Research Article

Profit-function of two - identical cold standby aircraft system subject to failure due to damaged by explosive device/weapons and failure due to extreme cold

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Abstract

What is happening with International Space Station While some may dismiss such claims as hype, there is a real fear among NASA officials that, should the orbiting space station need to be abandoned, the risk for a Catastrophic failure greatly increases. The situation arose when Russia's Soyuz rocket, carrying an automated re-supply ship for the station, had a failure of its third stage and crashed into Siberia before achieving orbit. While the Russian space agency investigates the crash the Soyuz rockets have been grounded indefinitely until both a cause and, if applicable, a solution to the problem has been found. Since the retirement of the U.S. Space Shuttle, the Soyuz is currently the only method of getting astronauts to the ISS. ISS currently has a 6-person crew on board that is scheduled to be reduced to three in late September, with the remaining three crew members returning in November. Each 3-person crew will come back to Earth in one of the two Soyuz capsules currently docked to the station. If the Russian rocket has not been cleared to return to flight the station will be abandoned once those last three crew members depart. The station is able to be operated remotely from the ground for a time, but without astronauts present, any critical issues that may arise that can't be corrected via a remote link could cause a serious problem for the ISS. Any critical failure that requires repair could result in system damage or even the loss of the space station altogether. NASA risk assessment indicates that there is a 10% of losing the station if it stays abandoned up to 6 months but that risk balloons to 50% is it stays unmanned for 12 months. While this is something the world's space agency fervently wishes to avoid, there is also an obvious consensus that the safety of crews has to be the top priority. This means that not only can no replacement crew be launched until Soyuz is deemed safe for flight again, but also that the current crew can stay no longer than November to satisfy the "6 month" rule for staying aboard the station. The concern comes from exposure to radiation as well as the service life of the Soyuz capsules they'll be returning home in (Soyuz capsules are rated for 200 days in space). The largest risk of evacuating the station is the possibility that some major component could break, or a significant anomaly could occur, and no astronauts would be available on orbit to fix it. In this paper we have taken risk of **catastrophic and critical failure of ISS without astronauts.** When the main unit fails due to catastrophic and critical failure of ISS without astronauts then cold standby system becomes operative. Critical failure of ISS without astronauts cannot occur simultaneously in both the units and after failure the unit undergoes Type-I or Type-II or Type-III repair facility immediately. Applying the regenerative point technique with renewal process theory the various reliability parameters MTSF, Availability, Busy period, Benefit-Function analysis have been evaluated.

Keywords: Cold Standby, risk of catastrophic and critical failure of ISS without astronauts, first come first serve, MTSF, Availability, Busy period, Benefit -Function.

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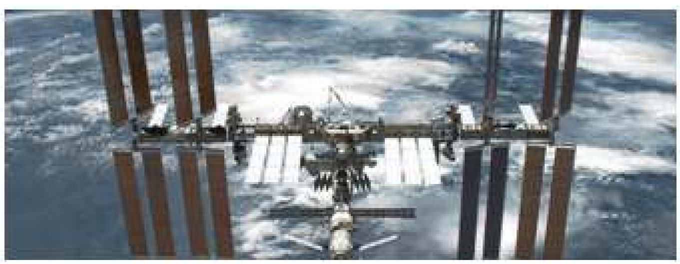
INTRODUCTION

Two factors could prompt an unprecedented de-staffing of the outpost: the certified orbital lifetimes of the two Soyuz lifeboats now at the station, and flight rules that call for crews to return to Earth during daylight. As it stands, three station crew mates scheduled to return to Earth on Sept. 8 will remain onboard the outpost at least an extra week while the investigation into the loss or a robotic Progress space freighter continues. A third-stage Soyuz rocket shutdown caused the failure. The planned Sept. 21 launch of a new station crew is being postponed indefinitely. A Russian space official said few days ago that once the mammoth International Space Station is no longer needed it will be sent into the Pacific Ocean. It's a plan that's long been in the works and is a step to avoid the station becoming dangerous space junk. It was supposed to plunge into the ocean as early as 2015. The US recently extended its life until at least 2020, and there's been talk of keeping it going even longer.



Russia sank its Mir space station in the Pacific in 2001 after 15 years in operation. Skylab, America's first space station fell from orbit in 1979 after six years in space. The International Space Station is the biggest orbiting outpost ever built and can sometimes be seen from the Earth with the naked eye. It's now big enough for six residents. (News.com.au) The six-member crew aboard the International Space Station has plenty of food and fuel and will not be immediately affected by the crash of a Russian supply ship, according to NASA. Astronauts may need to leave the International Space Station if Russian Soyuz rockets remain grounded beyond mid-November, believes NASA. The supply ship had been carrying about three tons of supplies toward the orbiting research lab. The retirement of the US space shuttle program earlier this year has left Russia as the sole nation capable of toting crew to the ISS aboard its Soyuz space capsules. Cargo missions can also be sent to the ISS on Japan's HTV and Europe's ATV supply ships, and two such missions are scheduled for early next year. The Interfax news agency cited a Russian space analyst, Sergei Puzanov, as saying the space station had supplies already aboard that could last two to three months and "the situation with the loss of the Progress cannot be called critical." In July of 2010, a Progress supply ship failed in its first automatic docking attempt due to equipment malfunction, but was connected with the orbiting laboratory two days later. Last month's final mission by the US shuttle Atlantis, known as STS-135, carried up more than five tons of provisions for the outpost. "**STS-135 delivered extra supplies. With those + planned ATV and HTV launches, space station will have enough supplies for all of 2012,**" the US space agency said in a message on the micro blogging site Twitter. "Our Russian colleagues will assess Progress data and determine root cause. Meanwhile, space station operations will continue normally." It is kind of strange that they are saying the ISS needs to be abandoned because they don't have supplies, after Atlantis just delivered 5 tons of food. If Russian Soyuz rockets remain grounded beyond mid-November, there will be no way to launch new crews before the current residents are supposed to leave. A joint board of the world's space agencies can also operate the orbiting science project indefinitely without astronauts on board. The space agency hasn't considered de-manning the ISS since the Columbia disaster in the 80s. Russian officials made worldwide headlines in July with a controversial comment about de-orbiting the space station in 2020 -- comments the space agency backpedaled from a day later. Autonomous operation is a very real possibility, however. Abandoning the space station, even for a short period, would be an unpleasant last resort for the world's five space agencies that have spent decades working on the project. Astronauts have been living aboard the space station since 2000, and the goal is to keep it going until 2020. Even if the space shuttles still were flying, space station crews still would need Soyuz-launched capsules to serve as lifeboats. The capsules are certified for no more than 6? months in space, thus the need to regularly rotate crews. Complicating matters is the need to land the capsules during daylight hours in Kazakhstan, resulting in weeks of blackout periods. Currently there are two Soyuz spacecrafts docked at the ISS. One is scheduled to bring back three astronauts in September and another is scheduled to bring the other three back in November. The spacecrafts have to come back because they were designed to spend only 200 days in space. Without any Soyuz or Progress modules docked, the only way to maneuver will be Zvezda's engines. Each use degrades the engines so they've been trying not to use them unless they have to. Nonetheless, the longer Zvezda has to spend doing every maneuver, the more likely it is to fail. The space station evacuation is one possibility following the failure of the unmanned Russian supply spacecraft just after its Aug. 24 launch — a surprise given the reliable track record of its workhorse Soyuz rocket. Only once before, after the 2003 space shuttle Columbia disaster, which destroyed that orbiter and its crew, did NASA consider evacuating the International Space Station — a process agency officials call "de-manning." In that case, though, managers decided to continue sending crews to the station on Soyuz rockets until the shuttle was ready to fly again. The largest risk of evacuating the station is the possibility that some major component could break, or a significant anomaly could occur, and no astronauts would be available on orbit to fix it. Russia now delayed the return to Earth of three astronauts. The RIA news agency said those members of the crew would now return to Earth around Sept. 16 instead of Sept. 8 and new crew members would blast off in late October or early November instead of on Sept. 22. The Soyuz has been extremely reliable over the decades; this was the first failure in 44 Russian supply hauls for the space station. Even with such a good track record, many in and outside NASA were concerned about retiring the space shuttles before a replacement was ready to fly astronauts. Late this year, commercial company SpaceX in California plans to launch its own rocket and supply ship to the space station, something NASA encourages. But a smaller, three-man crew on board the space station may require special training for the SpaceX docking. (FOXnews) How Would NASA and Russia Evacuate the International Space Station? The space station evacuation is one possibility following the failure of the unmanned Russian supply spacecraft just after its Aug. 24 launch — a surprise given the reliable track record of its workhorse Soyuz rocket. Only once before, after the 2003 space shuttle Columbia disaster, which destroyed that orbiter and its crew, did NASA consider evacuating the International Space Station — a process agency officials call "de-manning." In that case, though, managers decided to continue sending crews to the station on Soyuz rockets until the shuttle was ready to fly again. The largest risk of evacuating the station is the possibility that some major component could break, or a significant anomaly could occur, and no astronauts would be available on orbit to fix it.

In a new NASA-sponsored report, the National Research Council finds that the space agency needs a better plan for dealing with space junk before it gets out of hand and damages valuable spacecrafts. "*The current space environment is growing increasingly hazardous to spacecraft and astronauts,*" said Donald Kessler, chair of the committee that wrote the report and retired head of NASA's Orbital Debris Program Office. "*NASA needs to determine the best path forward for tackling the multifaceted problems caused by meteoroids and orbital debris that put human and robotic space operations at risk*." According to NASA, more than 500,000 pieces of debris, or "space junk," is tracked as its orbits the Earth, 20,000 pieces of which are larger than a softball. These pieces are not just meandering by, however; they can travel at speeds up to 17,500 miles per hour. Even something as small as a paint fleck can damage a satellite or spacecraft, NASA said. These objects can be anything from dead satellites to fragments that broke off other devices after a crash. The report finds that while NASA has used its space junk-related resources well, it can't really keep pace with the increasing hazards posed by this debris. NASA has several meteoroid and orbital debris programs, but at this point, there is no single management and budget structure that can efficiently coordinate all the efforts. NASA needs to come up with a strategic plan for how to handle the growing amount of space debris, including short-term and long-term goals, the report found. To keep track of new debris, meanwhile, the report recommends a more organized record of spacecraft problems, so officials can be kept abreast of what's out there. In June, the crew aboard the International Space Station got a scare when a piece of space debris got a bit too close for comfort, prompting them to take cover inside Russian space capsules. Lucky for them, the debris only came within 850 feet of the ISS before continuing on its way. In April, a piece of orbital debris making its way through the universe gave officials a similar scare. The debris, from the Chinese FENGYUN 1C satellite, could have affected the Expedition 27 spacecraft, which had launched from Earth the night before. (PCMag) Stochastic behavior of systems operating under changing environments has widely been studied. Dhillon , B.S. and Natesan, J. (1983) studied an outdoor power systems in fluctuating environment . Kan Cheng (1985) has studied reliability analysis of a system in a randomly changing environment. Jinhua Cao (1989) has studied a man machine system operating under changing environment subject to a Markov process with two states. The change in operating conditions viz. fluctuations of voltage, corrosive atmosphere, very low gravity etc. may make a system completely inoperative. Severe environmental conditions can make the actual mission duration longer than the ideal mission duration. In this paper we have taken **catastrophic and critical failure of ISS without astronauts.** When the main operative unit fails then cold standby system becomes operative. Critical failure of ISS without astronauts cannot occur simultaneously in both the units and after failure the unit undergoes repair facility of Type- II by ordinary repairman or Type III by multispecialty repairman in case of critical failure due to ISS without astronauts immediately. The repair is done on the basis of first fail first repaired.

Assumptions

1. $\lambda_{1,}\ \lambda_2$ are constant failure rates for catastrophic and critical failure of ISS without astronauts respectively. The CDF of repair time distribution of Type I, Type II and multispecialty repairmen Type-III are $G_1(t)$, $G_2(t)$ and $G_3(t)$.
2. The catastrophic and critical failure of ISS without astronauts is non-instantaneous and it cannot come simultaneously in both the units.
3. The repair starts immediately after critical failure of ISS without astronauts and works on the principle of first fail first repaired basis.
4. The repair facility does no damage to the units and after repair units are as good as new.
5. The switches are perfect and instantaneous.
6. All random variables are mutually independent.
7. When both the units fail, we give priority to operative unit for repair.
8. Repairs are perfect and failure of a unit is detected immediately and perfectly.
9. The system is down when both the units are non-operative.

Notations

λ_1 , λ_2 - failure rates for catastrophic and critical failure of ISS without astronauts respectively.

$G_1(t)$, $G_2(t)$, $G_3(t)$ – repair time distribution Type –I, Type-II, Type III due to risk of catastrophic and critical failure of ISS without astronauts, repair by the multispecialty repairman respectively.

p, q - probability of critical failure of ISS without astronauts respectively such that p+ q=1

$M_i(t)$ System having started from state i is up at time t without visiting any other regenerative state

$A_i(t)$ state is up state at instant t

$R_i(t)$ System having started from state i is busy for repair at time t without visiting any other regenerative state.

$B_i(t)$ the server is busy for repair at time t.

$H_i(t)$ Expected number of visits by the server for repairing given that the system initially starts from regenerative state i

SYMBOLS FOR STATES OF THE SYSTEM

Superscripts: O, CS, CRF, CATF, Operative, Cold Standby, critical failure, catastrophic of ISS without astronauts respectively

Subscripts: ncrf, crf, catf, ur, wr, uR

No critical failure of ISS with astronauts, critical failure of ISS without astronauts, catastrophic failure of ISS without astronauts, under repair, waiting for repair, under repair continued from previous state respectively

Up States: 0, 1, 2, 3, 8, 9;

Down states: 4, 5, 6, 7

Regeneration point: 0, 1, 2, 3, 8, 9

States of the System

0(O_{ncrf}, CS_{ncrf})
One unit is operative and the other unit is cold standby and there is no critical failure of ISS with astronauts in both the units.
1($CRF_{crf, urI}$, O_{ncrf})
The operating unit fails due to critical failure of ISS without astronauts and is under repair immediately of Type- I and standby unit starts operating with no critical failure of ISS with astronauts.
2($CATF_{catf, urII}$, O_{ncrf})
The operative unit fails due to catastrophic failure of ISS without astronauts and undergoes repair of type II and the standby unit becomes operative with no critical failure of ISS with astronauts.
3($CATF_{catf, urIII}$, O_{ncrf})
The first unit fails due to catastrophic failure of ISS without astronauts and under Type-III multispecialty repairman and the other unit is operative with no critical failure of ISS with astronauts.
4($CRF_{crf,uR1}$, $CRF_{crf,wrI}$)
The unit failed due to CRF resulting from critical failure of ISS without astronauts is under repair of Type- I continued from state 1and the other unit failed due to CRF resulting from critical failure of ISS without astronauts is waiting for repair of Type-I.
5($CRF_{crf,uR1}$, $CATF_{catf, wrII}$)
The unit failed due to CRF resulting from critical failure of ISS without astronauts is under repair of Type- I continued from state 1and the other unit fails due to catastrophic failure of ISS without astronauts is waiting for repair of Type- II.
6($CATF_{catf, uRII}$, $CRF_{crf ,wrI}$)
The operative unit fails due to catastrophic failure of ISS without astronauts and under repair continues from state 2 of Type –II and the other unit is failed due to CRF resulting from critical failure of ISS without astronauts and under Type-I
7($CATF_{catf ,uRII}$, $CRF_{crf,wrII}$)
The one unit fails due to catastrophic failure of ISS without astronauts is continued to be under repair of Type II and the other unit failed due to CRF resulting from critical failure of ISS without astronauts is waiting for repair of Type-II
8($CRF_{crf,urIII}$, $CATF_{catf, wrII}$)
The one unit critical failure of ISS without astronauts is under multispecialty repair of Type-III and the other unit is failed due to catastrophic failure of ISS without astronauts is waiting for repair of Type-II.
9($CRF_{crf,urIII}$, $CATF_{catf, wrI}$)
The one unit critical failure of ISS without astronauts is under multispecialty repair of Type-III and the other unit is failed due to catatrophic failure of ISS without astronauts is waiting for repair of Type-I

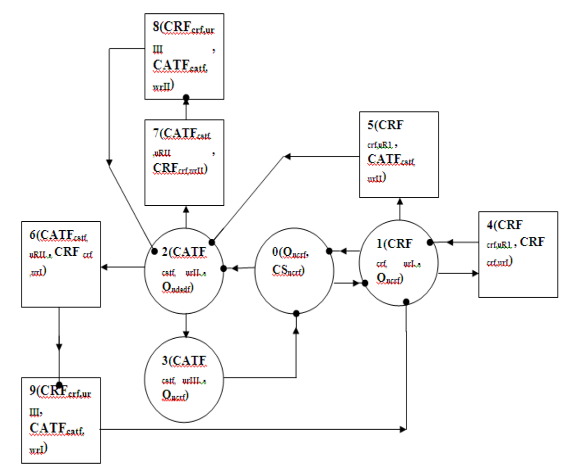


Figure 1: The State Transition Diagram

◯ Up-State ☐ Down-State ● Regeneration point

TRANSITION PROBABILITIES

Simple probabilistic considerations yield the following expressions:
$p_{01} = \lambda_1 / \lambda_1 + \lambda_2$, $p_{02} = \lambda_2 / \lambda_1 + \lambda_2$, $p_{10} = pG_1^*(\lambda_1)+q\, G_2^*(\lambda_2)$,
$p_{14} = p- pG_1^*(\lambda_1) = p_{11}^{(4)}$, $p_{15} = q- q\, G_1^*(\lambda_2) = p_{12}^{(5)}$,

$p_{23} = pG_2^*(\lambda_1)+q\,G_2^*(\lambda_2)$, $p_{26} = p- pG_2^*(\lambda_1) = p_{29}^{(6)}$,
$p_{27} = q- qG_2^*(\lambda_2) = p_{28}^{(7)}$, $p_{30} = p_{82} = p_{91} = 1$ (1)

We can easily verify that
$p_{01} + p_{02} = 1$, $p_{10} + p_{14}\ (=p_{11}^{(4)}) + p_{15}\ (=p_{12}^{(5)}) = 1$,
$p_{23} + p_{26}\ (=p_{29}^{(6)}) + p_{27}\ (=p_{28}^{(7)}) = 1$ (2)

And mean sojourn time is
$\mu_0 = E(T) = \int_0^\infty P[T > t]dt$

Mean Time To System Failure

$Ø_0(t) = Q_{01}(t)[s]\ Ø_1(t) + Q_{02}(t)[s]\ Ø_2(t)$
$Ø_1(t) = Q_{10}\ (t)[s]\ Ø_0(t) + Q_{14}(t) + Q_{15}(t)$
$Ø_2(t) = Q_{23}\ (t)[s]\ Ø_3(t) + Q_{26}(t) + Q_{27}(t)$
$Ø_3(t) = Q_{30}(t)[s]\ Ø_0(t)$ (3-6)

We can regard the failed state as absorbing
Taking Laplace-Stiljes transform of eq. (3-6) and solving for
$ø_0^*(s) = N_1(s) / D_1(s)$ (6)

where
$N_1(s) = Q_{01}^*[\ Q_{14}^*(s) + Q_{15}^*(s)\ ] + Q_{02}^*[\ Q_{26}^*(s) + Q_{27}^*(s)\ ]$
$D_1(s) = 1 - Q_{01}^*\ Q_{10}^* - Q_{02}^*\ Q_{23}^*\ Q_{30}^*$

Making use of relations (1) and (2) it can be shown that $ø_0^*(0) = 1$, which implies that $ø_0(t)$ is a proper distribution.

$$\text{MTSF} = E[T] = \left. \frac{d}{ds}\, ø_0^{*}(s) \right|_{s=0} = (D_1'(0) - N_1'(0)) / D_1(0)$$

$= (\ \mu_0 + p_{01}\ \mu_1 + p_{02}\ \mu_2\ ) / (1 - p_{01}\ p_{10} - p_{02}\ p_{23}\ )$

where
$\mu_0 = \mu_{01} + \mu_{02}$
$\mu_1 = \mu_{10} + \mu_{11}^{(4)} + \mu_{12}^{(5)}$,
$\mu_2 = \mu_{23} + \mu_{28}^{(7)} + \mu_{29}^{(6)}$

AVAILABILITY ANALYSIS

Let $M_i(t)$ be the probability of the system having started from state i is up at time t without making any other regenerative state. By probabilistic arguments, we have
$M_0(t) = e^{-\lambda_1 t} e^{-\lambda_2 t}$, $M_1(t) = p\,G_1(t)\, e^{-\lambda_1 t}$
$M_2(t) = q\,\overline{G_2(t)}$, $M_3(t) = G_3(t)$

The point wise availability $A_i(t)$ have the following recursive relations
$A_0(t) = M_0(t) + q_{01}(t)[c]A_1(t) + q_{02}(t)[c]A_2(t)$
$A_1(t) = M_1(t) + q_{10}(t)[c]A_0(t) + q_{12}^{(5)}(t)[c]A_2(t)+ q_{11}^{(4)}(t)[c]A_1(t)$,
$A_2(t) = M_2(t) + q_{23}(t)[c]A_3(t) + q_{28}^{(7)}(t)[c]\ A_8(t) + q_{29}^{(6)}(t)]\ [c]A_9(t)$
$A_3(t) = M_3(t) + q_{30}(t)[c]A_0(t)$
$A_8(t) = q_{82}(t)[c]A_2(t)$
$A_9(t) = q_{91}(t)[c]A_1(t)$ (7-11)

Taking Laplace Transform of eq. (7-11) and solving for $\hat{A}_0(s)$

$\hat{A}_0(s) = N_2(s) / D_2(s)$ (12)

where
$N_2(s) = \hat{M}_0 [\{1 - \hat{q}_{11}^{(4)}\}\{1- \hat{q}_{28}^{(7} \hat{q}_{82}\}- \hat{q}_{12}^{(5)} \hat{q}_{29}^{(6)} \hat{q}_{91}] + \hat{q}_{01}[\ \hat{M}_1\{1 - \hat{q}_{28}^{(7)} \hat{q}_{82}\} + \hat{q}_{12}^{(5)}\ \hat{q}_{23}\ \hat{M}_3] + \hat{q}_{02}[\{\hat{q}_{23}\ \hat{M}_3 + \hat{M}_2\}\{1 - \hat{q}_{11}^{(4)}\} + \hat{q}_{29}^{(6)})\ \hat{q}_{91}\ \hat{M}_1]$

$D_2(s) = \{1 - \hat{q}_{11}^{(4)}\}\{1- \hat{q}_{28}^{(7} \hat{q}_{82}\}- \hat{q}_{12}^{(5)} \hat{q}_{29}^{(6)} \hat{q}_{91} - \hat{q}_{01}[\ \hat{q}_{10}\ \{1 - \hat{q}_{28}^{(7)} \hat{q}_{82}\} + \hat{q}_{12}^{(5)}\ \hat{q}_{23}\ ] - \hat{q}_{02}[\{\hat{q}_{23}\ \hat{q}_{30}\ \{1 - \hat{q}_{11}^{(4)}\} + \hat{q}_{29}^{(6)}\ \hat{q}_{91}\ \hat{q}_{10}]$

(Omitting the arguments s for brevity)

The steady state availability

$A_0 = \lim_{t\to\infty}[A_0(t)] = \lim_{s\to 0}[s\,\hat{A}_0(s)] = \lim_{s\to 0}\frac{s\,N_2(s)}{D_2(s)}$

Using L' Hospitals rule, we get

$A_0 = \lim_{s\to 0}\frac{N_2(s)+s\,N_2'(s)}{D_2'(s)} = \frac{N_2(0)}{D_2'(0)}$ (13)

The expected up time of the system in (0, t] is

$\lambda_u(t) = \int_0^\infty A_0(z)\,dz$ So that $\bar{\lambda}_u(s) = \frac{\bar{A}_0(s)}{s} = \frac{N_2(S)}{SD_2(S)}$ (14)

The expected down time of the system in (0, t] is

$\lambda_d(t) = t - \lambda_u(t)$ So that $\bar{\lambda}_d(s) = \frac{1}{s^2} - \bar{\lambda}_u(s)$ (15)

The expected busy period of the server when there is catastrophic and critical failure of ISS without astronauts in (0, t]

$R_0(t) = q_{01}(t)[c]R_1(t) + q_{02}(t)[c]R_2(t)$

$R_1(t) = S_1(t) + q_{10}(t)[c]R_0(t) + q_{12}^{(5)}(t)[c]\,R_2(t) + q_{11}^{(4)}(t)[c]R_1(t)$

$R_2(t) = S_2(t) + q_{23}(t)[c]R_3(t) + q_{28}^{(7)}(t)\,R_8(t) + q_{29}^{(6)}(t)][c]R_9(t)$

$R_3(t) = S_3(t) + q_{30}(t)[c]R_0(t)$

$R_8(t) = S_8(t) + q_{82}(t)[c]R_2(t)$

$R_9(t) = S_9(t) + q_{91}(t)[c]R_1(t)$ (16-21)

where

$S_1(t) = p\,G_1\overline{(t)}\,e^{-\lambda_1 t}$, $S_1(t) = q\,G_2(t)\,e^{-\lambda_2 t}\,\overline{S_3}(t) = S_8(t) = S_9(t) = \overline{G_3(t)}$ (22)

Taking Laplace Transform of eq. (16-21) and solving for $\bar{R}_0(s)$

$\bar{R}_0(s) = N_3(s) / D_2(s)$ (23)

where

$N_3(s) = \hat{q}_{01}[\,\hat{S}_1(1 - \hat{q}_{28}^{(7)}\,\hat{q}_{82}) + \hat{q}_{12}^{(5)}[\,\hat{S}_2 + \hat{q}_{23}\hat{S}3 + \hat{q}_{28}^{(7)}(\,\hat{S}8 +$
$\hat{q}_{29}^{(6)}\,\hat{S}9)] + \hat{q}_{02}[\,(\,\hat{S}_2 + \hat{q}_{23}\hat{S}_3 + \hat{q}_{28}^{(7)}\,\hat{S}_8 + \hat{S}_9\,\hat{q}_{29}^{(6)})(1 - \hat{q}_{11}^{(4)}) + \hat{S}_1\,\hat{q}_{29}^{(6)}\,\hat{q}_{91}]$

and $D_2(s)$ is already defined.

(Omitting the arguments s for brevity)

In the long run, $R_0 = \frac{N_3(0)}{D_2'(0)}$ (24)

The expected period of the system under failure due to catatrophic and critical failure of ISS without astronauts

$\lambda_{rv}(t) = \int_0^\infty R_0(z)\,dz$ So that $\bar{\lambda}_{rv}(s) = \frac{\bar{R}_0(s)}{s}$

The expected number of visits by the repairman Type-I or Type-II for repairing the identical units in (0, t]

$H_0(t) = Q_{01}(t)[s][1 + H_1(t)] + Q_{02}(t)[s][1 + H_2(t)]$

$H_1(t) = Q_{10}(t)[s]H_0(t)] + Q_{12}^{(5)}(t)[s]\,H_8(t) + Q_{11}^{(4)}(t)]\,[s]H_1(t)$,

$H_2(t) = Q_{23}(t)[s]H_3(t) + Q_{28}^{(7)}(t)\,[s]\,H_8(t) + Q_{29}^{(6)}(t)]\,[c]H_9(t)$

$H_3(t) = Q_{30}(t)[s]H_0(t)$

$H_8(t) = Q_{82}(t)[s]H_2(t)$

$H_9(t) = Q_{91}(t)[s]H_1(t)$ (25-30)

Taking Laplace Transform of eq. (25-30) and solving for $H_0^*(s)$

$H_0^*(s) = N_4(s) / D_3(s)$ (31)

$N_4(s) = \{\,Q_{01}^* + Q_{02}^*\}[\,\{\,1 - Q_{28}^{(7)*}\,Q_{82}^*\,\} - Q_{12}^{(5)*}\,Q_{29}^{(6)*}\,Q_{91}^*\,]$

And

$D_3(s) = \{1 - Q_{11}^{(4)*}\}\,\{\,1 - Q_{28}^{(7)*}\,Q_{82}^*\} - Q_{12}^{(5)*}\,Q_{29}^{(6)*}\,Q_{91}^* - Q_{01}^*[\,Q_{10}^*\{\,1 - Q_{28}^{(7)*}\,Q_{82}^*\,\} + Q_{12}^{(5)*}\,Q_{23}^*\,Q_{30}^*] - Q_{02}^*\,Q_{30}^*\{1 - Q_{11}^{(4)*}\} + Q_{29}^{(6)*}\,Q_{91}^*\,Q_{10}^*]$

(Omitting the arguments s for brevity)

In the long run,

$H_0 = N_4(0) / D_3'(0)$ (32)

where
$N_4(0) = \{1 - p_{11}^{(4)}\} \{ 1- p_{28}^{(7)} \} - p_{12}^{(5)} p_{29}^{(6)}$

The expected number of visits by the multispecialty repairman Type-III for repairing the identical units in (0, t]

$W_0(t) = Q_{01}(t)[s][1+ W_1(t)] + Q_{02}(t)[s][1+ W_2(t)]$
$W_1(t) = Q_{10}(t)[s]W_0(t)] + Q_{12}^{(5)}(t)[s] W_8(t) + Q_{11}^{(4)}(t)] [s]W_1(t)$,
$W_2(t) = Q_{23}(t)[s]W_3(t) + Q_{28}^{(7)}(t) [s] W_8(t) +Q_{29}^{(6)}(t)] [c]W_9(t)$
$W_3(t) = Q_{30}(t)[s]W_0(t)$
$W_8(t) = Q_{82}(t)[s]W_2(t)$
$W_9(t) = Q_{91}(t)[s]W_1(t)$ (33-38)

Taking Laplace Transform of eq. (33-38) and solving for $H_0^*(s)$

$H_0^*(s) = N_5(s) / D_3(s)$ (39)

$N_5(s) = Q_{01}^* Q_{12}^{(5)*} [Q_{23}^* Q_{30}^* + Q_{28}^{(5)*} Q_{82}^* + Q_{29}^{(6)*} Q_{91}^*] + Q_{02}^* [Q_{23}^* Q_{30}^* + Q_{28}^{(5)*} Q_{82}^* + Q_{29}^{(6)*} Q_{91}^*]\{1 - Q_{11}^{(4)*}\}]$
(Omitting the arguments s for brevity)
In the long run,
$W_0 = N_5(0) / D_3'(0)$ (40)
Where
$N_5(0) = p_{01} p_{12}^{(5)} + p_{02}\{1 - p_{11}^{(4)}\}$

BENEFIT- FUNCTION ANALYSIS

The Benefit-Function analysis of the system considering mean up-time, expected busy period of the system under failure due to catastrophic and critical failure of ISS without astronauts, expected number of visits by the repairman for unit failure. The expected total Benefit-Function incurred in (0, t] is
C (t) = Expected total revenue in (0, t]

- expected busy period of the system under failure due to catastrophic and critical failure of ISS without astronauts for repairing the units in (0,t]
- expected number of visits by the repairman Type- I or Type- II for repairing of identical the units in (0,t]
- expected number of visits by the multispecialty repairman Type- III for repairing of identical the units in (0,t]

The expected total cost per unit time in steady state is

$C = \lim_{t\to\infty}(C(t)/t) = \lim_{s\to 0}(s^2 C(s))$
$= K_1A_0 - K_2R_0 - K_3H_0 - K_4W_0$

where
K_1: Revenue per unit up-time,
K_2: Cost per unit time for which the system is busy under repairing,
K_3: Cost per visit by the repairman type- I or type- II for units repair,
K_4: Cost per visit by the multispecialty repairman Type- III for units repair

CONCLUSION

After studying the system, we have analyzed graphically that when the failure rate catastrophic and critical failure of ISS without astronaut's increases, the MTSF, steady state availability decreases and the Profit-function decreased as the failure increases.

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