Research Article

Cost-benefit analysis of a two similar cold standby aircraft system with failure due to pilot error and failure caused due to dense fog

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Abstract

Pilot error (sometimes called **cockpit error**) is a decision, action or inaction by a pilot of an aircraft determined to be a cause or contributing factor in an accident or incident. Pilot error can be a mistake, oversight, lapse in judgment, or failure to exercise due diligence by pilots during the performance of their duties. A broader view of how human factors fit into a system is now considered standard practice by accident investigators when examining the chain of events that led to an accident. Description Usually in an accident caused by pilot error, it is assumed that the pilot in command (captain) makes an error unintentionally. However, an intentional disregard for a standard operating procedure (or warning) is still considered to be a pilot error, even if the pilot's actions justified criminal charges. The pilot may be a factor even during adverse weather conditions if the investigating body deems that the pilot did not exercise due diligence. The responsibility for the accident in such a case would depend upon whether the pilot could reasonably know of the danger and whether he or she took reasonable steps to avoid the weather problem. Flying into a hurricane (for other than legitimate research purposes) would be considered pilot error; flying into a microburst would not be considered pilot error if it was not detectable by the pilot, or in the time before this hazard was understood. Some weather phenomena (such as clear-air turbulence or mountain waves) are difficult to avoid, especially if the aircraft involved is the first aircraft to encounter the phenomenon in a certain area at a certain time. Placing pilot error as a cause of an aviation accident has often been controversial. For example, the NTSB ruled that the crash of American Airlines Flight 587 was because of the failure of the rudder, which was caused by "unnecessary and excessive rudder pedal inputs" on the part of the co-pilot who was operating the aircraft at the time. Attorneys for the co-pilot, who was killed in the crash, argue that American Airlines' pilots had never been properly trained concerning extreme rudder inputs. The attorneys also claimed that the rudder failure was actually caused by a flaw in the design of the Airbus A300 aircraft and that the co-pilot's rudder inputs should not have caused the catastrophic rudder failure that led to the accident that killed 265 people. We have taken units **Failure due to Pilot error and Failure caused due to dense fog** with failure time distribution as exponential and repair time distribution as General. We have find out MTSF, Availability analysis, the expected busy period of the server for repair when the failure caused due to Pilot error in (0,t], expected busy period of the server for repair when the failure caused due to dense fog in(0,t], the expected busy period of the server for repair when **Failure due to Pilot error and Failure caused due to dense fog** in (0,t], the expected number of visits by the repairman for failure of units due to Pilot error in (0,t], the expected number of visits by the repairman for failure caused due to dense fog in (0,t] and Cost-Benefit analysis using regenerative point technique. A special case using failure and repair distributions as exponential is derived and graphs have been drawn.

Keyword: Cold Standby, Failure due to Pilot error, Failure caused due to dense fog, MTSF, Availability, Busy period, Cost-Benefit Analysis

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INTRODUCTION

One of the most famous incidents of an aircraft disaster attributed to pilot error was the night time crash of Eastern Air Lines Flight 401 near Miami, Florida on December 29, 1972. The captain, first officer, and flight engineer had become fixated on a faulty landing gear light and had failed to realize that the flight controls had been bumped by one of the crew altering the autopilot settings from level flight to a slow descent. Told by ATC to hold over a sparsely populated area away from the airport while they dealt with the problem (with, as a result, very few lights on the ground visible to act as an external reference), the distracted flight crew did not notice the plane losing height and the aircraft eventually struck the ground in the Everglades, killing 101 out of 176 passengers and crew. The subsequent National Transportation Safety Board (NTSB) report on the incident blamed the flight crew for failing to monitor the aircraft's instruments properly. Details of the incident are now frequently used as a case study in training exercises by aircrews and air traffic controllers. During 2004 in the United States, pilot error was listed as the primary cause of 78.6% of fatal general aviation accidents, and as the primary cause of 75.5% of general aviation accidents overall. For scheduled air transport, pilot error typically accounts for just over half of worldwide accidents with a known cause. 28 July 1945 – a United States Army Air Forces B-25 bomber bound for Newark Airport crashed into the 79th floor of the Empire State Building after the pilot became lost in a heavy fog bank situated over Manhattan. All three crewmen were killed as well as eleven office workers in the building. 24 December 1958 – BOAC Bristol Britannia 312, registration G-AOVD, crashed as a result of a controlled flight into terrain, (CFIT), near Winkton, England while on a test flight. The crash was caused by a combination of bad weather and a failure on the part of both pilots to read the altimeter correctly. The First Officer and 2 other people survived. 3 January 1961 - Aero Flight 311 crashed near Kvevlax, Finland. All twenty-five occupants were killed in the crash, the worst in Finnish history. An investigation later determined that both pilots were intoxicated during the flight, and may have been interrupted by a passenger at the time of the crash. 28 February 1966 American astronauts Elliot See and Charles Bassett were killed when their T-38 Talon crashed into a building at Lambert-St. Louis International Airport during bad weather. A NASA investigation concluded that See had been flying too low on his landing approach. 29 December 1972 - Eastern Air Lines Flight 401 crashed into the Florida Everglades after the flight crew failed to notice the deactivation of the plane's autopilot, having been distracted by their own attempts to solve a problem with the landing gear. Out of 163 occupants, 75 survived the crash. 27 March 1977 – the Tenerife disaster; a senior KLM pilot failed to hear, understand or follow tower instructions, causing two Boeing 747s to collide on the runway at Tenerife; 583 people were killed in the worst-ever air disaster. 28 December 1978 United Airlines Flight 173; a flight simulator instructor Captain allowed his Douglas DC-8 to run out of fuel while investigating a landing gear problem. United Airlines subsequently changed their policy to disallow "simulator instructor time" in calculating a pilot's "total flight time". It was thought that a contributory factor to the accident is that an instructor can control the amount of fuel in simulator training so that it never runs out. 13 January 1982 – Air Florida Flight 90, a Boeing 737-200 with 79 passengers and crew, crashed into the 14th Street Bridge and careened into the Potomac River shortly after taking off from Washington National Airport. Seventy-five passengers and crew, and four motorists on the bridge were killed. The NTSB report blamed the flight crew for not properly employing the plane's de-icing system. 19 February 1985 – above the Pacific Ocean the crew of China Airlines Flight 006 lost control of their Boeing 747SP after the No. 4 engine flamed out. The aircraft fell 10,000 feet in twenty seconds and lost a total of 30,000 feet in two-and-a-half minutes before control was regained. There were no fatalities but the aircraft was badly damaged. 28 August 1988 – the Ramstein air show disaster; a member of an Italian aerobatic team misjudged a manoeuvre, causing a mid-air collision. Three pilots and 67 spectators on the ground were killed. 31 August 1988 – Delta Air Lines Flight 1141 crashed on takeoff after the crew forgot to deploy the flaps for increased lift. Of the 108 crew and passengers on board, fourteen were killed. 8 January 1989 – in the Kegworth air disaster, a fan blade broke off in the left engine of a new Boeing 737-400, but the pilots mistakenly shut down the right engine. The left engine eventually failed completely and the crew could not restart the right engine before the aircraft crashed. Instrumentation on the 737-400 was different from earlier models, but no flight simulator for the new model was available in Britain. 3 September 1989 – The crew of Varig Flight 254 made a series of mistakes so that their Boeing 737 ran out of fuel hundreds of miles off-course above the Amazon jungle. Thirteen died in the ensuing crash-landing. 21 October 1989 – Tan-Sahsa Flight 414 crashed into a hill near Toncontin International Airport in Tegucigalpa, Honduras, because of a bad landing procedure by the pilot. 127 people died in the accident. 23 March 1994 – Aeroflot Flight 593 crashed on its way to Hong Kong. The captain, Yaroslav Kudrinsky, invited his two children into the cockpit, and permitted them to sit at the controls, against airline regulations. His fifteen-year-old son, Eldar Kudrinsky, accidentally disconnected the autopilot, causing the plane to bank to the right before diving. The co-pilot brought up the plane too far, causing it to stall and start a flat spin. The pilots recovered the plane but it crashed into a forest, killing all 75 people on board. June 24 1994 - B-52 crashes in Fairchild Air Force Base. The crash was largely attributed to the personality and behavior of the pilot in command and delayed reactions to the earlier incidents involving this pilot. After past histories, Mark McGeehan, a USAF squadron commander, refused to allow any of his squadron members to fly with that pilot unless he (McGeehan) was also on the aircraft. McGeehan's attempts to save the plane immediately before the crash were not successful. This crash is now used in military and civilian aviation environments as a case study in teaching crew resource management. 30 June 1994 - Airbus Industrie Flight 129, a certification test flight of the Airbus 330-300, crashed at Toulouse-Blagnac Airport. While simulating an engine-out emergency just after takeoff with an extreme center of gravity location, the pilots chose improper manual settings which rendered the autopilot incapable of keeping the plane in the air, and by the time the PIC regained manual control, it was too late. The aircraft was destroyed, killing the flight crew, a test engineer, and four passengers which included Airbus and airline customer VIPs. The investigative board concluded the PIC was overworked from earlier flight testing that day, and was unable to devote sufficient time to the preflight briefing. As a result, Airbus had to revise the engine-out emergency procedures. 20 December 1995, American Airlines Flight 965 Boeing 757-200 with 155 passengers and a crew of eight, departed Miami approximately two hours behind schedule at 1835 Eastern Standard Time (EST). The investigators believe that the pilot's unfamiliarity with the modern technology installed in the Boeing 757-200 may have played a role. The pilots did not know their location in relation to a radio beacon in Tulua. The aircraft was equipped to provide that information electronically, but according to sources familiar with the investigation, the pilot apparently did not know how to access the information. 12 October 1997 – Singer John Denver died when his newly-bought Rutan Long-EZ home-built aircraft crashed into the Pacific Ocean off Pacific Grove, California. The NTSB indicated that Denver lost control of the aircraft while attempting to manipulate the fuel selector handle, which had been placed in a hard-to-reach position by the aircraft's builder. The NTSB cited his unfamiliarity with the aircraft's design as a cause of the crash. 16 July 1999 – John F. Kennedy, Jr. died when the Piper Saratoga light aircraft he was piloting crashed into the Atlantic Ocean off the coast of Martha's Vineyard, Massachusetts. The NTSB declared the crash was caused by "the pilot's failure to maintain control of his airplane during a descent over water at night, which was a result of spatial disorientation". Kennedy did not hold a certification for IFR flight, but did continue to fly after weather conditions obscured visual landmarks. 31 August 1999 – 65 people died after Lineas Aéreas Privadas Argentinas (LAPA) flight 3142 crashed after an attempted take-off with the flaps retracted. 31 October 2000 - Singapore Airlines Flight 006 was a Boeing 747-412 that took off from the wrong runway at the then Chiang Kai-Shek International Airport. It then collided with construction equipment on the runway, bursting into flames and killing 83 of 179 occupants. 12 November 2001 – American Airlines Flight 587 encountered heavy turbulence and the co-pilot over-applied the rudder pedal, turning the Airbus A300 side to side. Due to the excessive stress, the rudder failed. The A300 spun and hit a residential area, crushing 5 houses and killing 265. Contributing factors included wake turbulence and pilot training. 24 November 2001 – Crossair Flight 3597 crashed into a forest on approach to runway 28 at Zurich Airport. This was caused by Captain Lutz descending below the minimum safe altitude of 2400 feet on approach to runway 28 at Zurich. 15 April 2002 – Air China Flight 129, a Boeing 767-200, crashed near Pusan, South Korea killing 128 of the 166 people aboard. The co-pilot had been flying too low. 25 October 2002 – eight people, including US Senator Paul Wellstone, were killed in a crash near Eveleth, Minnesota. The NTSB concluded that "the flight crew did not monitor and maintain minimum speed." 26 February 2004 – a Beech 200 carrying Macedonian President Boris Trajkovski crashed, killing Trajkovski and eight other passengers. The crash investigation ruled that the accident was caused by "procedural mistakes by the crew" during the landing approach. 3 January 2004 – Flash Airlines Flight 604 dived into the Red Sea shortly after take off. All 148 people were killed. The captain had encountered vertigo, his control column was slanted to the right, and the captain did not notice. The 737 banked until it was unable to stay in the air. It is Egypt's worst air disaster. 14 August 2005 – the pilots of Helios Airways Flight 522 lost consciousness, most likely due to hypoxia caused by failure to switch the cabin pressurization to "Auto" during the pre-flight preparations. The Boeing 737-300 crashed after running out of fuel, killing all on board. 3 May 2006 – Armavia Flight 967 performed a CFIT, killing all on board, after the pilot lost spatial awareness during a simultaneous turn and climb. 27 August 2006 – Comair Flight 191 operating a Bombardier CRJ-100ER aircraft, crashed while taking off from Lexington's Blue Grass Airport. 49 of the 50 on board, including all 47 passengers, were killed. 1 January 2007 – Adam Air Flight 574; The crew's preoccupation with a malfunction of the inertial reference system diverted their attention from the flight instruments and allowed the increasing descent and bank angle to go unnoticed. Appearing to have become spatially disoriented, the pilots did not detect and appropriately arrest the descent soon enough to prevent loss of control. This caused the aircraft to impact the water at high speed and a steep angle and disintegrate, killing all 102 people on board.[6] 7 March 2007 – Garuda Indonesia Flight 200; poor Crew Resource Management and the failure to extend the flaps led the aircraft to run off the end of the runway after landing. Twenty-two of the 140 occupants were killed. 12 February 2009 - Colgan Air Flight 3407 flying as Continental Connection entered a stall and crashed in to a house in Clarence Center, NY due to lack of situational awareness of air speed by the captain and first officer and the captain's improper reaction to the plane's stick shaker stall warning system. All 49 people in the plane died, along with one person inside the house. 1 June 2009 - Air France Flight 447 entered a stall and crashed into the Atlantic Ocean following pitot tube failure and improper control inputs by the first officer. All 216 passengers and 12 crew members died. 10 April 2010 – 2010 Polish Air Force Tu-154 crash; during a descent towards Russia's Smolensk North Airport, the flight crew of the Polish presidential jet ignored automatic warnings and attempted a risky landing in heavy fog. The Tupolev Tu-154M descended too low and crashed into a nearby forest; all of the occupants were killed, including Polish president Lech Kaczynski, his wife Maria Kaczynska, and numerous government and military officials. 22 May 2010 - Air India Express Flight 812; overshot the runway at Mangalore Airport killing 158 people. The plane touched down 610 metres from the usual touchdown point after a steep descent. CVR recordings showed that the Brit-Serb captain of the aircraft was sleeping and woke up just minutes before landing. His lack of alertness made the plane land very fast and steep and it ran off the end of the tabletop runway. 28 July 2010 – Airblue Flight 202 crashed into the Margalla Hills due to the pilot going the wrong way, killing all 152 occupants aboard. 20 June 2011 – Rus Air Flight 9605 crashed onto a motorway while on final approach to Petrozavodsk Airport in western Russia, after the intoxicated navigator encouraged the captain to land in heavy fog. Forty-three people died in the crash, while only five survived. In this paper, we have failure caused due to dense fog which is non-instantaneous in nature. Here, we investigate a two identical cold standby –a system in which offline unit cannot fail. The failure is due to Pilot error and failure caused due to dense fog. When there is failure caused due to dense fog less degree, that is, within specified limit, it operates as normal as before but if these are beyond the specified degree the operation of the unit is stopped to avoid excessive damage of the unit and as the failure caused due to dense fog going on some characteristics of the unit change which we call failure of the unit. After failure due to Pilot error and failure caused due to dense fog failed unit undergoes repair immediately according to first come first served discipline.

ASSUMPTIONS

1. The system consists of two similar cold standby units. The failure time distributions of the operation of the unit stopped there is the failure due to dense fog, failure due to dense fog and failure caused due to Pilot error are exponential with rates $\lambda_{1,}$ λ_2 and λ_3 whereas the repairing rates for repairing the failed system due to Pilot error and due to dense fog are arbitrary with CDF $G_1(t)$ & $G_2(t)$ respectively.
2. When there is failure caused due to dense fog less degree that is within specified limit, it operates as normal as before but if these are beyond the specified degree the operation of the unit is avoided and as the failure caused due to dense fog goes on some characteristics of the unit change which we call failure of the unit.
3. The failure due to Pilot error and failure caused due to dense fog failed the units. The failure caused due to dense fog is non-instantaneous and it cannot occur simultaneously in both the units.
4. The repair facility works on the first fail first repaired (FCFS) basis.
5. The switches are perfect and instantaneous.
6. All random variables are mutually independent.

SYMBOLS FOR STATES OF THE SYSTEM

Superscripts: O, CS, SO, PEF, FDF

Operative, cold Standby, Stops the operation, Failure due to Pilot error and Failure caused due to dense fog respectively

Subscripts: ndf, udf, pe, ur, wr, uR

No failure due to dense fog, under failure due to dense fog, failure due to Pilot error, under repair, waiting for repair, under repair continued respectively

Up states: 0, 1, 3;

Down states: 2, 4, 5, 6, 7

States of the System

0(O_{ndf}, CS_{ndf})

One unit is operative and the other unit is cold standby and there is no failure due to dense fog in both the units.

1(SO_{udf}, O_{ndf})

The operation of the first unit stops automatically when there is failure due to dense fog and cold standby unit starts operating with no failure due to dense fog.

2(SO_{udf}, $FDF_{udf,ur}$)

The operation of the first unit stops automatically when there is failure due to dense fog and the other unit fails due to dense fog and undergoes repair.

3(PEF_{ur}, O_{udf})

The first unit fails due to Pilot error and undergoes repair and the other unit continues to be operative with no failure due to dense fog.

4(PEF_{ur}, SO_{udf})

The one unit fails due to Pilot error and undergoes repair and the other unit also stops automatically when there is failure due to dense fog.

5(PEF_{uR}, PEF_{wr})

The repair of the first unit is continued from state 4 and the other unit failed due to failure due to Pilot error is waiting for repair.

6(PEF_{uR}, SO_{udf})

he repair of the first unit is continued from state 3 and unit fails due to Pilot error and operation of other unit stops automatically when there is failure due to dense fog.

7(PEF_{wr}, $FDF_{df, uR}$)

The repair of failed unit due to dense fog is continued from state 2 and the first unit failed due to Pilot error is waiting for repair.

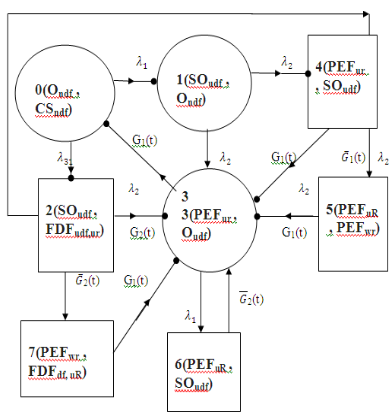

Figure 1: The State Transition Diagram

● Regeneration point ○Up State □Down State

TRANSITION PROBABILITIES

Simple probabilistic considerations yield the following expressions:

$p_{01} = \frac{\lambda 1}{\lambda 1+\lambda 3}$, $p_{02} = \frac{\lambda 3}{\lambda 1+\lambda 3}$

$p_{13} = \frac{\lambda 2}{\lambda 1+\lambda 2}$, $p_{14} = \frac{\lambda 1}{\lambda 1+\lambda 2}$

$p_{23} = \lambda_1 G_2^*(\lambda_2)$, $p_{23}^{(7)} = \lambda_2 G_2^*(\lambda_2)$, $p_{24} = \overline{G}_2^*(\lambda_2)$,

$p_{30} = G_1^*(\lambda_1)$, $p_{33}^{(6)} = \overline{G}_1^*(\lambda_1)$

$p_{43} = G_1^*(\lambda_2)$, $p_{43}^{(5)} = G_1^*(\lambda_2)$ (1)

we can easily verify that

$p_{01} + p_{02} = 1$, $p_{13} + p_{14} = 1$, $p_{23} + p_{23}^{(7)} + p_{24} = 1$, $p_{30} + p_{33}^{(6)} = 1$,

$p_{43} + P_{43}^{(5)} = 1$ (2)

And mean sojourn time are

$\mu_0 = E(T) = \int_0^\infty P[T > t]dt = -1/\lambda_1$

Similarly

$\mu_1 = 1/\lambda_2$, $\mu_2 = \int_0^\infty e^{-}\lambda 1\overline{G}1(t)dt$,

$\mu_4 = \int_0^\infty e^{-}\lambda 2\overline{G}1(t)dt$ (3)

Mean Time to System Failure

We can regard the failed state as absorbing

$\theta_0(t) = Q_{01}(t)[s]\theta_1(t) + Q_{02}(t)$

$\theta_1(t) = Q_{13}(t)[s]\theta_3(t) + Q_{14}(t)$, $\theta_3(t) = Q_{30}(t)[s]\theta_0(t) + Q_{33}^{(6)}(t)$ (4-6)

Taking Laplace-Stieltjes transforms of eq. (4-6) and solving for

$Q_0^*(s) = N_1(s) / D_1(s)$ (7)

Where

$N_1(s) = Q_{01}^*(s) \{ Q_{13}^*(s) Q_{33}^{(6)*}(s) + Q_{14}^*(s) \} + Q_{02}^*(s)$

$D_1(s) = 1 - Q_{01}^*(s) \quad Q_{13}^*(s) Q_{30}^*(s)$

Making use of relations (1) & (2) it can be shown that $Q_0^*(0) = 1$, which implies that $\theta_1(t)$ is a proper distribution.

$\text{MTSF} = E[T] = \frac{d}{ds}\theta_0(s)\Big|_{s=0} = (D_1'(0) - N_1'(0)) / D_1(0)$

$= (\mu_0 + p_{01}\mu_1 + p_{01} p_{13} \mu_3) / (1 - p_{01} p_{13} p_{30})$ (8)

Where

$\mu_0 = \mu_{01} + \mu_{02}$, $\mu_1 = \mu_{13} + \mu_{14}$, $\mu_2 = \mu_{23} + \mu_{23}^{(1)} + \mu_{24}$,

$\mu_3 = \mu_{30} + \mu_{33}^{(6)}$

$\mu_3 = \mu_{43} + \mu_{43}^{(5)}$

AVAILABILITY ANALYSIS

Let $M_i(t)$ be the probability of the system having started from state i is up at time t without making any other regenerative state. By probabilistic arguments, we have

The value of $M_0(t) = e^{-}\lambda_1 t\, e^{-}\lambda_3 t$, $M_1(t) = e^{-}\lambda_1 t\, e^{-}\lambda_2 t$

$M_3(t) = e^{-}\lambda_1\overline{G}_1(t)$. (9)

The point wise availability $A_i(t)$ have the following recursive relations

$A_0(t) = M_0(t) + q_{01}(t)[c]A_1(t) + q_{02}(t)[c]A_2(t)$

$A_1(t) = M_1(t) + q_{13}(t)[c]A_3(t) + q_{14}(t)[c]A_4(t)$,

$A_2(t) = \{q_{23}(t) + q_{23}^{(7)}(t)\}[c]A_3(t) + q_{33}^{(6)}(t)[c]A_3(t)$

$A_4(t) = \{q_{43}(t) + q_{43}^{(5)}(t)[c]A_3(t)$ (10 - 14)

Taking Laplace Transform of eq. (10-14) and solving for $\hat{A}_0(s)$

$\hat{A}_0(s) = N_2(s) / D_2(s)$ (15)

where

$N_2(s) = (1 - \hat{q}_{33}^{(6)}(s))\, \widehat{M}_0(s) + [\, \hat{q}_{01}(s) \{ \widehat{M}_1(s) + (\hat{q}_{13}(s) + \hat{q}_{14}(s)\, (\hat{q}_{43}(s) + \widehat{q}_{43}^{(5)}(s)\,))\} + \hat{q}_{02}(s)\{\hat{q}_{23}(s)) + \hat{q}_{23}^{(1)}(s)) + \hat{q}_{24}(s)\,(\hat{q}_{43}(s) + \hat{q}_{43}^{(5)}(s))\}]\, \widehat{M}_3(s)$

$D_2(s) = (1 - \hat{q}_{33}^{(6)}(s)) - \hat{q}_{30}(s)\, [\hat{q}_{01}(s)\{\hat{q}_{13}(s) + \hat{q}_{14}(s)\,(\hat{q}_{43}(s) + \hat{q}_{43}^{(5)}(s))\} + \widehat{q}_{20}(s)\{\hat{q}_{23}(s) + \widehat{q}_{23}^{(7)}(s) + \hat{q}_{24}(s)(\hat{q}_{43}(s)) + \hat{q}_{43}^{(5)}(s)\,)\}]$

The steady state availability

$A_0 = \lim_{t\to\infty}[A_0(t)] = \lim_{s\to 0}[s\,\hat{A}_0(s)] = \lim_{s\to 0}\frac{s\,N_2(s)}{D_2(s)}$

Using L' Hospitals rule, we get

$A_0 = \lim_{s\to 0} \frac{N_2(s)+s\ N_2'(s)}{D_2'(s)} = \frac{N_2(0)}{D_2'(0)}$ (16)

Where

$N_2(0)= p_{30}\,\widehat{M}_0(0) + p_{01}\widehat{M}_1(0)\,\widehat{M}_3(0)\,)$

$D_2'(0) = \mu_3 + [\,\mu_0 + p_{01}\,(\,\mu_1 + p_{14}\,\mu_4 + p_{02}(\,\mu_2 + p_{24}\,\mu_4\,)]\,p_{30}$

The expected up time of the system in (0,t] is

$\lambda_u(t) = \int_0^{\infty} A_0\,(z)dz$ So that $\widehat{\lambda_u}\,(s) = \frac{\widehat{A}_0\,(s)}{s} = \frac{N_2(S)}{SD_2(S)}$ (17)

The expected down time of the system in (0, t] is

$\lambda_d(t) = t - \lambda_u(t)$ So that $\widehat{\lambda_d}\,(s) = \frac{1}{s^2} - \widehat{\lambda_u}\,(s)$ (18)

The expected busy period of the server when the operation of the unit stops automatically failed when there is failure due to dense fog in (0, t]

$R_0(t) = q_{01}(t)[c]R_1(t) + q_{02}(t)[c]R_2(t)$

$R_1(t) = S_1(t) + q_{13}(t)[c]R_3(t) + q_{14}(t)[c]R_4(t),$

$R_2(t) = S_2(t) + q_{23}(t)[c]R_3(t) + q_{23}^{(7)}(t)[c]R_3(t) + q_{24}(t)[c]R_4(t)$

$R_3(t) = q_{30}(t)[c]R_0(t) + q_{33}^{(6)}(t)[c]R_3(t),$

$R_4(t) = S_4(t) + (q_{43}(t)+ q_{43}^{(5)}(t))\,[c]R_3(t)$ (19-23)

Where

$S_1(t)= e^{-\lambda_1 t}\,e^{-\lambda_2 t},\ \ S_2(t) = e^{-\lambda_1 t}\,\overline{G}_2(t),\ \ S_4(t) = e^{-\lambda_1 t}\,\overline{G}_1(t)$ (24)

Taking Laplace Transform of eq. (19-23) and solving for $\widehat{R_0}(s)$

$\widehat{R_0}(s) = N_3(s) / D_2(s)$ (25)

where

$N_3(s) = (1 - \hat{q}_{33}^{(6)}(s))\,[\,\hat{q}_{01}(s)(\,\hat{S}_1(s) + \hat{q}_{14}(s)\,\hat{S}_4(s)+ \hat{q}_{02}(s)(\,\hat{S}_2(s) + \hat{q}_{24}(s)\,\hat{S}_4(s))]$ and $D_2(s)$ is already defined.

In the long run, $R_0 = \frac{N_3(0)}{D_2'(0)}$ (26)

where $N_3(0)= p_{30}\,[\,p_{01}\,(\hat{S}_1(0) + p_{14}\,\hat{S}_4(0)\,) + p_{02}\,(\hat{S}_2(0) + p_{24}\,\hat{S}_4(0)\,)$ and $D_2'(0)$ is already defined.

The expected busy period of the server when the operation of the unit stops automatically failed when there is failure due to dense fog in (0,t] is

$\lambda_{rv}(t) = \int_0^{\infty} R_0\,(z)dz$ So that $\widehat{\lambda_{rv}}\,(s) = \frac{\widehat{R}_0\,(s)}{s}$ (27)

The expected busy period of the server for repair when failure is caused due to dense fog in (0, t]

$B_0(t) = q_{01}(t)[c]B_1(t) + q_{02}(t)[c]B_2(t)$

$B_1(t) = q_{13}(t)[c]B_3(t) + q_{14}(t)[c]B_4(t),$

$B_2(t) = q_{23}(t)[c]\,B_3(t) + q_{23}^{(7)}(t)[c]B_3(t) + q_{24}(t)[c]\,B_4(t)$

$B_3(t) = T_3\,(t)+ q_{30}(t)[c]\,B_0(t) + q_{33}^{(6)}(t)[c]B_3(t)$

$B_4(t) = T_4\,(t)+ \{\,q_{43}(t) + q_{43}^{(5)}(t)\}\,[c]B_3(t)$ (28-32)

Where

$T_3\,(t) = e^{-\lambda_2 t}\,\overline{G}_1(t) \qquad T_4\,(t) = e^{-\lambda_1 t}\,\overline{G}_1(t)$ (33)

Taking Laplace Transform of eq. (28-32) and solving for $\widehat{B_0}\,(s)$

$\widehat{B_0}\,(s) = N_4(s) / D_2(s)$ (34)

where

$N_4(s) = \widehat{T}_3(s)\,[\,\hat{q}_{01}(s)\,\{\,\hat{q}_{13}(s)+ \hat{q}_{14}\,(s)\,(\,\hat{q}_{43}(s) + \hat{q}_{43}^{(5)}(s))\}+ \hat{q}_{02}(s)\,\{\,\hat{q}_{23}(s)+ \hat{q}_{23}^{(7)}(s)+ \hat{q}_{24}(s)\,(\,\hat{q}_{43}(s) + \hat{q}_{43}^{(5)}(s))\}]+ \widehat{T}_4(s)\,[\hat{q}_{01}(s)\,\hat{q}_{44}(s)(1 - \hat{q}_{33}^{(6)}(s)\,) + (\,\hat{q}_{02}(s)\,\hat{q}_{24}\,(s)(1- \hat{q}_{33}^{(6)}(s)\,)]$

And $D_2(s)$ is already defined.

In steady state, $B_0 = \frac{N_4(0)}{D_2'(0)}$ (35)

where $N_4(0)= \hat{T}_3(0)+ \hat{T}_4(0)\,\{\,p_{30}\,(p_{01}p_{14} + p_{02}\,p_{24})\,\}$ and $D_2'(0)$ is already defined.

The expected busy period of the server for repair in (0, t] is

$\lambda_{ru}(t) = \int_0^{\infty} B_0\,(z)dz$ So that $\widehat{\lambda_{ru}}\,(s) = \frac{\widehat{B}_0\,(s)}{s}$ (36)

The expected busy period of the server for repair when failure caused due to Pilot error in (o, t]

$P_0(t) = q_{01}(t)[c]P_1(t) + q_{02}(t)[c]P_2(t)$

$P_1(t) = q_{13}(t)[c]P_3(t) + q_{14}(t)[c]P_4(t),$
$P_2(t) = L_2(t) + q_{23}(t)[c]P_3(t) + q_{23}^{(7)}(t)[c]P_3(t) + q_{24}(t)[c]P_4(t)$
$P_3(t) = q_{30}(t)[c]P_0(t) + q_{33}^{(6)}(t)[c]P_3(t),$
$P_4(t) = (q_{43}(t) + q_{43}^{(5)}(t))\,[c]P_3(t)$ (37-41)
Where $L_2(t) = e^{-\lambda_1 t}\,\overline{G}_2(t)$
Taking Laplace Transform of eq. (37-41) and solving for $\widehat{P_0}(s)$
$\widehat{P_0}(s) = N_5(s) / D_2(s)$ (43)
where $N_5(s) = \widehat{q}_{02}(s)\,\widehat{L}_2(s)\,(1 - \widehat{q}_{33}^{(6)}(s))$ and $D_2(s)$ is defined earlier.
In the long run, $P_0 = \frac{N_5(0)}{D_2'(0)}$ (44)
where $N_5(0) = p_{30}\, p_{02}\, \widehat{L}_2(0)$
and $D_2'(0)$ is already defined.
The expected Busy period of the server for repair when failure caused due to Pilot error in (o, t] is
$\square_{\square\square}(\tau) = \int_0^{\propto} \square_0(\square)\square\square$ Σo τηατ $\widehat{\square_{\square\square}}(s) = \frac{\hat{P}_0(s)}{s}$ (45)

The expected number of visits by the repairman for repairing the when failure due to dense fog in (0, t]

$H_0(t) = Q_{01}(t)[s]H_1(t) + Q_{02}(t)[s]H_2(t)$
$H_1(t) = Q_{13}(t)[s][1+H_3(t)] + Q_{14}(t)[s][1+H_4(t)],$
$H_2(t) = [Q_{23}(t) + Q_{23}^{(7)}(t)]\,[s][1+H_3(t)] + Q_{24}(t)[s][1+H_4(t)]$
$H_3(t) = Q_{30}(t)[s]H_0(t) + Q_{33}^{(6)}(t)[s]H_3(t),$
$H_4(t) = (Q_{43}(t) + Q_{43}^{(5)}(t))\,[s]H_3(t)$ (46-50)
Taking Laplace Transform of eq. (46-50) and solving for $H_0^*(s)$
$H_0^*(s) = N_6(s) / D_3(s)$ (51)
where
$N_6(s) = (1 - Q_{33}^{(6)*}(s))\{Q^*_{01}(s)(Q^*_{13}(s) + Q^*_{14}(s)) + Q^*_{02}(s)(Q^*_{24}(s) + Q^*_{23}(s) + Q^*_{23}{}^{(7)}(s))\}$

$D_3(s) = (1 - Q_{33}^{(6)*}(s)) - Q^*_{30}(s)[\,Q^*_{01}(s)\,\{\,Q^*_{13}(s) + Q^*_{14}(s)\,(Q_{43}^*(s)\,Q^*_{43}{}^{(5)}(s))\} + Q^*_{02}(s)\{\,Q^*_{23}(s) + Q^*_{23}{}^{(7)}(s)) + Q^*_{24}(s)(\,Q^*_{43}(s)) + Q^*_{43}{}^{(5)}(s))\,\}]$
In the long run,
$H_0 = \frac{N_6(0)}{D_3'(0)}$ (52)
where $N_6(0) = p_{30}$ and $D'_3(0)$ is already defined.

The expected number of visits by the repairman for repairing when failure is caused due to Pilot error in (0, t]

$V_0(t) = Q_{01}(t)[s]V_1(t) + Q_{02}(t)[s][1+V_2(t)]$
$V_1(t) = Q_{13}(t)[s]V_3(t) + Q_{14}(t)[s]V_4(t),$
$V_2(t) = Q_{24}(t)[s][1+V_4(t)] + [\,Q_{23}(t) + Q_{23}^{(7)}(t)[s][1+V_3(t)]$
$V_3(t) = Q_{30}(t)[s]V_0(t) + Q_{33}^{(6)}(t)[s]V_3(t)$ (53-57)
Taking Laplace-Stieltjes transform of eq. (53-57) and solving for $V_0{}^*(s)$
$V_0{}^*(s) = N_7(s) / D_4(s)$ (58)
where $N_7(s) = (1 - Q_{33}^{(6)*}(s))\{Q^*_{01}(s)(Q^*_{14}(s) + Q^*_{43}(s)) + Q^*_{02}(s)(Q^*_{24}(s) + Q^*_{02}(s)\,(Q^*_{23}(s) + Q^*_{23}{}^{(7)}(s))\}$
and $D_4(s)$ is the same as $D_3(s)$
In the long run, $V_0 = \frac{N_7(0)}{D_4'(0)}$ (59)
where $N_7(0) = p_{30}\,[p_{01}\, p_{14}\, p_{43} + p_{02}]$ and $D'_3(0)$ is already defined.

Cost Benefit Analysis

The cost-benefit function of the system considering mean up-time, expected busy period of the system under failure due to dense fog when the units stops automatically, expected busy period of the server for repair when failure due to Pilot error, expected number of visits by the repairman when failure is caused due to dense fog, expected number of visits by the repairman for Pilot error. The expected total cost-benefit incurred in (0, t] is
C (t) = Expected total revenue in (0, t] - expected total repair cost for failure due to dense fog when the units stops automatically in (0, t]

- expected total repair cost for repairing the units when failure is caused due to dense fog in (0,t]

- expected busy period of the system when there is failure due to Pilot error in (0,t]
- expected number of visits by the repairman for repairing when failure caused due to dense fog in (0,t]
- expected number of visits by the repairman for repairing the units when failure is due to Pilot error in (0,t]

The expected total cost per unit time in steady state is

$C = \lim_{t\to\infty}(C(t)/t) \quad = \lim_{s\to 0}(s^2 C(s))$

$= K_1 A_0 - K_2 R_0 - K_3 B_0 - K_4 P_0 - K_5 H_0 - K_6 V_0$

Where

K_1: Revenue per unit up-time,

K_2: Cost per unit time for which the system is under failure due to dense fog when units automatically stop.

K_3: Cost per unit time for which the system failure due to dense fog

K_4: Cost per unit time for which the system failure due to Pilot error

K_5: Cost per visit by the repairman for units repair when failure caused due to dense fog

K_6: Cost per visit by the repairman for units repair when failure is due to Pilot error.

CONCLUSION

After studying the system, we have analyzed graphically that when the failure rate due to operation of the unit stops automatically, due to Pilot error and failure due to dense fog rate increases, the MTSF and steady state availability decreases and the cost function decreased as the failure increases.

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