left(\frac{c-a+1}{2}\right) }$

+$\frac{-19498313510630240a^{11}c^{3}-283614779255808a^{12}c^{3}+424869773328a^{13}c^{3}+34884157440a^{14}c^{3}}{Γ\left(\frac{c+a+40}{2}\right)Γ\left(\frac{c-a+1}{2}\right) }$

+$\frac{223718880a^{15}c^{3}+168960a^{16}c^{3}-1320a^{17}c^{3}+41926857473108338574031323136c^{4}}{Γ\left(\frac{c+a+40}{2}\right)Γ\left(\frac{c-a+1}{2}\right) }$

+$\frac{-36530194263573519025608425472ac^{4}+10073161895102447793343156224a^{2}c^{4}}{Γ\left(\frac{c+a+40}{2}\right)Γ\left(\frac{c-a+1}{2}\right) }$

+$\frac{-982994215510126414330402816a^{3}c^{4}+3525971281070213051043584a^{4}c^{4}}{Γ\left(\frac{c+a+40}{2}\right)Γ\left(\frac{c-a+1}{2}\right) }$

+$\frac{3495316104378035584424960a^{5}c^{4}-34614294900669968848000a^{6}c^{4}-5838058520962342170496a^{7}c^{4}}{Γ\left(\frac{c+a+40}{2}\right)Γ\left(\frac{c-a+1}{2}\right) }$

+$\frac{-14056364908366038544a^{8}c^{4}+4211580331661514240a^{9}c^{4}+59896230910401600a^{10}c^{4}}{Γ\left(\frac{c+a+40}{2}\right)Γ\left(\frac{c-a+1}{2}\right) }$

+$\frac{-632023790990592a^{11}c^{4}-18889044222048a^{12}c^{4}-97887229440a^{13}c^{4}+486678720a^{14}c^{4}+5005440a^{15}c^{4}}{Γ\left(\frac{c+a+40}{2}\right)Γ\left(\frac{c-a+1}{2}\right) }$

+$\frac{7920a^{16}c^{4}+12760684127037351373307904000c^{5}-8660513420106141663756288000ac^{5}}{Γ\left(\frac{c+a+40}{2}\right)Γ\left(\frac{c-a+1}{2}\right) }$

+$\frac{1833562634531636673901756416a^{2}c^{5}-121841366024253671046283264a^{3}c^{5}}{Γ\left(\frac{c+a+40}{2}\right)Γ\left(\frac{c-a+1}{2}\right) }$

+$\frac{-2710623730807837944479744a^{4}c^{5}+400665605741716065773568a^{5}c^{5}+4070405183765017889792a^{6}c^{5}}{Γ\left(\frac{c+a+40}{2}\right)Γ\left(\frac{c-a+1}{2}\right) }$

+$\frac{-483555425073296576000a^{7}c^{5}-8068276861929453312a^{8}c^{5}+175566838422353664a^{9}c^{5}}{Γ\left(\frac{c+a+40}{2}\right)Γ\left(\frac{c-a+1}{2}\right) }$

+$\frac{4847389137566976a^{10}c^{5}+12230678363904a^{11}c^{5}-506820297984a^{12}c^{5}-4312019712a^{13}c^{5}}{Γ\left(\frac{c+a+40}{2}\right)Γ\left(\frac{c-a+1}{2}\right) }$

+$\frac{-5246208a^{14}c^{5}+25344a^{15}c^{5}+2773341755995029808758128640c^{6}-1485763907930506933037694976ac^{6}}{Γ\left(\frac{c+a+40}{2}\right)Γ\left(\frac{c-a+1}{2}\right) }$

+$\frac{240108485818220184763269120a^{2}c^{6}-9852957179482591512805376a^{3}c^{6}-534557977754408653846528a^{4}c^{6}}{Γ\left(\frac{c+a+40}{2}\right)Γ\left(\frac{c-a+1}{2}\right) }$

+$\frac{28149670762417281645568a^{5}c^{6}+807433260599691934208a^{6}c^{6}-21267233510805970688a^{7}c^{6}}{Γ\left(\frac{c+a+40}{2}\right)Γ\left(\frac{c-a+1}{2}\right) }$

+$\frac{-704829460473991424a^{8}c^{6}+979730718430464a^{9}c^{6}+178653618054912a^{10}c^{6}+1396938326784a^{11}c^{6}}{Γ\left(\frac{c+a+40}{2}\right)Γ\left(\frac{c-a+1}{2}\right) }$

+$\frac{-3506350848a^{12}c^{6}-60732672a^{13}c^{6}-109824a^{14}c^{6}+448917741602817129568010240c^{7}}{Γ\left(\frac{c+a+40}{2}\right)Γ\left(\frac{c-a+1}{2}\right) }$

+$\frac{-190651892544240900597874688ac^{7}+23175844810777525124071424a^{2}c^{7}}{Γ\left(\frac{c+a+40}{2}\right)Γ\left(\frac{c-a+1}{2}\right) }$

+$\frac{-459438476789254584680448a^{3}c^{7}-54629207556008653860864a^{4}c^{7}+1106835970971998046208a^{5}c^{7}}{Γ\left(\frac{c+a+40}{2}\right)Γ\left(\frac{c-a+1}{2}\right) }$

+$\frac{64294963263526927360a^{6}c^{7}-284458551615546880a^{7}c^{7}-30896984328477696a^{8}c^{7}-210676113483264a^{9}c^{7}}{Γ\left(\frac{c+a+40}{2}\right)Γ\left(\frac{c-a+1}{2}\right) }$

+$\frac{2959838069760a^{10}c^{7}+36144176640a^{11}c^{7}+62819328a^{12}c^{7}-219648a^{13}c^{7}+55635447236064914194825216c^{8}}{Γ\left(\frac{c+a+40}{2}\right)Γ\left(\frac{c-a+1}{2}\right) }$

+$\frac{-18698302219224664564629504ac^{8}+1667782924502252287827968a^{2}c^{8}-2661783539860427913216a^{3}c^{8}}{Γ\left(\frac{c+a+40}{2}\right)Γ\left(\frac{c-a+1}{2}\right) }$

+$\frac{-3587755914140667563008a^{4}c^{8}+9036117795554403840a^{5}c^{8}+2978068356617319680a^{6}c^{8}}{Γ\left(\frac{c+a+40}{2}\right)Γ\left(\frac{c-a+1}{2}\right) }$

+$\frac{18798345017227776a^{7}c^{8}-729494915525376a^{8}c^{8}-8879247336960a^{9}c^{8}+6645999360a^{10}c^{8}+364396032a^{11}c^{8}}{Γ\left(\frac{c+a+40}{2}\right)Γ\left(\frac{c-a+1}{2}\right) }$

+$\frac{768768a^{12}c^{8}+5376995734268194219622400c^{9}-1420850828358443319951360ac^{9}}{Γ\left(\frac{c+a+40}{2}\right)Γ\left(\frac{c-a+1}{2}\right) }$

+$\frac{89401407848249932644352a^{2}c^{9}+1332525643898270728192a^{3}c^{9}-159187142187653099520a^{4}c^{9}}{Γ\left(\frac{c+a+40}{2}\right)Γ\left(\frac{c-a+1}{2}\right) }$

+$\frac{-1515726793160785920a^{5}c^{9}+83805625001978880a^{6}c^{9}+1141818425459712a^{7}c^{9}-7537462732800a^{8}c^{9}}{Γ\left(\frac{c+a+40}{2}\right)Γ\left(\frac{c-a+1}{2}\right) }$

+$\frac{-156551915520a^{9}c^{9}-374133760a^{10}c^{9}+1025024a^{11}c^{9}+410085684500756095303680c^{10}}{Γ\left(\frac{c+a+40}{2}\right)Γ\left(\frac{c-a+1}{2}\right) }$

+$\frac{-84274841045028033134592ac^{10}+3508253808010892328960a^{2}c^{10}+110678463999328276480a^{3}c^{10}}{Γ\left(\frac{c+a+40}{2}\right)Γ\left(\frac{c-a+1}{2}\right) }$

+$\frac{-4732151243543060480a^{4}c^{10}-92483291170350080a^{5}c^{10}+1309095400797184a^{6}c^{10}+28730141440000a^{7}c^{10}}{Γ\left(\frac{c+a+40}{2}\right)Γ\left(\frac{c-a+1}{2}\right) }$

+$\frac{25820354560a^{8}c^{10}-1214653440a^{9}c^{10}-3075072a^{10}c^{10}+24846733566518671441920c^{11}}{Γ\left(\frac{c+a+40}{2}\right)Γ\left(\frac{c-a+1}{2}\right) }$

+$\frac{-3909374352436847443968ac^{11}+95793392223282364416a^{2}c^{11}+5055276670898741248a^{3}c^{11}}{Γ\left(\frac{c+a+40}{2}\right)Γ\left(\frac{c-a+1}{2}\right) }$

+$\frac{-87138747741380608a^{4}c^{11}-2773143309920256a^{5}c^{11}+5939823984640a^{6}c^{11}+376457768960a^{7}c^{11}}{Γ\left(\frac{c+a+40}{2}\right)Γ\left(\frac{c-a+1}{2}\right) }$

+$\frac{1241210880a^{8}c^{11}-2795520a^{9}c^{11}+1198625180760970100736c^{12}-141347017399724670976ac^{12}}{Γ\left(\frac{c+a+40}{2}\right)Γ\left(\frac{c-a+1}{2}\right) }$

+$\frac{1545233538725773312a^{2}c^{12}+152069777125081088a^{3}c^{12}-688545627521024a^{4}c^{12}-49137876336640a^{5}c^{12}}{Γ\left(\frac{c+a+40}{2}\right)Γ\left(\frac{c-a+1}{2}\right) }$

+$\frac{-143115714560a^{6} c^{12}+2355691520a^{7}c^{12}+7454720a^{8}c^{12}+45936354549510963200c^{13}}{Γ\left(\frac{c+a+40}{2}\right)Γ\left(\frac{c-a+1}{2}\right) }$

+$\frac{-3945389459851182080ac^{13}+1759899720613888a^{2}c^{13}+3107013266767872a^{3}c^{13}+7880455618560a^{4}c^{13}}{Γ\left(\frac{c+a+40}{2}\right)Γ\left(\frac{c-a+1}{2}\right) }$

+$\frac{-506274119680a^{5}c^{13}-2399272960a^{6}c^{13}+4587520a^{7}c^{13}+1388679268736696320c^{14}}{Γ\left(\frac{c+a+40}{2}\right)Γ\left(\frac{c-a+1}{2}\right) }$

+$\frac{-83551983410085888ac^{14}-610672412721152a^{2}c^{14}+42209659453440a^{3}c^{14}+264701870080a^{4}c^{14}}{Γ\left(\frac{c+a+40}{2}\right)Γ\left(\frac{c-a+1}{2}\right) }$

+$\frac{-2640445440a^{5}c^{14}-11141120a^{6}c^{14}+32680067255500800c^{15}-1304276194295808ac^{15}}{Γ\left(\frac{c+a+40}{2}\right)Γ\left(\frac{c-a+1}{2}\right) }$

+$\frac{-16405851799552a^{2}c^{15}+355682484224a^{3}c^{15}+2682781696a^{4}c^{15}-4456448a^{5}c^{15}+585809145102336c^{16}}{Γ\left(\frac{c+a+40}{2}\right)Γ\left(\frac{c-a+1}{2}\right) }$

+$\frac{-14310271746048ac^{16}-221392994304a^{2}c^{16}+1584267264a^{3}c^{16}+10027008a^{4} c^{16}+7722133094400c^{17}}{Γ\left(\frac{c+a+40}{2}\right)Γ\left(\frac{c-a+1}{2}\right) }$

+$\frac{-101512642560ac^{17}-1606680576a^{2}c^{17}+2359296a^{3}c^{17}+70527221760c^{18}-393478144ac^{18}-4980736a^{2}c^{18}}{Γ\left(\frac{c+a+40}{2}\right)Γ\left(\frac{c-a+1}{2}\right) }$

+$\frac{398458880c^{19}-524288ac^{19}+1048576c^{20}}{Γ\left(\frac{c+a+40}{2}\right)Γ\left(\frac{c-a+1}{2}\right) }$ +$\frac{335367096786357081410764800000}{Γ\left(\frac{c+a+41}{2}\right)Γ\left(\frac{c-a}{2}\right) }$

+$\frac{-586058735458048022906351616000 a+312146142012827622963103334400a^{2}}{Γ\left(\frac{c+a+41}{2}\right)Γ\left(\frac{c-a}{2}\right) }$

+$\frac{-66199862346792081420924541440a^{3}+4548866988247932940955571456a^{4}}{Γ\left(\frac{c+a+41}{2}\right)Γ\left(\frac{c-a}{2}\right) }$

+$\frac{227018175022003605939171360a^{5}-29944950177747597574144080a^{6}-659105654914824606149840a^{7}}{Γ\left(\frac{c+a+41}{2}\right)Γ\left(\frac{c-a}{2}\right) }$

+$\frac{75791087035872782536936a^{8}+2276608375387101064530a^{9}-66184234273950611325a^{10}}{Γ\left(\frac{c+a+41}{2}\right)Γ\left(\frac{c-a}{2}\right) }$

+$\frac{-3525199213630512990a^{11}-20282520330520539a^{12}+1382803279676260a^{13}+29983302492190a^{14}}{Γ\left(\frac{c+a+41}{2}\right)Γ\left(\frac{c-a}{2}\right) }$

+$\frac{128128444260a^{15}-2708725854a^{16}-37872150a^{17}-157185a^{18}+10a^{19}+a^{20}}{Γ\left(\frac{c+a+41}{2}\right)Γ\left(\frac{c-a}{2}\right) }$

+$\frac{831600805538859803215921152000 c-1052987018520593812723767705600 a c}{Γ\left(\frac{c+a+41}{2}\right)Γ\left(\frac{c-a}{2}\right) }$

+$\frac{426733904516661688776588656640a^{2}c-66770364740738418141395779584a^{3}c}{Γ\left(\frac{c+a+41}{2}\right)Γ\left(\frac{c-a}{2}\right) }$

+$\frac{2348085345321128686474834944a^{4}c+310009236027494578843051008a^{5}c}{Γ\left(\frac{c+a+41}{2}\right)Γ\left(\frac{c-a}{2}\right) }$

+$\frac{-15389298678996674662503232a^{6}c-921268164207427190765888a^{7}c+24753995876431093595232a^{8}c}{Γ\left(\frac{c+a+41}{2}\right)Γ\left(\frac{c-a}{2}\right) }$

+$\frac{1762507274982653088864a^{9}c+2990096423654681484a^{10}c-1301859240291902292a^{11}c}{Γ\left(\frac{c+a+41}{2}\right)Γ\left(\frac{c-a}{2}\right) }$

+$\frac{-24267427207871536a^{12}c+82526341522768a^{13}c+7034302070088a^{14}c+73302270984a^{15}c}{Γ\left(\frac{c+a+41}{2}\right)Γ\left(\frac{c-a}{2}\right) }$

+$\frac{85538160a^{16}c-2925840a^{17}c-16180a^{18}c-20a^{19}c+826273441765035683802670694400c^{2}}{Γ\left(\frac{c+a+41}{2}\right)Γ\left(\frac{c-a}{2}\right) }$

+$\frac{-803752902887255179892410613760ac^{2}+252024396308373373971963199488a^{2}c^{2}}{Γ\left(\frac{c+a+41}{2}\right)Γ\left(\frac{c-a}{2}\right) }$

+$\frac{-28489035748750676632029828096a^{3}c^{2}+143532631387480624722044928a^{4}c^{2}}{Γ\left(\frac{c+a+41}{2}\right)Γ\left(\frac{c-a}{2}\right) }$

+$\frac{140142407875774280669379648a^{5}c^{2}-1865560153509788547915712a^{6}c^{2}-345420571903967656413408a^{7}c^{2}}{Γ\left(\frac{c+a+41}{2}\right)Γ\left(\frac{c-a}{2}\right) }$

+$\frac{-551239531237627323936a^{8}c^{2}+406065609377486142084a^{9}c^{2}+7101041412627891300a^{10}c^{2}}{Γ\left(\frac{c+a+41}{2}\right)Γ\left(\frac{c-a}{2}\right) }$

+$\frac{-118726440826380816a^{11}c^{2}-4640236215012112a^{12}c^{2}-34256508277992a^{13}c^{2}+345239519064a^{14}c^{2}}{Γ\left(\frac{c+a+41}{2}\right)Γ\left(\frac{c-a}{2}\right) }$

+$\frac{6665495760a^{15}c^{2}+31122960a^{16}c^{2}-1980a^{17}c^{2}-220a^{18}c^{2}+462007088894186241001814753280c^{3}}{Γ\left(\frac{c+a+41}{2}\right)Γ\left(\frac{c-a}{2}\right) }$

+$\frac{-354582225664640572957069934592ac^{3}+86346699390299085548205883392a^{2}c^{3}}{Γ\left(\frac{c+a+41}{2}\right)Γ\left(\frac{c-a}{2}\right) }$

+$\frac{-6732765627651442139291314176a^{3}c^{3}-170892347817276835698473984a^{4}c^{3}}{Γ\left(\frac{c+a+41}{2}\right)Γ\left(\frac{c-a}{2}\right) }$

+$\frac{31556442577814191193371264a^{5}c^{3}+354192263042024047280640a^{6}c^{3}-58575583216943264122560a^{7}c^{3}}{Γ\left(\frac{c+a+41}{2}\right)Γ\left(\frac{c-a}{2}\right) }$

+$\frac{-1173534835070915089344a^{8}c^{3}+37107444878219989416a^{9}c^{3}+1314641080731080960 a^{10}c^{3}}{Γ\left(\frac{c+a+41}{2}\right)Γ\left(\frac{c-a}{2}\right) }$

+$\frac{3724085398748000a^{11}c^{3}-316469507722368a^{12}c^{3}-4376491046928a^{13}c^{3}-9460450560a^{14}c^{3}+171647520a^{15}c^{3}}{Γ\left(\frac{c+a+41}{2}\right)Γ\left(\frac{c-a}{2}\right) }$

+$\frac{1066560a^{16}c^{3}+1320a^{17}c^{3}+167709453188902576587109564416c^{4}-102825071534689323090904055808ac^{4}}{Γ\left(\frac{c+a+41}{2}\right)Γ\left(\frac{c-a}{2}\right) }$

+$\frac{19343633758355617130561900544a^{2}c^{4}-946172152672754707373983744a^{3}c^{4}}{Γ\left(\frac{c+a+41}{2}\right)Γ\left(\frac{c-a}{2}\right) }$

+$\frac{-60290005312197029135708416a^{4}c^{4}+3984033685438780048624640a^{5}c^{4}}{Γ\left(\frac{c+a+41}{2}\right)Γ\left(\frac{c-a}{2}\right) }$

+$\frac{136673012945841535479680a^{6}c^{4}-4884451546970599187584a^{7}c^{4}-202269720518538044944a^{8}c^{4}}{Γ\left(\frac{c+a+41}{2}\right)Γ\left(\frac{c-a}{2}\right) }$

+$\frac{582289238867381760a^{9}c^{4}+102748140882953280a^{10}c^{4}+1170749667639552a^{11}c^{4}-5984812081248a^{12}c^{4}}{Γ\left(\frac{c+a+41}{2}\right)Γ\left(\frac{c-a}{2}\right) }$

+$\frac{-186633807360a^{13}c^{4}-995945280a^{14}c^{4}+63360a^{15}c^{4}+7920a^{16}c^{4}+42736925839918853495822745600c^{5}}{Γ\left(\frac{c+a+41}{2}\right)Γ\left(\frac{c-a}{2}\right) }$

+$\frac{-21026745656045496348463595520ac^{5}+3014771937094573629763485696a^{2}c^{5}}{Γ\left(\frac{c+a+41}{2}\right)Γ\left(\frac{c-a}{2}\right) }$

+$\frac{-72964755005552301942603776a^{3}c^{5}-10309096016026560048185344a^{4}c^{5}+269991041072289608128512a^{5}c^{5}}{Γ\left(\frac{c+a+41}{2}\right)Γ\left(\frac{c-a}{2}\right) }$

+$\frac{19154701981218139528192a^{6}c^{5}-129709603231218250240a^{7}c^{5}-16633374581983940352a^{8}c^{5}}{Γ\left(\frac{c+a+41}{2}\right)Γ\left(\frac{c-a}{2}\right) }$

+$\frac{-151281781107435264a^{9}c^{5}+3635690749506816a^{10}c^{5}+71857903493376a^{11}c^{5}+233564787456a^{12}c^{5}}{Γ\left(\frac{c+a+41}{2}\right)Γ\left(\frac{c-a}{2}\right) }$

+$\frac{-2883648768a^{13}c^{5}-20452608a^{14}c^{5}-25344a^{15}c^{5}+8024013991163045633253703680c^{6}}{Γ\left(\frac{c+a+41}{2}\right)Γ\left(\frac{c-a}{2}\right) }$

+$\frac{-3165473238864637987565338624ac^{6}+338023411250974242200125440a^{2}c^{6}}{Γ\left(\frac{c+a+41}{2}\right)Γ\left(\frac{c-a}{2}\right) }$

+$\frac{-825702749831203882287104a^{3}c^{6}-1095805112655400995248128a^{4}c^{6}+4327620278143835825152a^{5}c^{6}}{Γ\left(\frac{c+a+41}{2}\right)Γ\left(\frac{c-a}{2}\right) }$

+$\frac{1518645340045653467648a^{6}c^{6}+11967751984128548608a^{7}c^{6}-736369410218334464a^{8}c^{6}}{Γ\left(\frac{c+a+41}{2}\right)Γ\left(\frac{c-a}{2}\right) }$

+$\frac{-13167640519697664a^{9}c^{6}+23950343569152a^{10}c^{6}+1940990827776a^{11}c^{6}+12084264192a^{12}c^{6}}{Γ\left(\frac{c+a+41}{2}\right)Γ\left(\frac{c-a}{2}\right) }$

+$\frac{-768768a^{13}c^{6}-109824a^{14}c^{6}+1146015991167187352194908160c^{7}-360711651690653897029844992ac^{7}}{Γ\left(\frac{c+a+41}{2}\right)Γ\left(\frac{c-a}{2}\right) }$

+$\frac{27652829243691726536310784a^{2}c^{7}+494928666960181650014208a^{3}c^{7}-77565160960545084862464a^{4}c^{7}}{Γ\left(\frac{c+a+41}{2}\right)Γ\left(\frac{c-a}{2}\right) }$

+$\frac{-912683876126578935808a^{5}c^{7}+73131042518682536960a^{6}c^{7}+1367869214613890560a^{7}c^{7}}{Γ\left(\frac{c+a+41}{2}\right)Γ\left(\frac{c-a}{2}\right) }$

+$\frac{-15075948986182656a^{8}c^{7}-499536365492736a^{9}c^{7}-2289441623040a^{10}c^{7}+21421171200a^{11}c^{7}}{Γ\left(\frac{c+a+41}{2}\right)Γ\left(\frac{c-a}{2}\right) }$

+$\frac{177036288a^{12}c^{7}+219648a^{13}c^{7}+127238803468552105074098176c^{8}-31674770312859992507744256ac^{8}}{Γ\left(\frac{c+a+41}{2}\right)Γ\left(\frac{c-a}{2}\right) }$

+$\frac{1641293325061130488537088a^{2}c^{8}+64416791110452922791936a^{3}c^{8}-3682862114057569451008a^{4}c^{8}}{Γ\left(\frac{c+a+41}{2}\right)Γ\left(\frac{c-a}{2}\right) }$

+$\frac{-94654416978676938240a^{5}c^{8}+2011302801680057600a^{6}c^{8}+64755135689147904a^{7}c^{8}}{Γ\left(\frac{c+a+41}{2}\right)Γ\left(\frac{c-a}{2}\right) }$

+$\frac{71706791380224a^{8}c^{8}-9704950295040a^{9}c^{8}-72506353920a^{10}c^{8}+4612608a^{11}c^{8}+768768a^{12}c^{8}}{Γ\left(\frac{c+a+41}{2}\right)Γ\left(\frac{c-a}{2}\right) }$

+$\frac{11143915551184914481152000c^{9}-2166501342863241856942080ac^{9}+67945566007433385869312a^{2}c^{9}}{Γ\left(\frac{c+a+41}{2}\right)Γ\left(\frac{c-a}{2}\right) }$

+$\frac{4590308763633539268608a^{3}c^{9}-110433016064669675520a^{4}c^{9}-4847055932530606080a^{5}c^{9}}{Γ\left(\frac{c+a+41}{2}\right)Γ\left(\frac{c-a}{2}\right) }$

+$\frac{18414268492139520a^{6}c^{9}+1724296281996288a^{7}c^{9}+11059342694400a^{8}c^{9}-83303700480a^{9}c^{9}}{Γ\left(\frac{c+a+41}{2}\right)Γ\left(\frac{c-a}{2}\right) }$

+$\frac{-825144320a^{10}c^{9}-1025024a^{11}c^{9}+777139863716630479503360c^{10}-115983160550649716932608ac^{10}}{Γ\left(\frac{c+a+41}{2}\right)Γ\left(\frac{c-a}{2}\right) }$

+$\frac{1694186473196915343360a^{2}c^{10}+218855822547202846720a^{3}c^{10}-1518822066052628480a^{4}c^{10}}{Γ\left(\frac{c+a+41}{2}\right)Γ\left(\frac{c-a}{2}\right) }$

+$\frac{-153679418618485760a^{5}c^{10}-701774131874816a^{6}c^{10}+25879857203200a^{7}c^{10}+241690408960a^{8}c^{10}}{Γ\left(\frac{c+a+41}{2}\right)Γ\left(\frac{c-a}{2}\right) }$

+$\frac{-15375360a^{9}c^{10}-3075072a^{10}c^{10}+43366015594351810314240c^{11}-4856370236805384241152ac^{11}}{Γ\left(\frac{c+a+41}{2}\right)Γ\left(\frac{c-a}{2}\right) }$

+$\frac{4835027675975417856a^{2}c^{11}+7324612714279657472a^{3}c^{11}+25274194347630592a^{4}c^{11}}{Γ\left(\frac{c+a+41}{2}\right)Γ\left(\frac{c-a}{2}\right) }$

+$\frac{-3100057245702144a^{5}c^{11}-28833388380160a^{6}c^{11}+181751664640a^{7}c^{11}+2247598080a^{8}c^{11}+2795520a^{9}c^{11}}{Γ\left(\frac{c+a+41}{2}\right)Γ\left(\frac{c-a}{2}\right) }$

+$\frac{1938152320641559166976c^{12}-157973724927696830464ac^{12}-1571292441388187648a^{2}c^{12}}{Γ\left(\frac{c+a+41}{2}\right)Γ\left(\frac{c-a}{2}\right) }$

+$\frac{172459215396995072a^{3}c^{12}+1763389309566976a^{4}c^{12}-37643413749760a^{5}c^{12}-468737884160a^{6}c^{12}}{Γ\left(\frac{c+a+41}{2}\right)Γ\left(\frac{c-a}{2}\right) }$

+$\frac{29818880a^{7}c^{12}+7454720a^{8}c^{12}+69156318341182259200c^{13}-3935423552794132480ac^{13}}{Γ\left(\frac{c+a+41}{2}\right)Γ\left(\frac{c-a}{2}\right) }$

+$\frac{-73410966430154752a^{2}c^{13}+2772134366281728a^{3}c^{13}+41276683714560a^{4}c^{13}-223692062720a^{5}c^{13}}{Γ\left(\frac{c+a+41}{2}\right)Γ\left(\frac{c-a}{2}\right) }$

+$\frac{-3683778560a^{6}c^{13}-4587520a^{7}c^{13}+1954648912706928640c^{14}-73191898739638272ac^{14}}{Γ\left(\frac{c+a+41}{2}\right)Γ\left(\frac{c-a}{2}\right) }$

+$\frac{-1872627520765952a^{2}c^{14}+28129133199360a^{3}c^{14}+525404078080a^{4}c^{14}-33423360a^{5}c^{14}-11141120a^{6}c^{14}}{Γ\left(\frac{c+a+41}{2}\right)Γ\left(\frac{c-a}{2}\right) }$

+$\frac{43162334574673920c^{15}-971633225367552ac^{15}-30480918904832a^{2}c^{15}+144865755136a^{3}c^{15}}{Γ\left(\frac{c+a+41}{2}\right)Γ\left(\frac{c-a}{2}\right) }$

+$\frac{3574071296a^{4}c^{15}+4456448a^{5}c^{15}+728267259641856c^{16}-8438736617472ac^{16}-315245789184a^{2}c^{16}}{Γ\left(\frac{c+a+41}{2}\right)Γ\left(\frac{c-a}{2}\right) }$

+$\frac{20054016a^{3}c^{16}+10027008a^{4}c^{16}+9060954931200c^{17}-38346424320ac^{17}-1889796096a^{2}c^{17}-2359296a^{3}c^{17}}{Γ\left(\frac{c+a+41}{2}\right)Γ\left(\frac{c-a}{2}\right) }$

+$\frac{78297169920c^{18}-4980736ac^{18}-4980736a^{2}c^{18}+419430400c^{19}+524288ac^{19}+1048576c^{20}}{Γ\left(\frac{c+a+41}{2}\right)Γ\left(\frac{c-a}{2}\right) }$ ] (5)

**Derivations of main result:**

Putting b= -a-40 , z=$ \frac{1}{2}$ in known result (2) , we get

(2a+40) 2F1(a, -a-40 ; c; $\frac{1}{2}$) = a 2F1(a+1,-a-40; c; $\frac{1}{2}$) + (a+40) 2F1(a,-a-39; c; $\frac{1}{2}$)

Now involving the same parallel method of Ref [5], the main result is derived.

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