

A Summation Formula of Half Argument Influence with Recurrence Relation

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Research Article

Abstract: The main aim of the present paper is to evaluate a summation formula of half argument associated with contiguous relation and recurrence relation of gamma function.

Key words: Gaussian Hypergeometric function, Recurrence relation, Bailey summation theorem, Contiguous relation.

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Introduction:

Generalized Gaussian Hypergeometric function of one variable is defined by

$${}_A F_B(a_1, a_2, \dots, a_A; b_1, b_2, \dots, b_B; z) = \sum_{k=0}^{\infty} \frac{(a_1)_k (a_2)_k \dots (a_A)_k z^k}{(b_1)_k (b_2)_k \dots (b_B)_k k!} \dots (1)$$

where the parameters b_1, b_2, \dots, b_B are neither zero nor negative integers and A, B are non negative integers.

Contiguous Relation is defined by

[Andrews p.363(9.16)]

$$(a-b) {}_2F_1(a, b; c; z) = a {}_2F_1(a+1, b; c; z) - b {}_2F_1(a, b+1; c; z) \dots (2)$$

Recurrence relation is defined by

$$\Gamma(\xi+1) = \xi \Gamma(\xi) \dots (3)$$

Bailey summation theorem [Prud, p.491(7.3.7.3)]

$${}_2F_1(a, 1-a; c; \frac{1}{2}) = \frac{\Gamma(\frac{c}{2}) \Gamma(\frac{c+1}{2})}{\Gamma(\frac{c+a}{2}) \Gamma(\frac{c+1-a}{2})} = \frac{\sqrt{\pi} \Gamma(c)}{2^{c-1} \Gamma(\frac{c+a}{2}) \Gamma(\frac{c+1-a}{2})} \dots (4)$$

Main Summation Formula:

$$\begin{aligned} {}_2F_1(a, -a-40; c; \frac{1}{2}) &= \frac{\sqrt{\pi} \Gamma(c)}{2^{c+40}} \left[\frac{-66875101180623272603049984000 a^7 1357280184447789909895884800 a^2}{\Gamma(\frac{c+a+40}{2}) \Gamma(\frac{c-a+1}{2})} \right. \\ &+ \frac{-25714790950373338695805785600 a^3 + 3649155977880410544364179456 a^4}{\Gamma(\frac{c+a+40}{2}) \Gamma(\frac{c-a+1}{2})} \\ &+ \frac{-104982886696471128317302560 a^5 - 17081120544451777974736080 a^6 + 731764160260783185841360 a^7}{\Gamma(\frac{c+a+40}{2}) \Gamma(\frac{c-a+1}{2})} \\ &+ \frac{49058956189087369506136 a^8 - 1127898659648383594530 a^9 - 89322308991894454125 a^{10}}{\Gamma(\frac{c+a+40}{2}) \Gamma(\frac{c-a+1}{2})} \\ &+ \frac{-267351987755153730 a^{11} + 63311743532688261 a^{12} + 1230183880910140 a^{13} - 3243688031810 a^{14}}{\Gamma(\frac{c+a+40}{2}) \Gamma(\frac{c-a+1}{2})} \\ &+ \frac{-341178590820 a^{15} - 3651511854 a^{16} - 5077050 a^{17} + 139215 a^{18} + 790 a^{19} + a^{20}}{\Gamma(\frac{c+a+40}{2}) \Gamma(\frac{c-a+1}{2})} \\ &+ \frac{66875101180744917703458816000 c - 189985004451228730616119296000 ac}{\Gamma(\frac{c+a+40}{2}) \Gamma(\frac{c-a+1}{2})} \\ &+ \frac{127461722050600090647554949120 a^2 c - 32698614748137402381734117376 a^3 c}{\Gamma(\frac{c+a+40}{2}) \Gamma(\frac{c-a+1}{2})} \\ &+ \left. \frac{3112488390013770587916985344 a^4 c + 17435208969056442656858112 a^5 c}{\Gamma(\frac{c+a+40}{2}) \Gamma(\frac{c-a+1}{2})} \right] \end{aligned}$$

$$\begin{aligned}
 & \frac{-15453151414910521210524992a^6c+97642175088798664593728a^7c+36361145615195178372192a^8c}{\Gamma\left(\frac{c+a+40}{2}\right)\Gamma\left(\frac{c-a+1}{2}\right)} \\
 & + \frac{207062572140949520736a^9c-39702606806587503636a^{10}c-801954460495433388a^{11}c}{\Gamma\left(\frac{c+a+40}{2}\right)\Gamma\left(\frac{c-a+1}{2}\right)} \\
 & + \frac{9766776111021904a^{12}c+461063762814512a^{13}c+3875318999688a^{14}c-28428184584a^{15}c}{\Gamma\left(\frac{c+a+40}{2}\right)\Gamma\left(\frac{c-a+1}{2}\right)} \\
 & + \frac{-645434640a^{16}c-3251760a^{17}c-980a^{18}c+20a^{19}c+118627724267212505853041049600c^2}{\Gamma\left(\frac{c+a+40}{2}\right)\Gamma\left(\frac{c-a+1}{2}\right)} \\
 & + \frac{-193639952718842374558148198400ac^2+93016963884979881869293559808a^2c^2}{\Gamma\left(\frac{c+a+40}{2}\right)\Gamma\left(\frac{c-a+1}{2}\right)} \\
 & + \frac{-17422819402724518077246311424a^3c^2+1031610380484178452170147328a^4c^2}{\Gamma\left(\frac{c+a+40}{2}\right)\Gamma\left(\frac{c-a+1}{2}\right)} \\
 & + \frac{46196414077865484231694272a^5c^2-4959237589489857015278272a^6c^2-100719136266782788084512a^7c^2}{\Gamma\left(\frac{c+a+40}{2}\right)\Gamma\left(\frac{c-a+1}{2}\right)} \\
 & + \frac{8587061429047147285824a^8c^2+226898390603909225916a^9c^2-4552047674626168860a^{10}c^2}{\Gamma\left(\frac{c+a+40}{2}\right)\Gamma\left(\frac{c-a+1}{2}\right)} \\
 & + \frac{-208525479763087344a^{11}c^2-1124881537937872a^{12}c^2+42278077673832a^{13}c^2+684327983064a^{14}c^2}{\Gamma\left(\frac{c+a+40}{2}\right)\Gamma\left(\frac{c-a+1}{2}\right)} \\
 & + \frac{2194766640a^{15}c^2-21386640a^{16}c^2-156420a^{17}c^2-220a^{18}c^2+91893021619284232336740188160c^3}{\Gamma\left(\frac{c+a+40}{2}\right)\Gamma\left(\frac{c-a+1}{2}\right)} \\
 & + \frac{-105894362587221112530214453248ac^3+38184824036314771057292378112a^2c^3}{\Gamma\left(\frac{c+a+40}{2}\right)\Gamma\left(\frac{c-a+1}{2}\right)} \\
 & + \frac{-5217954699738689318667147264a^3c^3+154802720535224701053833216a^4c^3}{\Gamma\left(\frac{c+a+40}{2}\right)\Gamma\left(\frac{c-a+1}{2}\right)} \\
 & + \frac{18035890704304867606142336a^5c^3-720787628093566157429760a^6c^3-3777743825222068429120a^7c^3}{\Gamma\left(\frac{c+a+40}{2}\right)\Gamma\left(\frac{c-a+1}{2}\right)} \\
 & + \frac{748744915379634640896a^8c^3+46358887174170154584a^9c^3+116326659272412160a^{10}c^3}{\Gamma\left(\frac{c+a+40}{2}\right)\Gamma\left(\frac{c-a+1}{2}\right)} \\
 & + \frac{-19498313510630240a^{11}c^3-283614779255808a^{12}c^3+424869773328a^{13}c^3+34884157440a^{14}c^3}{\Gamma\left(\frac{c+a+40}{2}\right)\Gamma\left(\frac{c-a+1}{2}\right)} \\
 & + \frac{223718880a^{15}c^3+168960a^{16}c^3-1320a^{17}c^3+41926857473108338574031323136c^4}{\Gamma\left(\frac{c+a+40}{2}\right)\Gamma\left(\frac{c-a+1}{2}\right)} \\
 & + \frac{-36530194263573519025608425472ac^4+10073161895102447793343156224a^2c^4}{\Gamma\left(\frac{c+a+40}{2}\right)\Gamma\left(\frac{c-a+1}{2}\right)} \\
 & + \frac{-982994215510126414330402816a^3c^4+3525971281070213051043584a^4c^4}{\Gamma\left(\frac{c+a+40}{2}\right)\Gamma\left(\frac{c-a+1}{2}\right)} \\
 & + \frac{3495316104378035584424960a^5c^4-34614294900669968848000a^6c^4-5838058520962342170496a^7c^4}{\Gamma\left(\frac{c+a+40}{2}\right)\Gamma\left(\frac{c-a+1}{2}\right)} \\
 & + \frac{-14056364908366038544a^8c^4+4211580331661514240a^9c^4+59896230910401600a^{10}c^4}{\Gamma\left(\frac{c+a+40}{2}\right)\Gamma\left(\frac{c-a+1}{2}\right)} \\
 & + \frac{-632023790990592a^{11}c^4-18889044222048a^{12}c^4-97887229440a^{13}c^4+486678720a^{14}c^4+5005440a^{15}c^4}{\Gamma\left(\frac{c+a+40}{2}\right)\Gamma\left(\frac{c-a+1}{2}\right)} \\
 & + \frac{7920a^{16}c^4+12760684127037351373307904000c^5-8660513420106141663756288000ac^5}{\Gamma\left(\frac{c+a+40}{2}\right)\Gamma\left(\frac{c-a+1}{2}\right)} \\
 & + \frac{1833562634531636673901756416a^2c^5-121841366024253671046283264a^3c^5}{\Gamma\left(\frac{c+a+40}{2}\right)\Gamma\left(\frac{c-a+1}{2}\right)} \\
 & + \frac{-2710623730807837944479744a^4c^5+400665605741716065773568a^5c^5+4070405183765017889792a^6c^5}{\Gamma\left(\frac{c+a+40}{2}\right)\Gamma\left(\frac{c-a+1}{2}\right)} \\
 & + \frac{-48355425073296576000a^7c^5-8068276861929453312a^8c^5+175566838422353664a^9c^5}{\Gamma\left(\frac{c+a+40}{2}\right)\Gamma\left(\frac{c-a+1}{2}\right)} \\
 & + \frac{4847389137566976a^{10}c^5+12230678363904a^{11}c^5-506820297984a^{12}c^5-4312019712a^{13}c^5}{\Gamma\left(\frac{c+a+40}{2}\right)\Gamma\left(\frac{c-a+1}{2}\right)} \\
 & + \frac{-5246208a^{14}c^5+25344a^{15}c^5+2773341755995029808758128640c^6-1485763907930506933037694976ac^6}{\Gamma\left(\frac{c+a+40}{2}\right)\Gamma\left(\frac{c-a+1}{2}\right)}
 \end{aligned}$$

$$\begin{aligned}
 & \frac{240108485818220184763269120a^2c^6 - 9852957179482591512805376a^3c^6 - 534557977754408653846528a^4c^6}{\Gamma\left(\frac{c+a+40}{2}\right)\Gamma\left(\frac{c-a+1}{2}\right)} \\
 & + \frac{28149670762417281645568a^5c^6 + 807433260599691934208a^6c^6 - 21267233510805970688a^7c^6}{\Gamma\left(\frac{c+a+40}{2}\right)\Gamma\left(\frac{c-a+1}{2}\right)} \\
 & + \frac{-704829460473991424a^8c^6 + 979730718430464a^9c^6 + 178653618054912a^{10}c^6 + 1396938326784a^{11}c^6}{\Gamma\left(\frac{c+a+40}{2}\right)\Gamma\left(\frac{c-a+1}{2}\right)} \\
 & + \frac{-3506350848a^{12}c^6 - 60732672a^{13}c^6 - 109824a^{14}c^6 + 448917741602817129568010240c^7}{\Gamma\left(\frac{c+a+40}{2}\right)\Gamma\left(\frac{c-a+1}{2}\right)} \\
 & + \frac{-190651892544240900597874688ac^7 + 23175844810777525124071424a^2c^7}{\Gamma\left(\frac{c+a+40}{2}\right)\Gamma\left(\frac{c-a+1}{2}\right)} \\
 & + \frac{-459438476789254584680448a^3c^7 - 54629207556008653860864a^4c^7 + 1106835970971998046208a^5c^7}{\Gamma\left(\frac{c+a+40}{2}\right)\Gamma\left(\frac{c-a+1}{2}\right)} \\
 & + \frac{64294963263526927360a^6c^7 - 284458551615546880a^7c^7 - 30896984328477696a^8c^7 - 210676113483264a^9c^7}{\Gamma\left(\frac{c+a+40}{2}\right)\Gamma\left(\frac{c-a+1}{2}\right)} \\
 & + \frac{2959838069760a^{10}c^7 + 36144176640a^{11}c^7 + 62819328a^{12}c^7 - 219648a^{13}c^7 + 55635447236064914194825216c^8}{\Gamma\left(\frac{c+a+40}{2}\right)\Gamma\left(\frac{c-a+1}{2}\right)} \\
 & + \frac{-18698302219224664564629504ac^8 + 1667782924502252287827968a^2c^8 - 2661783539860427913216a^3c^8}{\Gamma\left(\frac{c+a+40}{2}\right)\Gamma\left(\frac{c-a+1}{2}\right)} \\
 & + \frac{-3587755914140667563008a^4c^8 + 9036117795554403840a^5c^8 + 2978068356617319680a^6c^8}{\Gamma\left(\frac{c+a+40}{2}\right)\Gamma\left(\frac{c-a+1}{2}\right)} \\
 & + \frac{18798345017227776a^7c^8 - 729494915525376a^8c^8 - 8879247336960a^9c^8 + 6645999360a^{10}c^8 + 364396032a^{11}c^8}{\Gamma\left(\frac{c+a+40}{2}\right)\Gamma\left(\frac{c-a+1}{2}\right)} \\
 & + \frac{768768a^{12}c^8 + 5376995734268194219622400c^9 - 1420850828358443319951360ac^9}{\Gamma\left(\frac{c+a+40}{2}\right)\Gamma\left(\frac{c-a+1}{2}\right)} \\
 & + \frac{89401407848249932644352a^2c^9 + 1332525643898270728192a^3c^9 - 159187142187653099520a^4c^9}{\Gamma\left(\frac{c+a+40}{2}\right)\Gamma\left(\frac{c-a+1}{2}\right)} \\
 & + \frac{-1515726793160785920a^5c^9 + 83805625001978880a^6c^9 + 1141818425459712a^7c^9 - 7537462732800a^8c^9}{\Gamma\left(\frac{c+a+40}{2}\right)\Gamma\left(\frac{c-a+1}{2}\right)} \\
 & + \frac{-156551915520a^9c^9 - 374133760a^{10}c^9 + 1025024a^{11}c^9 + 410085684500756095303680c^{10}}{\Gamma\left(\frac{c+a+40}{2}\right)\Gamma\left(\frac{c-a+1}{2}\right)} \\
 & + \frac{-84274841045028033134592ac^{10} + 3508253808010892328960a^2c^{10} + 110678463999328276480a^3c^{10}}{\Gamma\left(\frac{c+a+40}{2}\right)\Gamma\left(\frac{c-a+1}{2}\right)} \\
 & + \frac{-4732151243543060480a^4c^{10} - 92483291170350080a^5c^{10} + 1309095400797184a^6c^{10} + 28730141440000a^7c^{10}}{\Gamma\left(\frac{c+a+40}{2}\right)\Gamma\left(\frac{c-a+1}{2}\right)} \\
 & + \frac{25820354560a^8c^{10} - 1214653440a^9c^{10} - 3075072a^{10}c^{10} + 24846733566518671441920c^{11}}{\Gamma\left(\frac{c+a+40}{2}\right)\Gamma\left(\frac{c-a+1}{2}\right)} \\
 & + \frac{-3909374352436847443968ac^{11} + 95793392223282364416a^2c^{11} + 5055276670898741248a^3c^{11}}{\Gamma\left(\frac{c+a+40}{2}\right)\Gamma\left(\frac{c-a+1}{2}\right)} \\
 & + \frac{-87138747741380608a^4c^{11} - 2773143309920256a^5c^{11} + 5939823984640a^6c^{11} + 376457768960a^7c^{11}}{\Gamma\left(\frac{c+a+40}{2}\right)\Gamma\left(\frac{c-a+1}{2}\right)} \\
 & + \frac{1241210880a^8c^{11} - 2795520a^9c^{11} + 1198625180760970100736c^{12} - 141347017399724670976ac^{12}}{\Gamma\left(\frac{c+a+40}{2}\right)\Gamma\left(\frac{c-a+1}{2}\right)} \\
 & + \frac{1545233538725773312a^2c^{12} + 152069777125081088a^3c^{12} - 688545627521024a^4c^{12} - 49137876336640a^5c^{12}}{\Gamma\left(\frac{c+a+40}{2}\right)\Gamma\left(\frac{c-a+1}{2}\right)} \\
 & + \frac{-143115714560a^6c^{12} + 2355691520a^7c^{12} + 7454720a^8c^{12} + 45936354549510963200c^{13}}{\Gamma\left(\frac{c+a+40}{2}\right)\Gamma\left(\frac{c-a+1}{2}\right)} \\
 & + \frac{-3945389459851182080ac^{13} + 1759899720613888a^2c^{13} + 3107013266767872a^3c^{13} + 7880455618560a^4c^{13}}{\Gamma\left(\frac{c+a+40}{2}\right)\Gamma\left(\frac{c-a+1}{2}\right)} \\
 & + \frac{-506274119680a^5c^{13} - 2399272960a^6c^{13} + 4587520a^7c^{13} + 1388679268736696320c^{14}}{\Gamma\left(\frac{c+a+40}{2}\right)\Gamma\left(\frac{c-a+1}{2}\right)} \\
 & + \frac{-83551983410085888ac^{14} - 610672412721152a^2c^{14} + 42209659453440a^3c^{14} + 264701870080a^4c^{14}}{\Gamma\left(\frac{c+a+40}{2}\right)\Gamma\left(\frac{c-a+1}{2}\right)} \\
 & + \frac{-2640445440a^5c^{14} - 11141120a^6c^{14} + 32680067255500800c^{15} - 1304276194295808ac^{15}}{\Gamma\left(\frac{c+a+40}{2}\right)\Gamma\left(\frac{c-a+1}{2}\right)}
 \end{aligned}$$

$$\begin{aligned}
 & \frac{-16405851799552a^2c^{15} + 355682484224a^3c^{15} + 2682781696a^4c^{15} - 4456448a^5c^{15} + 585809145102336c^{16}}{\Gamma\left(\frac{c+a+40}{2}\right)\Gamma\left(\frac{c-a+1}{2}\right)} \\
 & + \frac{-14310271746048ac^{16} - 221392994304a^2c^{16} + 1584267264a^3c^{16} + 10027008a^4c^{16} + 7722133094400c^{17}}{\Gamma\left(\frac{c+a+40}{2}\right)\Gamma\left(\frac{c-a+1}{2}\right)} \\
 & + \frac{-101512642560ac^{17} - 1606680576a^2c^{17} + 2359296a^3c^{17} + 70527221760c^{18} - 393478144ac^{18} - 4980736a^2c^{18}}{\Gamma\left(\frac{c+a+40}{2}\right)\Gamma\left(\frac{c-a+1}{2}\right)} \\
 & + \frac{398458880c^{19} - 524288ac^{19} + 1048576c^{20}}{\Gamma\left(\frac{c+a+40}{2}\right)\Gamma\left(\frac{c-a+1}{2}\right)} + \frac{335367096786357081410764800000}{\Gamma\left(\frac{c+a+41}{2}\right)\Gamma\left(\frac{c-a}{2}\right)} \\
 & + \frac{-586058735458048022906351616000a + 312146142012827622963103334400a^2}{\Gamma\left(\frac{c+a+41}{2}\right)\Gamma\left(\frac{c-a}{2}\right)} \\
 & + \frac{-66199862346792081420924541440a^3 + 4548866988247932940955571456a^4}{\Gamma\left(\frac{c+a+41}{2}\right)\Gamma\left(\frac{c-a}{2}\right)} \\
 & + \frac{227018175022003605939171360a^5 - 29944950177747597574144080a^6 - 659105654914824606149840a^7}{\Gamma\left(\frac{c+a+41}{2}\right)\Gamma\left(\frac{c-a}{2}\right)} \\
 & + \frac{75791087035872782536936a^8 + 2276608375387101064530a^9 - 66184234273950611325a^{10}}{\Gamma\left(\frac{c+a+41}{2}\right)\Gamma\left(\frac{c-a}{2}\right)} \\
 & + \frac{-3525199213630512990a^{11} - 20282520330520539a^{12} + 1382803279676260a^{13} + 29983302492190a^{14}}{\Gamma\left(\frac{c+a+41}{2}\right)\Gamma\left(\frac{c-a}{2}\right)} \\
 & + \frac{128128444260a^{15} - 2708725854a^{16} - 37872150a^{17} - 157185a^{18} + 10a^{19} + a^{20}}{\Gamma\left(\frac{c+a+41}{2}\right)\Gamma\left(\frac{c-a}{2}\right)} \\
 & + \frac{831600805538859803215921152000c - 1052987018520593812723767705600ac}{\Gamma\left(\frac{c+a+41}{2}\right)\Gamma\left(\frac{c-a}{2}\right)} \\
 & + \frac{426733904516661688776588656640a^2c - 66770364740738418141395779584a^3c}{\Gamma\left(\frac{c+a+41}{2}\right)\Gamma\left(\frac{c-a}{2}\right)} \\
 & + \frac{2348085345321128686474834944a^4c + 310009236027494578843051008a^5c}{\Gamma\left(\frac{c+a+41}{2}\right)\Gamma\left(\frac{c-a}{2}\right)} \\
 & + \frac{-15389298678996674662503232a^6c - 921268164207427190765888a^7c + 24753995876431093595232a^8c}{\Gamma\left(\frac{c+a+41}{2}\right)\Gamma\left(\frac{c-a}{2}\right)} \\
 & + \frac{1762507274982653088864a^9c + 2990096423654681484a^{10}c - 1301859240291902292a^{11}c}{\Gamma\left(\frac{c+a+41}{2}\right)\Gamma\left(\frac{c-a}{2}\right)} \\
 & + \frac{-24267427207871536a^{12}c + 82526341522768a^{13}c + 7034302070088a^{14}c + 73302270984a^{15}c}{\Gamma\left(\frac{c+a+41}{2}\right)\Gamma\left(\frac{c-a}{2}\right)} \\
 & + \frac{85538160a^{16}c - 2925840a^{17}c - 16180a^{18}c - 20a^{19}c + 826273441765035683802670694400c^2}{\Gamma\left(\frac{c+a+41}{2}\right)\Gamma\left(\frac{c-a}{2}\right)} \\
 & + \frac{-803752902887255179892410613760ac^2 + 252024396308373373971963199488a^2c^2}{\Gamma\left(\frac{c+a+41}{2}\right)\Gamma\left(\frac{c-a}{2}\right)} \\
 & + \frac{-28489035748750676632029828096a^3c^2 + 143532631387480624722044928a^4c^2}{\Gamma\left(\frac{c+a+41}{2}\right)\Gamma\left(\frac{c-a}{2}\right)} \\
 & + \frac{140142407875774280669379648a^5c^2 - 1865560153509788547915712a^6c^2 - 345420571903967656413408a^7c^2}{\Gamma\left(\frac{c+a+41}{2}\right)\Gamma\left(\frac{c-a}{2}\right)} \\
 & + \frac{-551239531237627323936a^8c^2 + 406065609377486142084a^9c^2 + 7101041412627891300a^{10}c^2}{\Gamma\left(\frac{c+a+41}{2}\right)\Gamma\left(\frac{c-a}{2}\right)} \\
 & + \frac{-118726440826380816a^{11}c^2 - 4640236215012112a^{12}c^2 - 34256508277992a^{13}c^2 + 345239519064a^{14}c^2}{\Gamma\left(\frac{c+a+41}{2}\right)\Gamma\left(\frac{c-a}{2}\right)} \\
 & + \frac{6665495760a^{15}c^2 + 31122960a^{16}c^2 - 1980a^{17}c^2 - 220a^{18}c^2 + 462007088894186241001814753280c^3}{\Gamma\left(\frac{c+a+41}{2}\right)\Gamma\left(\frac{c-a}{2}\right)} \\
 & + \frac{-354582225664640572957069934592ac^3 + 86346699390299085548205883392a^2c^3}{\Gamma\left(\frac{c+a+41}{2}\right)\Gamma\left(\frac{c-a}{2}\right)} \\
 & + \frac{-6732765627651442139291314176a^3c^3 - 170892347817276835698473984a^4c^3}{\Gamma\left(\frac{c+a+41}{2}\right)\Gamma\left(\frac{c-a}{2}\right)} \\
 & + \frac{31556442577814191193371264a^5c^3 + 354192263042024047280640a^6c^3 - 58575583216943264122560a^7c^3}{\Gamma\left(\frac{c+a+41}{2}\right)\Gamma\left(\frac{c-a}{2}\right)} \\
 & + \frac{-1173534835070915089344a^8c^3 + 37107444878219989416a^9c^3 + 1314641080731080960a^{10}c^3}{\Gamma\left(\frac{c+a+41}{2}\right)\Gamma\left(\frac{c-a}{2}\right)}
 \end{aligned}$$

$$\begin{aligned}
 & \frac{3724085398748000a^{11}c^3 - 316469507722368a^{12}c^3 - 4376491046928a^{13}c^3 - 9460450560a^{14}c^3 + 171647520a^{15}c^3}{\Gamma\left(\frac{c+a+41}{2}\right)\Gamma\left(\frac{c-a}{2}\right)} \\
 & + \frac{1066560a^{16}c^3 + 1320a^{17}c^3 + 167709453188902576587109564416c^4 - 102825071534689323090904055808ac^4}{\Gamma\left(\frac{c+a+41}{2}\right)\Gamma\left(\frac{c-a}{2}\right)} \\
 & + \frac{19343633758355617130561900544a^2c^4 - 946172152672754707373983744a^3c^4}{\Gamma\left(\frac{c+a+41}{2}\right)\Gamma\left(\frac{c-a}{2}\right)} \\
 & + \frac{-60290005312197029135708416a^4c^4 + 3984033685438780048624640a^5c^4}{\Gamma\left(\frac{c+a+41}{2}\right)\Gamma\left(\frac{c-a}{2}\right)} \\
 & + \frac{136673012945841535479680a^6c^4 - 4884451546970599187584a^7c^4 - 202269720518538044944a^8c^4}{\Gamma\left(\frac{c+a+41}{2}\right)\Gamma\left(\frac{c-a}{2}\right)} \\
 & + \frac{582289238867381760a^9c^4 + 102748140882953280a^{10}c^4 + 1170749667639552a^{11}c^4 - 5984812081248a^{12}c^4}{\Gamma\left(\frac{c+a+41}{2}\right)\Gamma\left(\frac{c-a}{2}\right)} \\
 & + \frac{-186633807360a^{13}c^4 - 995945280a^{14}c^4 + 63360a^{15}c^4 + 7920a^{16}c^4 + 42736925839918853495822745600c^5}{\Gamma\left(\frac{c+a+41}{2}\right)\Gamma\left(\frac{c-a}{2}\right)} \\
 & + \frac{-21026745656045496348463595520ac^5 + 3014771937094573629763485696a^2c^5}{\Gamma\left(\frac{c+a+41}{2}\right)\Gamma\left(\frac{c-a}{2}\right)} \\
 & + \frac{-7296475500552301942603776a^3c^5 - 10309096016026560048185344a^4c^5 + 269991041072289608128512a^5c^5}{\Gamma\left(\frac{c+a+41}{2}\right)\Gamma\left(\frac{c-a}{2}\right)} \\
 & + \frac{19154701981218139528192a^6c^5 - 129709603231218250240a^7c^5 - 16633374581983940352a^8c^5}{\Gamma\left(\frac{c+a+41}{2}\right)\Gamma\left(\frac{c-a}{2}\right)} \\
 & + \frac{-151281781107435264a^9c^5 + 3635690749506816a^{10}c^5 + 71857903493376a^{11}c^5 + 233564787456a^{12}c^5}{\Gamma\left(\frac{c+a+41}{2}\right)\Gamma\left(\frac{c-a}{2}\right)} \\
 & + \frac{-2883648768a^{13}c^5 - 20452608a^{14}c^5 - 25344a^{15}c^5 + 8024013991163045633253703680c^6}{\Gamma\left(\frac{c+a+41}{2}\right)\Gamma\left(\frac{c-a}{2}\right)} \\
 & + \frac{-3165473238864637987565338624ac^6 + 338023411250974242200125440a^2c^6}{\Gamma\left(\frac{c+a+41}{2}\right)\Gamma\left(\frac{c-a}{2}\right)} \\
 & + \frac{-825702749831203882287104a^3c^6 - 1095805112655400995248128a^4c^6 + 4327620278143835825152a^5c^6}{\Gamma\left(\frac{c+a+41}{2}\right)\Gamma\left(\frac{c-a}{2}\right)} \\
 & + \frac{1518645340045653467648a^6c^6 + 11967751984128548608a^7c^6 - 736369410218334464a^8c^6}{\Gamma\left(\frac{c+a+41}{2}\right)\Gamma\left(\frac{c-a}{2}\right)} \\
 & + \frac{-13167640519697664a^9c^6 + 23950343569152a^{10}c^6 + 1940990827776a^{11}c^6 + 12084264192a^{12}c^6}{\Gamma\left(\frac{c+a+41}{2}\right)\Gamma\left(\frac{c-a}{2}\right)} \\
 & + \frac{-768768a^{13}c^6 - 109824a^{14}c^6 + 1146015991167187352194908160c^7 - 360711651690653897029844992ac^7}{\Gamma\left(\frac{c+a+41}{2}\right)\Gamma\left(\frac{c-a}{2}\right)} \\
 & + \frac{27652829243691726536310784a^2c^7 + 494928666960181650014208a^3c^7 - 77565160960545084862464a^4c^7}{\Gamma\left(\frac{c+a+41}{2}\right)\Gamma\left(\frac{c-a}{2}\right)} \\
 & + \frac{-912683876126578935808a^5c^7 + 73131042518682536960a^6c^7 + 1367869214613890560a^7c^7}{\Gamma\left(\frac{c+a+41}{2}\right)\Gamma\left(\frac{c-a}{2}\right)} \\
 & + \frac{-15075948986182656a^8c^7 - 499536365492736a^9c^7 - 2289441623040a^{10}c^7 + 21421171200a^{11}c^7}{\Gamma\left(\frac{c+a+41}{2}\right)\Gamma\left(\frac{c-a}{2}\right)} \\
 & + \frac{177036288a^{12}c^7 + 219648a^{13}c^7 + 12723880346852105074098176c^8 - 31674770312859992507744256ac^8}{\Gamma\left(\frac{c+a+41}{2}\right)\Gamma\left(\frac{c-a}{2}\right)} \\
 & + \frac{1641293325061130488537088a^2c^8 + 64416791110452922791936a^3c^8 - 3682862114057569451008a^4c^8}{\Gamma\left(\frac{c+a+41}{2}\right)\Gamma\left(\frac{c-a}{2}\right)} \\
 & + \frac{-94654416978676938240a^5c^8 + 2011302801680057600a^6c^8 + 64755135689147904a^7c^8}{\Gamma\left(\frac{c+a+41}{2}\right)\Gamma\left(\frac{c-a}{2}\right)} \\
 & + \frac{71706791380224a^8c^8 - 9704950295040a^9c^8 - 72506353920a^{10}c^8 + 4612608a^{11}c^8 + 768768a^{12}c^8}{\Gamma\left(\frac{c+a+41}{2}\right)\Gamma\left(\frac{c-a}{2}\right)} \\
 & + \frac{11143915551184914481152000c^9 - 2166501342863241856942080ac^9 + 67945566007433385869312a^2c^9}{\Gamma\left(\frac{c+a+41}{2}\right)\Gamma\left(\frac{c-a}{2}\right)} \\
 & + \frac{4590308763633539268608a^3c^9 - 110433016064669675520a^4c^9 - 4847055932530606080a^5c^9}{\Gamma\left(\frac{c+a+41}{2}\right)\Gamma\left(\frac{c-a}{2}\right)} \\
 & + \frac{18414268492139520a^6c^9 + 1724296281996288a^7c^9 + 11059342694400a^8c^9 - 83303700480a^9c^9}{\Gamma\left(\frac{c+a+41}{2}\right)\Gamma\left(\frac{c-a}{2}\right)}
 \end{aligned}$$

$$\begin{aligned}
 & \frac{-825144320a^{10}c^9 - 1025024a^{11}c^9 + 777139863716630479503360c^{10} - 115983160550649716932608ac^{10}}{\Gamma\left(\frac{c+a+41}{2}\right)\Gamma\left(\frac{c-a}{2}\right)} \\
 & + \frac{1694186473196915343360a^2c^{10} + 218855822547202846720a^3c^{10} - 1518822066052628480a^4c^{10}}{\Gamma\left(\frac{c+a+41}{2}\right)\Gamma\left(\frac{c-a}{2}\right)} \\
 & + \frac{-153679418618485760a^5c^{10} - 701774131874816a^6c^{10} + 25879857203200a^7c^{10} + 241690408960a^8c^{10}}{\Gamma\left(\frac{c+a+41}{2}\right)\Gamma\left(\frac{c-a}{2}\right)} \\
 & + \frac{-15375360a^9c^{10} - 3075072a^{10}c^{10} + 43366015594351810314240c^{11} - 4856370236805384241152ac^{11}}{\Gamma\left(\frac{c+a+41}{2}\right)\Gamma\left(\frac{c-a}{2}\right)} \\
 & + \frac{4835027675975417856a^2c^{11} + 7324612714279657472a^3c^{11} + 25274194347630592a^4c^{11}}{\Gamma\left(\frac{c+a+41}{2}\right)\Gamma\left(\frac{c-a}{2}\right)} \\
 & + \frac{-3100057245702144a^5c^{11} - 28833388380160a^6c^{11} + 181751664640a^7c^{11} + 2247598080a^8c^{11} + 2795520a^9c^{11}}{\Gamma\left(\frac{c+a+41}{2}\right)\Gamma\left(\frac{c-a}{2}\right)} \\
 & + \frac{1938152320641559166976c^{12} - 157973724927696830464ac^{12} - 1571292441388187648a^2c^{12}}{\Gamma\left(\frac{c+a+41}{2}\right)\Gamma\left(\frac{c-a}{2}\right)} \\
 & + \frac{172459215396995072a^3c^{12} + 1763389309566976a^4c^{12} - 37643413749760a^5c^{12} - 468737884160a^6c^{12}}{\Gamma\left(\frac{c+a+41}{2}\right)\Gamma\left(\frac{c-a}{2}\right)} \\
 & + \frac{29818880a^7c^{12} + 7454720a^8c^{12} + 69156318341182259200c^{13} - 3935423552794132480ac^{13}}{\Gamma\left(\frac{c+a+41}{2}\right)\Gamma\left(\frac{c-a}{2}\right)} \\
 & + \frac{-73410966430154752a^2c^{13} + 2772134366281728a^3c^{13} + 41276683714560a^4c^{13} - 223692062720a^5c^{13}}{\Gamma\left(\frac{c+a+41}{2}\right)\Gamma\left(\frac{c-a}{2}\right)} \\
 & + \frac{-3683778560a^6c^{13} - 4587520a^7c^{13} + 1954648912706928640c^{14} - 73191898739638272ac^{14}}{\Gamma\left(\frac{c+a+41}{2}\right)\Gamma\left(\frac{c-a}{2}\right)} \\
 & + \frac{-1872627520765952a^2c^{14} + 28129133199360a^3c^{14} + 525404078080a^4c^{14} - 33423360a^5c^{14} - 11141120a^6c^{14}}{\Gamma\left(\frac{c+a+41}{2}\right)\Gamma\left(\frac{c-a}{2}\right)} \\
 & + \frac{43162334574673920c^{15} - 971633225367552ac^{15} - 30480918904832a^2c^{15} + 144865755136a^3c^{15}}{\Gamma\left(\frac{c+a+41}{2}\right)\Gamma\left(\frac{c-a}{2}\right)} \\
 & + \frac{3574071296a^4c^{15} + 4456448a^5c^{15} + 728267259641856c^{16} - 8438736617472ac^{16} - 315245789184a^2c^{16}}{\Gamma\left(\frac{c+a+41}{2}\right)\Gamma\left(\frac{c-a}{2}\right)} \\
 & + \frac{20054016a^3c^{16} + 10027008a^4c^{16} + 9060954931200c^{17} - 38346424320ac^{17} - 1889796096a^2c^{17} - 2359296a^3c^{17}}{\Gamma\left(\frac{c+a+41}{2}\right)\Gamma\left(\frac{c-a}{2}\right)} \\
 & + \frac{78297169920c^{18} - 4980736ac^{18} - 4980736a^2c^{18} + 419430400c^{19} + 524288ac^{19} + 1048576c^{20}}{\Gamma\left(\frac{c+a+41}{2}\right)\Gamma\left(\frac{c-a}{2}\right)}] \quad (5)
 \end{aligned}$$

Derivations of main result:

Putting $b = -a-40$, $z = \frac{1}{2}$ in known result (2), we get

$$\begin{aligned}
 & (2a+40) {}_2F_1\left(a, -a-40; c; \frac{1}{2}\right) = a {}_2F_1\left(a+1, -a-40; c; \frac{1}{2}\right) \\
 & + (a+40) {}_2F_1\left(a, -a-39; c; \frac{1}{2}\right)
 \end{aligned}$$

Now involving the same parallel method of Ref [5], the main result is derived.

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