# Gain-function of two non-identical warm standby system with failure due to nonavailability of sunlight and switch failure 

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#### Abstract

Introduction: In Solar System the sunlight plays aa important and vital role. The non-conventional renewable solar energy which is cheap and readily available for use in institutions, hospitals, industries and all sort of equipments and places where energy is required. But for solar energy Sun is the prime source from where solar energy can be generated. During rainy and winter seasons the sun is under the cover of clouds regularly resulting solar penal cells are unable to receive sunlight which causing failure of the system. In the present paper we have taken two non-identical warm standby system with failure due to non- availability of sunlight. When there is non-availability of sunlight the working of unit stops automatically. The failure time distribution is taken as exponential and repair time distribution as general. Using Semi Markov regenerative point technique we have calculated different reliability characteristics such as MTSF, reliability of the system, availability analysis in steady state, busy period analysis of the system under repair, expected number of visits by the repairman in the long run and profit-function. Special case by taking repair as exponential has been derived and graphs are drawn.


Keywords: warm standby, non-availability of sunlight, MTSF, Availability, busy period, Gain-function.
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## INTRODUCTION

In Solar System the sunlight plays the pivotal and vital role. The non-conventional renewable solar energy which is cheap and easily available for use in institutions, hospitals, industries and all sort of places and equipments where energy is required. But for solar energy Sun is the primary source from where solar energy can be generated. The sun is under the cover of clouds almost every day during rainy and winter seasons causing solar penal cells not receiving sunlight and becomes inactive unable to generate solar energy resulting failure of the system.
Assumptions

1. The failure time distribution is exponential whereas the repair time distribution is arbitrary of two non-identical units.
2. The repairs are perfect and starts immediately upon failure of units with repair discipline are FCFS.

[^0]3. The operation of the unit stops as soon as there is non-availability of sunlight.
4. The failure of a unit is detected immediately and perfectly.
5. The switches are instantaneous but not perfect.
6. All random variables are mutually independent.

Symbols for states of the System
Superscripts: O, WS, SO, FNASL, SF
Operative, Warm Standby, Stops the operation, Failure due to non-availability of sunlight, Switch failure respectively Subscripts: nasl, asl, ur, wr, uR
Non-availability of sunlight, availability of sunlight, under repair, waiting for repair, under repair continued respectively Up states: 0, 1, 2, 9 ;
Down states: 3,4,5,6,7,8,10,11
Regeneration point: $0,1,2,4,7,10$
States of the System
$\mathbf{0}\left(\mathbf{O}_{\text {ass }}, \mathbf{W S}_{\text {nasl }}\right)$ The first unit is operative due to availability of sunlight and the second unit is warm standby with nonavailability of sunlight.
$\mathbf{1}\left(\mathrm{SO}_{\text {nast }}, \mathrm{O}_{\text {ass }}\right)$
The operation of the first unit stops automatically due to non-availability of sunlight and warm standby units starts operating due to availability of sunlight.
2(FNASL ${ }_{\text {ur }}, \mathrm{O}_{\text {asi }}$ )
The first unit fails due to non-availability of sunlight undergoes repair and the second unit continues to be operative due to availability of sunlight.
3(FNASL ${ }_{u r}$, SO $_{\text {nass }}$ )
The repair of the first unit is continued from state 2 and in the other unit the operation of the unit stops automatically due to non-availability of sunlight.
4(FNASL ${ }_{\text {ur }}$, SO $_{\text {nas }}$ )
The one unit fails due to non-availability of sunlight and undergoes repair and the other unit also stops automatically due to non-availability of sunlight.

## 5(FNASL ${ }_{u r}$, FNASL $_{w r}$ )

The repair of the first unit is continued from state 4 and the other unit is failed due to non-availability of sunlight in it and is waiting for repair.


Figure 1: The State Transition Diagram
$\bigcirc$ up state $\quad \square$ down state

## 6 ( $\mathbf{O}_{\text {asl }}$, FNASL $_{\text {ur }}$ )

The first unit is operative due to availability of sunlight and the second unit failed due to non-availability of sunlight is under repair.
7( $\mathrm{SO}_{\text {nast }} \mathrm{SF}_{\text {ur }}$ )
The operation of the first unit stops automatically due to non-availability of sunlight and during switchover to the second unit switch fails and undergoes repair.
8(FNASL ${ }_{\text {wr }}$, SF $_{\text {uR }}$ )
The repair of failed switch is continued from state 7 and the first unit is failed due to non-availability of sunlight is waiting for repair.
$\mathbf{9}\left(\mathrm{O}_{\text {ast }}, \mathrm{SO}_{\text {nass }}\right)$
The first unit is operative due to availability of sunlight and the operation of warm standby second unit is stopped due to non-availability of sunlight.
$\mathbf{1 0}\left(\mathrm{SO}_{\text {nasl }}, \mathrm{SF}_{\mathrm{ur}}\right)$
The operation of the first unit stops automatically due to non-availability of sunlight and in the second unit switch fails and undergoes repair.
11(FNASL ${ }_{w r}$, SF $_{\text {ur }}$ )
The repair of the second unit is continued from state 10 and the first unit is failed due to non-availability of sunlight is waiting for repair.

## TRANSITION PROBABILITIES

Simple probabilistic considerations yield the following expressions :
$\mathrm{p}_{01}=\frac{\lambda 1}{\lambda 1+\lambda_{2}+\lambda_{3}}, \mathrm{P}_{07}=\frac{\lambda 2}{\lambda 1+\lambda 2+\lambda_{3}}$
$\mathrm{p}_{09}=\frac{\lambda_{2}}{\lambda 1+\lambda 2+\lambda_{3}}, \mathrm{p}_{12}=\frac{\lambda_{1}}{\lambda 1+\lambda_{3}}, \mathrm{p}_{14}=\frac{\lambda_{3}}{\frac{\lambda 1 \mathrm{P}}{} \mathrm{P}_{3}}$
$\mathrm{P}_{20}=\mathrm{G}_{1}\left(\lambda_{1}\right), \mathrm{P}_{22}{ }^{(3)}=\mathrm{G}_{1}{ }^{*}\left(\lambda_{1}\right)=\mathrm{p}_{23}, \mathrm{P}_{72}=\mathrm{G}_{2}{ }^{*}\left(\lambda_{4}\right)$,
$\mathrm{P}_{72}{ }^{(8)}=\mathrm{G}_{2}{ }^{*}\left(\lambda_{4}\right)=\mathrm{P}_{78}$
We can easity verify that
$\mathrm{p}_{01}+\mathrm{p}_{07}+\mathrm{p}_{09}=1, \mathrm{p}_{12}+\mathrm{p}_{14}=1, \mathrm{p}_{20}+\mathrm{p}_{23}\left(=\mathrm{p}_{22}^{(3)}\right)=1, \mathrm{p}_{46}{ }^{(6)}=1 \mathrm{p}_{60}=1$,
$\mathrm{p}_{72}+\mathrm{P}_{72}{ }^{(5)}+\mathrm{p}_{74}=1, \mathrm{p}_{9,10}=1, \mathrm{p}_{10,2}+\mathrm{p}_{10,2}{ }^{(11)}=1$
We can easily verify that
$\mathrm{p}_{01}+\mathrm{p}_{07}+\mathrm{p}_{09}=1, \mathrm{p}_{12}+\mathrm{p}_{14}=1, \mathrm{p}_{20}+\mathrm{p}_{23}\left(=\mathrm{F}_{22}{ }^{(3)}\right)=1, \mathrm{p}_{46}{ }^{(6)}=1 \mathrm{p}_{60}=1$,
$\mathrm{p}_{72}+\mathrm{P}_{72}{ }^{(5)}+\mathrm{p}_{74}=1, \mathrm{p}_{9,10}=1, \mathrm{p}_{10,2}+\mathrm{p}_{10,2}{ }^{(11)}=1$ (1)
And mean sojourn time are
$\mu_{0}=\mathrm{E}(\mathrm{T})=\int_{0}^{\infty} P[T>t] d t$

## MEAN TIME TO SYSTEM FAILURE

We can regar̃d the failed state as absorbing
$\theta_{0}(t)=Q_{01}(t)[s] \theta_{1}(t)+Q_{09}(t)[s] \theta_{9}(t)+Q_{07}(t)$
$\theta_{1}(t)=Q_{12}(t)[s] \theta_{2}(t)+Q_{14}(t), \theta_{2}(t)=Q_{20}(t)[s] \theta_{0}(t)+Q_{22}^{(3)}(t)$
$\theta_{4}(t)=Q_{9,10}(t)$
Taking Laplace-Stiltjes transform of eq. (3-5) and solving for
$Q_{0}^{*}(s)=\mathrm{N}_{1}(\mathrm{~s}) / \mathrm{D}_{1}(\mathrm{~s})$
Where
$\mathrm{N}_{\mathrm{l}}(\mathrm{s})=Q_{01}^{*}(s)\left\{Q_{12}^{*}(s) Q_{22}^{(3) *}(s)+Q_{14}^{*}(s)\right\}+Q_{09}^{*}(s) Q_{9,10}^{*}(s)+Q_{07}^{*}(s)$
$\mathrm{D}_{1}(\mathrm{~s})=1-Q_{01}^{*}(s) \quad Q_{12}^{*}(s) Q_{20}^{*}(s)$
Making use of relations (1) and (2) it can be shown that $\theta_{0}(0)=1$, which implies that $\theta_{0}(\mathrm{t})$ is a proper distribution.
$\operatorname{MTSF}=\mathrm{E}[\mathrm{T}]=\mathrm{d} /\left.\mathrm{ds} \theta_{0}{ }^{*}(0)\right|_{\mathrm{s} \neq 0}=\left(\mathrm{D}_{1}^{\prime}(0)-\mathrm{N}_{1}^{\prime}(0)\right) / \mathrm{D}_{1}(0)$
$=\left(\mu_{0}+\mathrm{p}_{01} \mu_{1}+\mathrm{p}_{01} \mathrm{p}_{12} \mu_{2}+\mathrm{p}_{09} \mu_{9}\right) /\left(1-\mathrm{p}_{01} \mathrm{p}_{12} \mathrm{p}_{20}\right)$
where
$\mu_{0}=\mu_{01}+\mu_{07}+\mu_{09}, \mu_{1}=\mu_{12}+\mu_{14}, \mu_{2}=\mu_{20}+\mu_{22}{ }^{(3)}, \mu_{9}=\mu_{9,10}$

## AVAILABILITY ANALYSIS

Let $\mathrm{M}_{\mathrm{i}}(\mathrm{t})$ be the probability of the system having started from state I is up at time t without making any other regenerative state belonging to E . By probabilistic arguments, we have
The value of $\mathrm{M}_{0}(\mathrm{t}), \mathrm{M}_{1}(\mathrm{t}), \mathrm{M}_{2}(\mathrm{t}), \mathrm{M}_{4}(\mathrm{t})$ can be found easily.
The point wise availability $A_{i}(t)$ have the following recursive relations
$\mathrm{A}_{0}(\mathrm{t})=\mathrm{M}_{0}(\mathrm{t})+\mathrm{q}_{01}(\mathrm{t})[\mathrm{c}] \mathrm{A}_{1}(\mathrm{t})+\mathrm{q}_{07}(\mathrm{t})[\mathrm{c}] \mathrm{A}_{7}(\mathrm{t})+\mathrm{q}_{09}(\mathrm{t})[\mathrm{c}] \mathrm{A}_{9}(\mathrm{t})$
$\mathrm{A}_{1}(\mathrm{t})=\mathrm{M}_{1}(\mathrm{t})+\mathrm{q}_{12}(\mathrm{t})[\mathrm{c}] \mathrm{A}_{2}(\mathrm{t})+\mathrm{q}_{14}(\mathrm{t})[\mathrm{c}] \mathrm{A}_{4}(\mathrm{t}), \mathrm{A}_{2}(\mathrm{t})=\mathrm{M}_{2}(\mathrm{t})+\mathrm{q}_{20}(\mathrm{t})[\mathrm{c}] \mathrm{A}_{0}(\mathrm{t})+\mathrm{q}_{22}{ }^{(3)}(\mathrm{t})[\mathrm{c}] \mathrm{A}_{2}(\mathrm{t})$
$\mathrm{A}_{4}(\mathrm{t})=\mathrm{q}_{46}{ }^{(3)}(\mathrm{t})[\mathrm{c}] \mathrm{A}_{6}(\mathrm{t}), \mathrm{A}_{6}(\mathrm{t})=\mathrm{q}_{60}(\mathrm{t})[\mathrm{c}] \mathrm{A}_{0}(\mathrm{t})$
$\mathrm{A}_{7}(\mathrm{t})=\left(\mathrm{q}_{72}(\mathrm{t})+\mathrm{q}_{72}{ }^{(8)}(\mathrm{t})\right)[\mathrm{c}] \mathrm{A}_{2}(\mathrm{t})+\mathrm{q}_{74}(\mathrm{t})[\mathrm{c}] \mathrm{A}_{4}(\mathrm{t})$
$\mathrm{A}_{9}(\mathrm{t})=\mathrm{M}_{9}(\mathrm{t})+\mathrm{q}_{9,10}(\mathrm{t})[\mathrm{c}] \mathrm{A}_{10}(\mathrm{t}), \mathrm{A}_{10}(\mathrm{t})=\mathrm{q}_{10,2}(\mathrm{t})[\mathrm{c}] \mathrm{A}_{2}(\mathrm{t})+\mathrm{q}_{10,2}{ }^{(11)}(\mathrm{t})[\mathrm{c}] \mathrm{A}_{2}(\mathrm{t})$
Taking Laplace Transform of eq. (7-14) and solving for $\hat{A}_{0}(s)$

$$
\hat{A}_{0}(s)=\mathrm{N}_{2}(\mathrm{~s}) / \mathrm{D}_{2}(\mathrm{~s})
$$

Where
$\mathrm{N}_{2}(\mathrm{~s})=\left(1-\hat{q}_{22}{ }^{(3)}(\mathrm{s})\right)\left\{\widehat{M}_{0}(\mathrm{~s})+\hat{q}_{01}(\mathrm{~s}) \widehat{M}_{1}(\mathrm{~s})+\hat{q}_{09}(\mathrm{~s}) \widehat{M}_{9}(\mathrm{~s})\right\}+\widehat{M}_{2}(\mathrm{~s})\left\{\hat{q}_{01}(\mathrm{~s}) \hat{q}_{42}(\mathrm{~s})+\widehat{q}_{07}(\mathrm{~s})\left(\hat{q}_{72}(\mathrm{~s})+\hat{q}_{73}{ }^{(8)}(\mathrm{s})\right)+\hat{q}_{09}\right.$ (s) $\left.\hat{q}_{9,10}(\mathrm{~s})\left(\hat{q}_{10,2}(\mathrm{~s})+\hat{q}_{10,2}{ }^{(11)}(\mathrm{s})\right)\right\}$
$\mathrm{D}_{2}(\mathrm{~s})=\left(1-\hat{q}_{22}{ }^{(3)}(\mathrm{s})\right)\left\{1-\hat{q}_{46}{ }^{(5)}(\mathrm{s}) \hat{q}_{60}(\mathrm{~s})\left(\hat{q}_{01}(\mathrm{~s}) \hat{q}_{44}(\mathrm{~s})+\hat{q}_{07}(\mathrm{~s}) \hat{q}_{74}(\mathrm{~s})\right)\right.$
$-\widehat{q}_{20}(\mathrm{~s})\left\{\hat{q}_{01}(\mathrm{~s}) \widehat{q}_{12}(\mathrm{~s})+\hat{q}_{07}(\mathrm{~s})\left(\hat{q}_{72}(\mathrm{~s})\right)+\hat{q}_{72}{ }^{(8)}(\mathrm{s})+\hat{q}_{09}(\mathrm{~s}) \hat{q}_{9,10}(\mathrm{~s})\right.$
$\left.\left(\hat{q}_{10,2}(\mathrm{~s})+\hat{q}_{10,2}{ }^{(11)}(\mathrm{s})\right)\right\}$
The steady state availability
$\mathrm{A}_{0}=\lim _{t \rightarrow \infty}\left[A_{0}(t)\right]=\lim _{s \rightarrow 0}\left[s \hat{A}_{0}(s)\right]=\lim _{s \rightarrow 0} \frac{s N_{2}(s)}{D_{2}(s)}$
Using L' Hospitals rule, we get
$\mathrm{A}_{0}=\lim _{s \rightarrow 0} \frac{N_{2}(s)+s N_{2}{ }^{\prime}(s)}{D_{2}{ }^{\prime}(s)}=\frac{N_{2}(0)}{D_{2}{ }^{\prime}(0)}$
Where
$\mathrm{N}_{2}(0)=\mathrm{p}_{20}\left(\widehat{M}_{0}(0)+\mathrm{p}_{01} \widehat{M}_{1}(0)+\mathrm{p}_{09} \widehat{M}_{9}(0)\right)+\widehat{M}_{2}(0)\left(\mathrm{p}_{01} \mathrm{p}_{12}+\mathrm{p}_{07}\left(\mathrm{p}_{72}\right.\right.$ $\left.\left.+\mathrm{p}_{72}{ }^{(8)}+\mathrm{p}_{09}\right)\right)$
$\mathrm{D}_{2}^{\prime}(0)=\mathrm{p}_{20}\left\{\mu_{0}+\mathrm{p}_{01} \mu_{1}+\left(\mathrm{p}_{01} \mathrm{p}_{14}+\mathrm{p}_{07} \mathrm{p}_{74}\right) \mu_{4}+\mathrm{p}_{07} \mu_{7}+\mathrm{p}_{07} \mu_{7}+\mathrm{p}_{09}\left(\mu_{9}+\mu_{10}\right)\right.$
$+\mu_{2}\left\{1-\left(\left(\mathrm{p}_{01} \mathrm{p}_{14}+\mathrm{p}_{07} \mathrm{p}_{74}\right)\right\}\right.$
$\mu_{4}=\mu{ }_{46}{ }^{(5)}, \mu_{7}=\mu_{72}+\mu{ }_{72}^{(8)}+\mu_{74}, \mu_{10}=\mu_{10,2}+\mu{ }_{10,2}^{(11)}$
The expected up time of the system in $(0, t]$ is
$\lambda_{u}(\mathrm{t})=\int_{0}^{\infty} A_{0}(z) d z$ So that $\widehat{\lambda_{u}}(\mathrm{~s})=\frac{\widehat{\mathrm{A}}_{0}(\mathrm{~s})}{\mathrm{s}}=\frac{N_{2}(S)}{S D_{2}(S)}$
The expected down time of the system in $(0, t]$ is

$$
\lambda_{d}(\mathrm{t})=\mathrm{t}-\lambda_{u}(\mathrm{t}) \text { So that } \widehat{\lambda_{d}}(\mathrm{~s})=\frac{1}{\mathrm{~s}^{2}}-\widehat{\lambda_{u}}(\mathrm{~s})(18)
$$

The expected busy period of the server for repairing the failed unit under non-availability of sunlight in $(0, t]$
$\mathrm{R}_{0}(\mathrm{t})=\mathrm{S}_{0}(\mathrm{t})+\mathrm{q}_{01}(\mathrm{t})[\mathrm{c}] \mathrm{R}_{1}(\mathrm{t})+\mathrm{q}_{07}(\mathrm{t})[\mathrm{c}] \mathrm{R}_{7}(\mathrm{t})+\mathrm{q}_{09}(\mathrm{t})[\mathrm{c}] \mathrm{R}_{9}(\mathrm{t})$
$\mathrm{R}_{1}(\mathrm{t})=\mathrm{S}_{1}(\mathrm{t})+\mathrm{q}_{12}(\mathrm{t})[\mathrm{c}] \mathrm{R}_{2}(\mathrm{t})+\mathrm{q}_{14}(\mathrm{t})[\mathrm{c}] \mathrm{R}_{4}(\mathrm{t})$,
$\mathrm{R}_{2}(\mathrm{t})=\mathrm{q}_{20}(\mathrm{t})[\mathrm{c}] \mathrm{R}_{0}(\mathrm{t})+\mathrm{q}_{22}{ }^{(3)}(\mathrm{t})[\mathrm{c}] \mathrm{R}_{2}(\mathrm{t})$
$\mathrm{R}_{4}(\mathrm{t})=\mathrm{q}_{46}{ }^{(3)}(\mathrm{t})[\mathrm{c}] \mathrm{R}_{6}(\mathrm{t}), \mathrm{R}_{6}(\mathrm{t})=\mathrm{q}_{60}(\mathrm{t})[\mathrm{c}] \mathrm{R}_{0}(\mathrm{t})$
$\mathrm{R}_{7}(\mathrm{t})=\left(\mathrm{q}_{72}(\mathrm{t})+\mathrm{q}_{72}{ }^{(8)}(\mathrm{t})\right)[\mathrm{c}] \mathrm{R}_{2}(\mathrm{t})+\mathrm{q}_{74}(\mathrm{t})[\mathrm{c}] \mathrm{R}_{4}(\mathrm{t})$
$\mathrm{R}_{9}(\mathrm{t})=\mathrm{S}_{9}(\mathrm{t})+\mathrm{q}_{9,10}(\mathrm{t})[\mathrm{c}] \mathrm{R}_{10}(\mathrm{t}), \mathrm{R}_{10}(\mathrm{t})=\mathrm{q}_{10,2}(\mathrm{t})+\mathrm{q}_{10,2}{ }^{(11)}(\mathrm{t})[\mathrm{c}] \mathrm{R}_{2}(\mathrm{t})$
Taking Laplace Transform of eq. (19-26) and solving for $\widehat{R_{0}}(s)$
$\widehat{R_{0}}(s)=\mathrm{N}_{3}(\mathrm{~s}) / \mathrm{D}_{2}(\mathrm{~s})$
Where
$\mathrm{N}_{2}(\mathrm{~s})=\left(1-\hat{q}_{22}{ }^{(3)}(\mathrm{s})\right)\left\{\hat{S}_{0}(\mathrm{~s})+\hat{q}_{01}(\mathrm{~s}) \hat{S}_{1}(\mathrm{~s})+\hat{q}_{09}(\mathrm{~s}) \hat{S}_{9}(\mathrm{~s})\right\}$ and $\mathrm{D}_{2}(\mathrm{~s})$ is already defined.
In the long run, $\mathrm{R}_{0}=\frac{N_{3}(0)}{D_{2}{ }^{\prime}(0)}$
where $\mathrm{N}_{3}(0)=\mathrm{p}_{20}\left(\hat{S}_{0}(0)+\mathrm{p}_{01} \hat{S}_{1}(0)+\mathrm{p}_{09} \hat{S}_{9}(0)\right)$ and $\mathrm{D}_{2}{ }^{\prime}(0)$ is already defined.
The expected period of the system under non-availability of sunlight in $(0, t]$ is
$\lambda_{r v}(\mathrm{t})=\int_{0}^{\infty} R_{0}(z) d z$ So that $\widehat{\lambda_{r v}}(\mathrm{~s})=\frac{\widehat{\mathrm{R}}_{0}(\mathrm{~s})}{\mathrm{s}}$
The expected Busy period of the server for repair of dissimilar units by the repairman in $(0, t]$
$\mathrm{B}_{0}(\mathrm{t})=\mathrm{q}_{01}(\mathrm{t})[\mathrm{c}] \mathrm{B}_{1}(\mathrm{t})+\mathrm{q}_{07}(\mathrm{t})[\mathrm{c}] \mathrm{B}_{7}(\mathrm{t})+\mathrm{q}_{09}(\mathrm{t})[\mathrm{c}] \mathrm{B}_{9}(\mathrm{t})$
$\mathrm{B}_{1}(\mathrm{t})=\mathrm{q}_{12}(\mathrm{t})[\mathrm{c}] \mathrm{B}_{2}(\mathrm{t})+\mathrm{q}_{14}(\mathrm{t})[\mathrm{c}] \mathrm{B}_{4}(\mathrm{t}), \mathrm{B}_{2}(\mathrm{t})=\mathrm{q}_{20}(\mathrm{t})[\mathrm{c}] \mathrm{B}_{0}(\mathrm{t})+\mathrm{q}_{22}{ }^{(3)}(\mathrm{t})[\mathrm{c}] \mathrm{B}_{2}(\mathrm{t})$
$\mathrm{B}_{4}(\mathrm{t})=\mathrm{T}_{4}(\mathrm{t})+\mathrm{q}_{46}{ }^{(3)}(\mathrm{t})[\mathrm{c}] \mathrm{B}_{6}(\mathrm{t}), \mathrm{B}_{6}(\mathrm{t})=\mathrm{T}_{6}(\mathrm{t})+\mathrm{q}_{60}(\mathrm{t})[\mathrm{c}] \mathrm{B}_{0}(\mathrm{t})$
$\mathrm{B}_{7}(\mathrm{t})=\left(\mathrm{q}_{72}(\mathrm{t})+\mathrm{q}_{72}{ }^{(8)}(\mathrm{t})\right)[\mathrm{c}] \mathrm{B}_{2}(\mathrm{t})+\mathrm{q}_{74}(\mathrm{t})[\mathrm{c}] \mathrm{B}_{4}(\mathrm{t})$
$\mathrm{B}_{9}(\mathrm{t})=\mathrm{q}_{9,10}(\mathrm{t})[\mathrm{c}] \mathrm{B}_{10}(\mathrm{t}), \mathrm{B}_{10}(\mathrm{t})=\mathrm{T}_{10}(\mathrm{t})+\left(\mathrm{q}_{10,2}(\mathrm{t})+\mathrm{q}_{10,2}{ }^{(11)}(\mathrm{t})[\mathrm{c}] \mathrm{B}_{2}(\mathrm{t})\right.$
Taking Laplace Transform of eq. (29-36) and solving for $\widehat{B_{0}}(s)$
$\widehat{B_{0}}(s)=\mathrm{N}_{4}(\mathrm{~s}) / \mathrm{D}_{2}(\mathrm{~s})(37)$
Where

$$
\begin{aligned}
\mathrm{N}_{4}(\mathrm{~s})= & \left(1-\hat{q}_{22}{ }^{(3)}(\mathrm{s})\right)\left\{\hat{q}_{01}(\mathrm{~s}) \hat{q}_{14}(\mathrm{~s})\left(\widehat{T}_{4}(\mathrm{~s})+\hat{q}_{46}{ }^{(5)}(\mathrm{s}) \widehat{T}_{6}(\mathrm{~s})\right)+\hat{q}_{07}{ }^{(3)}(\mathrm{s}) \widehat{q}_{74}(\mathrm{~s})\left(\widehat{T}_{4}(\mathrm{~s})\right.\right. \\
& \left.\left.+\hat{q}_{46}{ }^{(5)}(\mathrm{s}) \widehat{T}_{6}(\mathrm{~s})\right)+\hat{q}_{09}(\mathrm{~s}) \widehat{q}_{09,10}(\mathrm{~s}) \widehat{T}_{10}(\mathrm{~s})\right)
\end{aligned}
$$

And $D_{2}(s)$ is already defined.
In steady state, $\mathrm{B}_{0}=\frac{N_{4}(0)}{D_{2}{ }^{\prime}(0)}$
where $\mathrm{N}_{4}(0)=\mathrm{p}_{20}\left\{\left(\mathrm{p}_{01} \mathrm{p}_{14}+\mathrm{p}_{07} \mathrm{p}_{74}\right)\left(\widehat{T}_{4}(0)+\widehat{T}_{6}(0)\right)+\mathrm{p}_{09} \widehat{T}_{10}(0)\right\}$ and $\mathrm{D}_{2}{ }^{\prime}(0)$ is already defined.
The expected busy period of the server for repair in $(0, t]$ is
$\lambda_{r u}(\mathrm{t})=\int_{0}^{\propto} B_{0}(z) d z$ So that $\widehat{\lambda_{r u}}(\mathrm{~s})=\frac{\widehat{\mathrm{B}}_{0}(\mathrm{~s})}{\mathrm{s}}$
The expected Busy period of the server for repair of switch in $(0, t]$
$\mathrm{P}_{0}(\mathrm{t})=\mathrm{q}_{01}(\mathrm{t})[\mathrm{c}] \mathrm{P}_{1}(\mathrm{t})+\mathrm{q}_{07}(\mathrm{t})[\mathrm{c}] \mathrm{P}_{7}(\mathrm{t})+\mathrm{q}_{09}(\mathrm{t})[\mathrm{c}] \mathrm{P}_{9}(\mathrm{t})$
$\mathrm{P}_{1}(\mathrm{t})=\mathrm{q}_{12}(\mathrm{t})[\mathrm{c}] \mathrm{P}_{2}(\mathrm{t})+\mathrm{q}_{14}(\mathrm{t})[\mathrm{c}] \mathrm{P}_{4}(\mathrm{t}), \mathrm{P}_{2}(\mathrm{t})=\mathrm{q}_{20}(\mathrm{t})[\mathrm{c}] \mathrm{P}_{0}(\mathrm{t})+\mathrm{q}_{22}{ }^{(3)}(\mathrm{t})[\mathrm{c}] \mathrm{P}_{2}(\mathrm{t})$
$\mathrm{P}_{4}(\mathrm{t})=\mathrm{q}_{46}{ }^{(3)}(\mathrm{t})[\mathrm{c}] \mathrm{P}_{6}(\mathrm{t}), \mathrm{P}_{6}(\mathrm{t})=\mathrm{q}_{60}(\mathrm{t})[\mathrm{c}] \mathrm{P}_{0}(\mathrm{t})$
$\mathrm{P}_{7}(\mathrm{t})=\mathrm{L}_{7}(\mathrm{t})+\left(\mathrm{q}_{72}(\mathrm{t})+\mathrm{q}_{72}{ }^{(8)}(\mathrm{t})\right)[\mathrm{c}] \mathrm{P}_{2}(\mathrm{t})+\mathrm{q}_{74}(\mathrm{t})[\mathrm{c}] \mathrm{P}_{4}(\mathrm{t})$
$\mathrm{P}_{9}(\mathrm{t})=\mathrm{q}_{9,10}(\mathrm{t})[\mathrm{c}] \mathrm{P}_{10}(\mathrm{t}), \mathrm{P}_{10}(\mathrm{t})=\left(\mathrm{q}_{10,2}(\mathrm{t})+\mathrm{q}_{10,2}{ }^{(11)}(\mathrm{t})\right)[\mathrm{c}] \mathrm{P}_{2}(\mathrm{t})$
Taking Laplace Transform of eq. (40-47) and solving for
$\widehat{P_{0}}(s)=\mathrm{N}_{5}(\mathrm{~s}) / \mathrm{D}_{2}(\mathrm{~s})$
where $\mathrm{N}_{2}(\mathrm{~s})=\widehat{q}_{07}(\mathrm{~s}) \hat{L}_{7}(\mathrm{~s})\left(1-\hat{q}_{22}{ }^{(3)}(\mathrm{s})\right)$ and $\mathrm{D}_{2}(\mathrm{~s})$ is defined earlier.
In the long run, $\mathrm{P}_{0}=\frac{N_{5}(0)}{D_{2}{ }^{\prime}(0)}$
where
$\mathrm{N}_{5}(0)=\mathrm{p}_{20} \mathrm{p}_{07} \hat{L}_{4}(0)$
and $D_{2}^{\prime}(0)$ is already defined.
The expected busy period of the server for repair of the switch in $(0, t]$ is
$\lambda_{r s}(\mathrm{t})=\int_{0}^{\alpha} P_{0}(z) d z$ So that $\widehat{\lambda_{r s}}(\mathrm{~s})=\frac{\widehat{\mathrm{P}}_{0}(\mathrm{~s})}{\mathrm{s}}$
The expected number of visits by the repairman for repairing the non-identical units in $(0, t]$
$\mathrm{H}_{0}(\mathrm{t})=\mathrm{Q}_{01}(\mathrm{t})[\mathrm{c}] \mathrm{H}_{1}(\mathrm{t})+\mathrm{Q}_{07}(\mathrm{t})[\mathrm{c}] \mathrm{H}_{7}(\mathrm{t})+\mathrm{Q}_{09}(\mathrm{t})[\mathrm{c}] \mathrm{H}_{9}(\mathrm{t})$
$\mathrm{H}_{1}(\mathrm{t})=\mathrm{Q}_{12}(\mathrm{t})[\mathrm{c}]\left[1+\mathrm{H}_{2}(\mathrm{t})\right]+\mathrm{Q}_{14}(\mathrm{t})[\mathrm{c}]\left[1+\mathrm{H}_{4}(\mathrm{t})\right], \mathrm{H}_{2}(\mathrm{t})=\mathrm{Q}_{20}(\mathrm{t})[\mathrm{c}] \mathrm{H}_{0}(\mathrm{t})+\mathrm{Q}_{22}{ }^{(3)}(\mathrm{t})[\mathrm{c}] \mathrm{H}_{2}(\mathrm{t})$
$\mathrm{H}_{4}(\mathrm{t})=\mathrm{Q}_{46}{ }^{(3)}(\mathrm{t})[\mathrm{c}] \mathrm{H}_{6}(\mathrm{t}), \mathrm{H}_{6}(\mathrm{t})=\mathrm{Q}_{60}(\mathrm{t})[\mathrm{c}] \mathrm{H}_{0}(\mathrm{t})$
$\mathrm{H}_{7}(\mathrm{t})=\left(\mathrm{Q}_{72}(\mathrm{t})+\mathrm{Q}_{72}{ }^{(8)}(\mathrm{t})\right)[\mathrm{c}] \mathrm{H}_{2}(\mathrm{t})+\mathrm{Q}_{74}(\mathrm{t})[\mathrm{c}] \mathrm{H}_{4}(\mathrm{t})$
$\mathrm{H}_{9}(\mathrm{t})=\mathrm{Q}_{9,10}(\mathrm{t})[\mathrm{c}]\left[1+\mathrm{H}_{10}(\mathrm{t})\right], \mathrm{H}_{10}(\mathrm{t})=\left(\mathrm{Q}_{10,2}(\mathrm{t})[\mathrm{c}]+\mathrm{Q}_{10,2}{ }^{(11)}(\mathrm{t})\right)[\mathrm{c}] \mathrm{H}_{2}(\mathrm{t})$
Taking Laplace Transform of eq. (51-58) and solving for $H_{0}^{*}(s)$
$H_{0}^{*}(s)=\mathrm{N}_{6}(\mathrm{~s}) / \mathrm{D}_{3}(\mathrm{~s})$
Where
$\mathrm{N}_{6}(\mathrm{~s})=\left(1-Q_{22}{ }^{(3)^{*}}(\mathrm{~s})\right)\left\{Q^{*}{ }_{01}(\mathrm{~s})\left(Q^{*}{ }_{12}(\mathrm{~s})+Q^{*}{ }_{14}(\mathrm{~s})\right)+Q^{*}{ }_{09}(\mathrm{~s}) Q^{*}{ }_{9,10}(\mathrm{~s})\right\}$
$\mathrm{D}_{3}(\mathrm{~s})=\left(1-Q_{22}{ }^{(3)^{*}}(\mathrm{~s})\right)\left\{1-\left(Q^{*}{ }_{01}(\mathrm{~s}) Q^{*}{ }_{14}(\mathrm{~s})+Q^{*}{ }_{07}(\mathrm{~s}) Q^{*}{ }_{74}(\mathrm{~s})\right) Q_{46}{ }^{(5)^{*}}(\mathrm{~s}) Q^{*}{ }_{60}(\mathrm{~s})\right\}$
$-Q^{*}{ }_{20}(\mathrm{~s})\left\{Q^{*}{ }_{01}(\mathrm{~s}) Q^{*}{ }_{12}(\mathrm{~s})+Q^{*}{ }_{07}(\mathrm{~s})\left(\underset{(11)^{*}}{Q^{*}}(\mathrm{~s})\right)+Q^{*}{ }_{72}^{(8)}(\mathrm{s})+\right.$ $\left.Q^{*}{ }_{09}(\mathrm{~s}) Q^{*}{ }_{9,10}(\mathrm{~s})\left(Q^{*}{ }_{10,2}(\mathrm{~s})+\mathrm{Q}_{10,2}{ }^{(11)^{*}}(\mathrm{~s})\right)\right\}$
In the long run, $\mathrm{H}_{0}=\quad \frac{N_{6}(0)}{D_{3}{ }^{\prime}(0)}(60)$
where $\mathrm{N}_{6}(0)=\mathrm{p}_{20}\left(\mathrm{p}_{01}+\mathrm{p}_{09}\right)$ and $\mathrm{D}^{\prime}(0)$ is already defined.
The expected number of visits by the repairman for repairing the switch in $(0, t]$
$\mathrm{V}_{0}(\mathrm{t})=\mathrm{Q}_{01}(\mathrm{t})[\mathrm{c}] \mathrm{V}_{1}(\mathrm{t})+\mathrm{Q}_{07}(\mathrm{t})[\mathrm{c}] \mathrm{V}_{7}(\mathrm{t})+\mathrm{Q}_{09}(\mathrm{t})[\mathrm{c}] \mathrm{V}_{9}(\mathrm{t})$
$\mathrm{V}_{1}(\mathrm{t})=\mathrm{Q}_{12}(\mathrm{t})[\mathrm{c}] \mathrm{V}_{2}(\mathrm{t})+\mathrm{Q}_{14}(\mathrm{t})[\mathrm{c}] \mathrm{V}_{4}(\mathrm{t}), \mathrm{V}_{2}(\mathrm{t})=\mathrm{Q}_{20}(\mathrm{t})[\mathrm{c}] \mathrm{V}_{0}(\mathrm{t})+\mathrm{Q}_{22}{ }^{(3)}(\mathrm{t})[\mathrm{c}] \mathrm{V}_{2}(\mathrm{t})$
$\mathrm{V}_{4}(\mathrm{t})=\mathrm{Q}_{46}{ }^{(3)}(\mathrm{t})[\mathrm{c}] \mathrm{V}_{6}(\mathrm{t}), \mathrm{V}_{6}(\mathrm{t})=\mathrm{Q}_{60}(\mathrm{t})[\mathrm{c}] \mathrm{V}_{0}(\mathrm{t})$
$\mathrm{V}_{7}(\mathrm{t})=\left(\mathrm{Q}_{72}(\mathrm{t})\left[1+\mathrm{V}_{2}(\mathrm{t})\right]+\mathrm{Q}_{72}{ }^{(8)}(\mathrm{t})\right)[\mathrm{c}] \mathrm{V}_{2}(\mathrm{t})+\mathrm{Q}_{74}(\mathrm{t})[\mathrm{c}] \mathrm{V}_{4}(\mathrm{t})$
$\mathrm{V}_{9}(\mathrm{t})=\mathrm{Q}_{9,10}(\mathrm{t})[\mathrm{c}] \mathrm{V}_{10}(\mathrm{t}), \mathrm{V}_{10}(\mathrm{t})=\left(\mathrm{Q}_{10,2}(\mathrm{t})+\mathrm{Q}_{10,2}{ }^{(11)}(\mathrm{t})\right)[\mathrm{c}] \mathrm{V}_{2}(\mathrm{t})(61-68)$
Taking Laplace-Stieltjes transform of eq. (61-68) and solving for $V_{0}{ }^{*}(s)$
$V_{0}{ }^{*}(s)=\mathrm{N}_{7}(\mathrm{~s}) / \mathrm{D}_{4}(\mathrm{~s})(69)$
where $\mathrm{N}_{7}(\mathrm{~s})=Q^{*}{ }_{07}(\mathrm{~s}) Q^{*}{ }_{72}(\mathrm{~s})\left(1-Q_{22}{ }^{(3)^{*}}(\mathrm{~s})\right)$ and $\mathrm{D}_{4}(\mathrm{~s})$ is the same as $\mathrm{D}_{3}(\mathrm{~s})$
In the long run, $\mathrm{V}_{0}=\frac{N_{7}(0)}{D_{4}(0)}(70)$
where $\mathrm{N}_{7}(0)=\mathrm{p}_{20} \mathrm{p}_{07} \mathrm{p}_{72}$ and $\mathrm{D}^{\prime}{ }_{3}(0)$ is already defined.

## GAIN-FUNCTION ANALYSIS

The Gain- function of the system considering mean up-time, expected busy period of the system under non-availability of sunlight when the units stops automatically, expected busy period of the server for repair of unit and switch, expected number of visits by the repairman for non-identical units failure, expected number of visits by the repairman for switch failure.
The expected total Gain-function incurred in $(0, t]$ is
$\mathrm{C}(\mathrm{t})=$ Expected total revenue in $(0, \mathrm{t}]$ - expected total repair cost for switch in $(0, \mathrm{t}]$

- expected total repair cost for repairing the units in $(0, \mathrm{t}$ ]
- expected busy period of the system under non-availability of sunlight when the units automatically stop in $(0, t]$
- expected number of visits by the repairman for repairing the switch in $(0, \mathrm{t}]$
- expected number of visits by the repairman for repairing of the non-identical units in $(0, t]$

The expected total cost per unit time in steady state is
$\mathrm{C}=\lim _{t \rightarrow \infty}(C(t) / t)=\lim _{s \rightarrow 0}\left(s^{2} C(s)\right)$
$=\mathrm{K}_{1} \mathrm{~A}_{0}-\mathrm{K}_{2} \mathrm{P}_{0}-\mathrm{K}_{3} \mathrm{~B}_{0}-\mathrm{K}_{4} \mathrm{R}_{0}-\mathrm{K}_{5} \mathrm{~V}_{0}-\mathrm{K}_{6} \mathrm{H}_{0}$
Where
$\mathbf{K}_{1}$ : revenue per unit up-time,
$\mathbf{K}_{2}$ : cost per unit time for which the system is under switch repair
$\mathbf{K}_{3}$ : cost per unit time for which the system is under unit repair
$\mathbf{K}_{4}$ : when units automatically stop cost per unit time for which the system is under non-availability of sunlight
$\mathbf{K}_{\mathbf{5}}$ : cost per visit by the repairman for which switch repair,
$\mathbf{K}_{6}$ : cost per visit by the repairman for non-identical units repair.

## CONCLUSION

After studying the system, we have analyzed graphically that when the failure rate due to non-availability of sunlight and failure rate due to switch failure increases, the MTSF and steady state availability decreases and the Gain-Function also decreased as the failure increases.

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