Research Article

Benefit-Function of Two-Non Identical Cold Standby System subject to failure due to High acoustics and Very low gravity

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Abstract

Reliability is a measure of how well a system performs or meets its design requirements. It is hence the prime concern of all scientists and engineers engaged in developing such a system. In this paper we have taken **FVLG- failure due to very low gravity** and **FHA-failure due to High acoustics** when the main unit fails due to High Acoustics then cold standby system becomes operative. High acoustics cannot occur simultaneously in both the units and after failure the unit undergoes very costly repair facility immediately. Applying the regenerative point technique with renewal process theory the various reliability parameters MTSF, Availability, Busy period, Benefit-Function analysis have been evaluated.
Keywords: Cold Standby, FVLG-failure due to Very low gravity, FHA-failure due to High acoustics, first come first serve, MTSF, Availability, Busy period, Benefit -Function.

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INTRODUCTION

Stochastic behaviour of systems operating under changing environments has widely been studied.. Dhillon, B.S. and Natesan, J. (1983) studied an outdoor power systems in fluctuating environment. Kan Cheng 1985) has studied reliability analysis of a system in a randomly changing environment. Jinhua Cao (1989) has studied a man machine system operating under changing environment subject to a Markov process with two states. The change in operating conditions viz. fluctuations of voltage, corrosive atmosphere, very low gravity etc. may make a system completely inoperative. Severe environmental conditions can make the actual mission duration longer than the ideal mission duration. In this paper we have taken **FVLG - Failure due to very low gravity** and other **FHA-failure due to High acoustics**. When the main operative unit **fails due to High acoustics-FHA** then cold standby system becomes operative. **High acoustics** cannot occur simultaneously in both the units and after failure the unit undergoes repair facility of very high cost in case of **FHA-failure due to High acoustics** immediately. Failure due to **Very low gravity** may be destructive. The repair is done on the basis of first fail first repaired.

Assumptions

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1. $F_1(t)$ and $F_2(t)$ are general failure time distributions due to **High acoustics** and **Very low gravity.** The repair is of two types -Type -I, Type-II with repair time distributions as $G_1(t)$ and $G_2(t)$ respectively.
2. The **High acoustics** are non-instantaneous and it cannot come simultaneously in both the units.
3. Whenever the **acoustics** occur within specified limit of the unit, it works as normal as before. But as soon as there occur **High acoustics** of magnitude beyond specified limit of the unit the operation of the unit stops automatically.
4. The repair starts immediately after the **High acoustics** of beyond specified limit of the unit are over and works on the principle of first fail first repaired basis.
5. The repair facility does no damage to the units and after repair units are as good as new.
6. The switches are perfect and instantaneous.
7. All random variables are mutually independent.
8. When both the units fail, we give priority to operative unit for repair.
9. **FVLG-** Failure due to Very low gravity when it is beyond specified limit.
10. Repairs are perfect and failure of a unit is detected immediately and perfectly.
11. The system is down when both the units are non-operative.

Symbols for states of the System

$F_1(t)$ and $F_2(t)$ are the **failure rates** due to **High acoustics** and **Very low gravity** respectively

$G_1(t)$, $G_2(t)$ – repair time distribution Type -I, Type-II due to **High acoustics** and **Very low gravity** respectively

Superscripts O, CS, FHA, FVLG

Operative, Cold Standby, failure due to High acoustics, Failure due to Very low gravity respectively

Subscripts nhaf, haf, vlg, ur, wr, uR

No High acoustics failure, High acoustics failure, very low gravity, under repair, waiting for repair, under repair continued from previous state respectively

Up states: 0, 1, 2;

Down states: 3, 4

Regeneration point: 0, 1, 2

Notations

$M_i(t)$ System having started from state I is up at time t without visiting any other regenerative state

$A_i(t)$ state is up state as instant t

$R_i(t)$ System having started from state I is busy for repair at time t without visiting any other regenerative state.

$B_i(t)$ The server is busy for repair at time t.

$H_i(t)$ Expected number of visits by the server for repairing given that the system initially starts from regenerative state i

By High acoustics we mean High acoustics beyond the specified limit

States of the System

$0(O_{nhaf}, CS_{nhaf})$

One unit is operative and the other unit is cold standby and there are no High acoustics in both the units.

$1(SOFHA_{haf, ur}, O_{nhaf})$

The operating unit fails due to High acoustics and is under repair immediately of very costly Type- I and standby unit starts operating with no High acoustics.

$2(FVLG_{nhaf,vlg, ur}, O_{nhaf})$

The operative unit fails due to FVLG resulting from Very low gravity and undergoes repair of type II and the standby unit becomes operative with no High acoustics.

$3(FHA_{haf,uR}, FVLG_{nhaf, vlg,wr})$

The first unit fails due to High acoustics and under very costly Type-! Repair is continued from state 1 and the other unit fails due to FVLG resulting from Very low gravity and is waiting for repair of Type -II.

$4(FHA_{haf,uR}, FHA_{haf,wr})$

The one unit fails due to High acoustics is continues under repair of very costly Type - I from state 1 and the other unit also fails due to High acoustics. is waiting for repair of very costly Type- I.

$5(FVLG_{nhaf, vlg, uR}, FHA_{haf, wr})$

The operating unit fails due to Very low gravity (FVLG mode) and under repair of Type - II continues from the state 2 and the other unit fails due to High acoustics is waiting for repair of very costly Type- I.

$6(FVLG_{nhaf,vlg,uR}, FVLG_{nhaf,vlg,wr})$

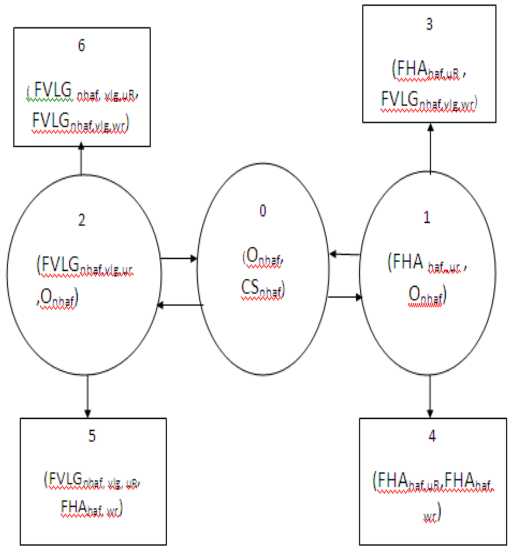

Figure 1: The State Transition Diagram

● Regeneration point -states 0, 1, 2

○ Up State

▭ Down State

The operative unit fails due to FVLG resulting from Very low gravity and under repair continues from state 2 of Type –II and the other unit is also failed due to FHVLG resulting from Very low gravity and is waiting for repair of Type-II and there is no High acoustics.

TRANSITION PROBABILITIES

Simple probabilistic considerations yield the following expressions :

$p_{01} = \int_0^\infty \overline{F}_2(t)dF_1(t)$, $p_{02} = \int_0^\infty \overline{F}_1(t)dF_2(t)$

$p_{10} = \int_0^\infty \overline{F}_2(t)dG_1(t)$, $p_{13} = p_{11}^{(3)} = p_{11}^{(4)} = \int_0^\infty \overline{G}_1(t)dF_2(t)$

$p_{25} = p_{22}^{(5)} = p_{22}^{(6)} = \int_0^\infty \overline{G}_2(t)dF_1(t)$

clearly

$p_{01} + p_{02} = 1,$

$p_{10} + p_{13} = (p_{11}^{(3)}) + p_{14} = (p_{11}^{(4)}) = 1,$

$p_{20} + p_{25} = (p_{22}^{(5)}) + p_{26} = (p_{22}^{(6)})$ (1)

And mean sojourn time are

$\mu_0 = E(T) = \int_0^\infty P[T > t]dt$ (2)

Mean Time To System Failure

$\emptyset_0(t) = Q_{01}(t)[s]\ \emptyset_1(t) + Q_{02}(t)[s]\ \emptyset_2(t)$

$\emptyset_1(t) = Q_{10}(t)[s]\ \emptyset_0(t) + Q_{13}(t) + Q_{14}(t)$

$\emptyset_2(t) = Q_{20}(t)[s]\ \emptyset_0(t) + Q_{25}(t) + Q_{26}(t)$ (3-5)

We can regard the failed state as absorbing

Taking Laplace-Stiljes transform of eq. (3-5) and solving for

$\phi_0^*(s) = N_1(s) / D_1(s)$ (6)

Where

$N_1(s) = Q_{01}^*[\ Q_{13}^*(s) + Q_{14}^*(s)\ ] + Q_{02}^*[\ Q_{25}^*(s) + Q_{26}^*(s)\ ]$

$D_1(s) = 1 - Q_{01}^*Q_{10}^* - Q_{02}^*Q_{20}^*$

Making use of relations (1) and (2) it can be shown that $ø_0^*(0) = 1$, which implies that $ø_0^*(t)$ is a proper distribution.

$$MTSF = E[T] = \left. \frac{d}{ds} ø_0^*(s) \right|_{s=0} = (D_1'(0) - N_1'(0)) / D_1(0)$$

$$= (\mu_0 + p_{01}\mu_1 + p_{02}\mu_2) / (1 - p_{01}p_{10} - p_{02}p_{20})$$

where

$$\mu_0 = \mu_{01} + \mu_{02},\ \mu_1 = \mu_{10} + \mu_{13} + \mu_{14}$$

$$\mu_2 = \mu_{20} + \mu_{25} + \mu_{26}$$

Availability analysis

Let $M_i(t)$ be the probability of the system having started from state I is up at time t without making any other regenerative state belonging to E. By probabilistic arguments, we have

The value of $M_0(t)$, $M_1(t)$, $M_2(t)$ can be found easily.

The point wise availability $A_i(t)$ have the following recursive relations

$A_0(t) = M_0(t) + q_{01}(t)[c]A_1(t) + q_{02}(t)[c]A_2(t)$

$A_1(t) = M_1(t) + q_{10}(t)[c]A_0(t) + q_{11}^{(3)}(t)[c]A_1(t) + q_{11}^{(4)}(t)[c]A_1(t)$,

$A_2(t) = M_2(t) + q_{20}(t)[c]A_0(t) + [q_{22}^{(5)}(t)[c] + q_{22}^{(6)}(t)]\,[c]A_2(t)$ (7-9)

Taking Laplace Transform of eq. (7-9) and solving for $\hat{A}_0(s)$

$\hat{A}_0(s) = N_2(s) / D_2(s)$ (10)

Where

$N_2(s) = \hat{M}_0(s)(1 - \hat{q}_{11}^{(3)}(s) - \hat{q}_{11}^{(4)}(s))\,(1 - \hat{q}_{22}^{(5)}(s) - \hat{q}_{22}^{(6)}(s)) + \bar{q}_{01}(s)\,\hat{M}_1(s)$

$[1 - \hat{q}_{22}^{(5)}(s) - \hat{q}_{22}^{(6)}(s)] + \hat{q}_{02}(s)\,\hat{M}_2(s)(1 - \hat{q}_{11}^{(3)}(s) - \hat{q}_{11}^{(4)}(s))$

$D_2(s) = (1 - \hat{q}_{11}^{(3)}(s) - \hat{q}_{11}^{(4)}(s))\ \{1 - \hat{q}_{22}^{(5)}(s) - \hat{q}_{22}^{(6)}(s)\,)[1-(\hat{q}_{01}(s)\,\hat{q}_{10}(s))(1-$

$\hat{q}_{11}^{(3)}(s) - \hat{q}_{11}^{(4)}(s))]$

The steady state availability

$$A_0 = \lim_{t\to\infty}[A_0(t)] = \lim_{s\to 0}[s\,\hat{A}_0(s)] = \lim_{s\to 0}\frac{s\,N_2(s)}{D_2(s)}$$

Using L' Hospitals rule, we get

$$A_0 = \lim_{s\to 0}\frac{N_2(s) + s\,N_2'(s)}{D_2'(s)} = \frac{N_2(0)}{D_2'(0)} \qquad (11)$$

The expected up time of the system in (0,t] is

$$\lambda_u(t) = \int_0^\infty A_0(z)\,dz \text{ So that } \bar{\lambda}_u(s) = \frac{\hat{A}_0(s)}{s} = \frac{N_2(S)}{SD_2(S)} \qquad (12)$$

The expected down time of the system in (0,t] is

$$\lambda_d(t) = t - \lambda_u(t) \text{ So that } \bar{\lambda}_d(s) = \frac{1}{s^2} - \bar{\lambda}_u(s) \qquad (13)$$

The expected busy period of the server when there is FVLG-failure resulting from Very low gravity in (0,t]

$R_0(t) = q_{01}(t)[c]R_1(t) + q_{02}(t)[c]R_2(t)$

$R_1(t) = S_1(t) + q_{01}(t)[c]R_1(t) + [q_{11}^{(3)}(t) + q_{11}^{(4)}(t)[c]R_1(t)$,

$R_2(t) = q_{20}(t)[c]R_0(t) + [q_{22}^{(6)}(t) + q_{22}^{(5)}(t)][c]R_2(t)$ (14-16)

Taking Laplace Transform of eq. (14-16) and solving for $\bar{R}_0(s)$

$\bar{R}_0(s) = N_3(s) / D_3(s)$ (17)

Where

$N_3(s) = \hat{q}_{01}(s)\,\hat{S}_1(s)$ and

$D_3(s) = (1 - \hat{q}_{11}^{(3)}(s) - \hat{q}_{11}^{(4)}(s)) - \ddot{q}_{01}(s)$ is already defined.

In the long run, $R_0 = \frac{N_3(0)}{D_2'(0)}$ (18)
The expected period of the system under FH-failure resulting from Very low gravity in (0,t] is
$\lambda_{rv}(t) = \int_0^\infty R_0(z)dz$ So that $\bar{\lambda}_{rv}(s) = \frac{\bar{R}_0(s)}{s}$

The expected Busy period of the server when there is High acoustics in (0,t]

$B_0(t) = q_{01}(t)[c]B_1(t) + q_{02}(t)[c]B_2(t)$
$B_1(t) = q_{01}(t)[c]B_1(t) + [q_{11}^{(3)}(t) + q_{11}^{(4)}(t)]\ [c]B_1(t),$
$B_2(t) = T_2(t) + q_{02}(t)[c]\ B_2(t) + [q_{22}^{(5)}(t) + q_{22}^{(6)}(t)]\ [c]B_2(t)$
$T_2(t) = e^{-\lambda_1 t}\,G_2(t)$ (19- 21)
Taking Laplace Transform of eq. (19-21) and solving for $\bar{B}_0(s)$
$\bar{B}_0(s) = N_4(s) / D_2(s)$ (22)
Where
$N_4(s) = \hat{q}_{02}(s)\,\hat{T}_2(s))$
And $D_2(s)$ is already defined.

In steady state, $B_0 = \frac{N_4(0)}{D_2'(0)}$ (23)
The expected busy period of the server for repair in (0,t] is
$\lambda_{ru}(t) = \int_0^\infty B_0(z)dz$ So that $\bar{\lambda}_{ru}(s) = \frac{\bar{B}_0(s)}{s}$ (24)

The expected number of visits by the repairman for repairing the non-identical units in (0,t]

$H_0(t) = Q_{01}(t)[s][1 + H_1(t)] + Q_{02}(t)[s][1 + H_2(t)]$
$H_1(t) = Q_{10}(t)[s]H_0(t)] + [Q_{11}^{(3)}(t) + Q_{11}^{(4)}(t)]\ [s]H_1(t),$
$H_2(t) = Q_{20}(t)[s]H_0(t) + [Q_{22}^{(5)}(t) + Q_{22}^{(6)}(t)]\ [c]H_2(t)$
(25-27)
Taking Laplace Transform of eq. (25-27)
and solving for $H_0^*(s)$
$H_0^*(s) = N_6(s) / D_3(s)$ (28)
In the long run, $H_0 = \frac{N_6(0)}{D_3'(0)}$ (29)

BENEFIT- FUNCTION ANALYSIS

The Benefit-Function analysis of the system considering mean up-time, expected busy period of the system under High acoustics when the units stops automatically, expected busy period of the server for repair of unit under Very low gravity expected number of visits by the repairman for unit failure.
The expected total Benefit-Function incurred in (0,t] is
C (t) = Expected total revenue in (0,t] - expected total repair cost in (0,t] due to High acoustics failure

- expected total repair cost due to FH- failure resulting from Very low gravity for repairing the units in (0,t]
- expected busy period of the system under High acoustics when the units automatically stop in (0,t]
- expected number of visits by the repairman for repairing of non-identical the units in (0,t]

The expected total cost per unit time in steady state is
$C = \lim_{t\to\infty}(C(t)/t) = \lim_{s\to 0}(s^2 C(s))$
$= K_1A_0 - K_2R_0 - K_3B_0 - K_4H_0$
Where
K_1: revenue per unit up-time,
K_2: cost per unit time for which the system is under repair of type- I
K_3: cost per unit time for which the system is under repair of type-II
K_4: cost per visit by the repairman for units repair.

CONCLUSION

After studying the system, we have analyzed graphically that when the failure rate due to Very low gravity and failure rate due to High acoustics increases, the MTSF and steady state availability decreases and the Benifit-function decreased as the failure increases.

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