Research Article

On the equivariant estimation of an uniform location model under general progressive type II right censored sample

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Abstract We consider the problem of obtaining Minimum Risk Equivariant (MRE) estimator for the parameter of the uniform model in three situations by assuming that a general progressive Type II right censored sample, these generalize the results of Chandrasekar *et al.* (2002) for progressive Type II right censored sample. The paper is organized as follows: Section 2 deals with the problem of equivariant estimation under Squared error loss function. In Section 3, the problem of equivariant estimation under Absolute error loss function discussed. Finally, we dealt with the problem of equivariant estimation of the parameter under Linex loss function (Varian, 1975) in Section 4.

Keywords: Equivariant Estimation, General Progressive Censored Sample, Linex Loss function, MRE, Uniform Location Model.

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INTRODUCTION

Progressive Type II right censored sampling is an important method of acquiring data in life-testing studies. Aggarwala and Balakrishnan (1998) points out that the scheme of progressive censoring enables us to use live units, removed early, in other tests. Balakrishnan and Sandhu (1996), by assuming a general progressive Type II right censored sample, derived the BLUE's for the parameters of one-and two-parameter exponential distributions. For the later, they also derived MLE's and shown that they are simply the BLUE's, adjusted for their bias. Let us consider the following general progressive Type II right censoring scheme (Balakrishnan and Sandhu, 1996) : Suppose N randomly selected units were placed on a life test; the failure times of the first r units to fail were not observed ; at the time of the $(r+1)^{th}$ failure, R_{r+1} number of surviving units are withdrawn from the test randomly, and so on; at the time of the $(r+i)^{th}$ failure, R_{r+i} number of surviving units are randomly withdrawn from the test ; finally, at the time of the n-th failure, the remaining $R_n = N - n - R_{r+1} - R_{r+2} - \ldots - R_{n-1}$ are withdrawn from the test. Suppose $X_{r+1:N} \leq X_{r+2:N} \leq \ldots. \leq X_{n:N}$ are the life-times of the completely observed units to fail, and $R_{r+1}, R_{r+2}, \ldots, R_n$ are the number of units withdrawn from the test at these failure

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times, respectively. It follows that $N = n + \sum_{i=r+1}^{n} Ri$. If the failure times are from a continuous population with the pdf and the distribution function F, then the joint pdf of ($X_{r+1:N}$, $X_{r+2:N}$, ... , $X_{n:N}$) is given by g_θ (x_{r+1}, ..., x_n)

$$= \mathbf{c}\ \{F_\theta (x_{r+1})\}^r \prod_{i=r+1}^{n} f_\theta(x_i)\{1 - F_\theta(x_i)\}^{R_i}, \tag{1.1}$$

where $\mathbf{c} = \binom{N}{r}(N-r)\prod_{j=r+2}^{n}\left(N - \sum_{i=r+1}^{j-1} R_i - j + 1\right)$.

Location Model

The pdf is taken to be

$$f_\xi(x) = \begin{cases} 1, & \xi \le x \le \xi + 1\ ;\ \xi \in R \\ 0, & \text{otherwise} \end{cases}.$$

and from (1.1) we get

$$g_\xi(x_{r+1},\ldots,x_n) = c(x_{x+1} - \xi)\prod_{i=1}^{n}(1 - x_i + \xi)^{R_i}, \quad \xi \le x_{r+1} \le x_n \le \xi + 1; \xi \in R. \tag{1.2}$$

Thus the distribution of $(X_{r+1:N},\ldots,X_{n:N})$ belongs to a location family with the location parameter ξ. We are interested in deriving MRE estimator of ξ by considering three loss functions. Following Lehmann and Casella (1998), the MRE estimator of ξ is given by $\delta^*(\mathbf{X}) = \delta_0(\mathbf{X}) - v^*(\mathbf{Y})$, where δ_0 is a location equivariant estimator, v(**y**) = v^*(**y**) minimizes E_0[$\rho\{\delta_0(\mathbf{x})$-v(**y**)}|**y**] and E_0 denotes E_ξ when $\xi = 0$. Here $Y_i = X_{i:N} - X_{r+1:N}, i = r+2,\ldots,n$, $\mathbf{Y} = (Y_{r+2},\ldots,Y_n)$ and ρ is an invariant loss function.

SQUARED ERROR LOSS

If the loss is squared error, then $v^* = E_0(\delta_0 \mid \mathbf{y})$. Take $\delta_0(\mathbf{X}) = \frac{X_{r+1:N} + X_{n:N}}{2}$. Clearly δ_0 is an equivariant estimator but not complete sufficient. Since we are interested in the evaluation of conditional distribution under $\xi = 0$, we take $\xi = 0$ in (1.2). In order to find v^*, consider the transformation $Y_{r+1} = (X_{r+1:N} + X_{n:N})/2$ and $Y_i = X_{i:N} - X_{r+1:N}, i = r+2,\ldots,n.$ Then $X_{r+1:N} = Y_{r+1} - Y_n/2$ and $X_{i:N} = Y_i + Y_{r+1} - Y_n/2, i = r+2,\ldots,n$ and the Jacobian of the transformation is J=1. Thus the joint pdf of $(Y_{r+1}, Y_{r+2},\ldots,Y_n)$ is given by

$$h(y_{r+1},\ldots,y_n) = c(y_{r+1} - y_n/2)^r\{1 - (y_{r+1} - y_n/2)\}^{R_{r+1}}$$

$$\pi_{i=r+2}^{n}\{1 - (y_i + y_{r+1} - y_n/2)\}^{R_i}, \qquad 0 < y_{r+2} < y_{r+3} < \ldots < y_n,\ y_n/2 < y_{r+1} < 1 - y_n/2,$$

$$0 < y_n < 1.$$

Also, the joint pdf of $(Y_{r+2},\ldots,Y_n)$ is given by

$$h_1(y_{r+2},...,y_n)$$
$$= c\int_{y_n/2}^{1-y_n/2}(y_{r+1}-y_n/2)^r\{1-(y_{r+1}-y_n/2)\}^{R_{r+1}}\times$$
$$\underset{i=r+2}{\overset{n}{\pi}}\{1-(y_i+y_{r+1}-y_n/2)\}^{R_i}\,dy_{r+1},\qquad 0<y_{r+2}<...<y_n<1.$$

Thus the conditional pdf of $\delta_0 = Y_{r+1}$ given $(Y_{r+2},...,y_n)$ is given by

$$h_2(y_{r+1}\mid y_{r+2},...,y_n)$$
$$=[(y_{r+1}-y_n/2)^r\{1-(y_{r+1}-y_n/2)\}^{R_{r+1}}\times$$
$$\underset{i=r+2}{\overset{n}{\pi}}\{1-(y_i+y_{r+1}-y_n/2)\}^{R_i}]/$$
$$[\int_{y_n/2}^{1-y_n/2}(y_{r+1}-y_n/2)^r\{1-(y_{r+1}-y_n/2)\}^{R_{r+1}}$$
$$\times\underset{i=r+2}{\overset{n}{\pi}}\{1-(y_i+y_{r+1}-y_n/2)\}^{R_i}\,dy_{r+1}]$$
$$y_n/2<y_{r+1}<1-y_n/2. \tag{2.1}$$

Now

$$v^* = E_0(\delta_0\mid\mathbf{y})$$
$$=[\int_{y_n/2}^{1-y_n/2}y_{r+1}(y_{r+1}-y_n/2)^r\{1-(y_{r+1}-y_n/2)\}^{R_{r+1}}\times$$
$$\underset{i=r+2}{\overset{n}{\pi}}\{1-(y_i+y_{r+1}-y_n/2)\}^{R_i}\,dy_{r+1}]/$$
$$[\int_{y_n/2}^{1-y_n/2}(y_{r+1}-y_n/2)^r\{1-(y_{r+1}-y_n/2)\}^{R_{r+1}}\times$$
$$\underset{i=r+2}{\overset{n}{\pi}}\{1-(y_i+y_{r+1}-y_n/2)\}^{R_i}\,dy_{r+1}]\qquad =$$

in view of (2.1).

$$v^* = [\int_{(X_{n:N}-X_{r+1:N})/2}^{1-(X_{n:N}-X_{r+1:N})/2}y_{r+1}\{y_{r+1}-(X_{n:N}-X_{r+1:N})/2\}^r\times$$
$$\underset{i=r+1}{\overset{n}{\pi}}\{1-X_{i:N}-y_{r+1}+(X_{r+1:N}+X_{n:N})/2\}^{R_i}\,dy_{r+1}]/$$
$$[\int_{(X_{n:N}-X_{r+1:N})/2}^{1-(X_{n:N}-X_{r+1:N})/2}\{y_{r+1}-(X_{n:N}-X_{r+1:N})/2\}^r\times$$

So, $$\underset{i=r+1}{\overset{n}{\pi}}\{1-X_{i:N}-y_{r+1}+(X_{r+1:N}+X_{n:N})/2\}^{R_i}\,dy_{r+1}] \tag{2.2}$$

Therefore the MRE estimator of ξ is given by

$$\delta^*(\mathbf{X}) = \delta_0(\mathbf{X}) - E_0(\delta_0 \mid \mathbf{y}) = \frac{X_{r+1:N} + X_{n:N}}{2} - v^*, \tag{2.3}$$

where v^* is as given in (2.2).

Remark 2.1: when the loss is squared error, the Pittman estimator of ξ is $\delta^*(\mathbf{X}) == \frac{X_{r+1:N} + X_{n:N}}{2} - v^*$, which coincides with (2.3).

Remark2.2: If r=0 and $R_i = r_i$ then the above estimator in (2.3) reduces to the one in Uniform Location model under Progressive Type-II censored sample case (Leo Alexander , 2000).

ABSOLUTE ERROR LOSS FUNCTION

If the loss is absolute error, then the MRE estimator of ξ is $\delta^*(\mathbf{X}) = \frac{X_{r+1:N} + X_{n:N}}{2} - v_0$,

where v_0 is the median of the distribution with pdf given in (2.1).

LINEX LOSS FUNCTION

Consider the location invariant Linex loss function (Varian, 1975),

$L(\xi ; \delta) = e^{a(\delta - \xi)} - a(\delta - \xi) - 1, a \in \mathbf{R} - \{0\}$. Take $\delta_0(\mathbf{X}) = (X_{r+1:N} + X_{n:N})/2$.

In order to find $v^* = E_0[\rho\{\delta_0(\mathbf{x}) - v\} \mid \mathbf{y}]$, consider

$$R(\delta \mid \mathbf{y}) = E_0\{(e^{a(\delta_0 - v))} - a(\delta_0 - v) - 1) \mid \mathbf{y}\} \qquad = e^{-av} E_0(e^{a\delta_0} \mid \mathbf{y}) - aE_0(\delta_0 \mid \mathbf{y}) + av - 1$$

$$= e^{-av} E_0(e^{a\delta_0} \mid \mathbf{y}) - av^* + av - 1,$$

where v^* is given by (2.2). Define $M(a;\mathbf{y}) = E_0(e^{a\delta_0} \mid \mathbf{y})$.

Now, $\frac{dR}{dv} = e^{-av}(-a)M(a;\mathbf{y}) + a$ and $\frac{d^2R}{dv^2} = e^{-av} a^2 M(a;\mathbf{y})$.

Note that $\frac{d^2R}{dv^2} \geq 0 \quad \forall$ v.

Thus $\frac{dR}{dv} = 0 \Rightarrow e^{-av} M(a;\mathbf{y}) = 1$

$$\Rightarrow v^* = \log M(a;\mathbf{y})/a$$

$$= 1/a\{cgf \text{ of } (\delta_0 \mid \mathbf{y}) \text{ at a}\}.$$

Therefore the MRE estimator of ξ is given by

$$\delta^*(\mathbf{X}) = \frac{X_{r+1:N} + X_{n:N}}{2} - \frac{1}{a}\{cgf \text{ of } (\delta_0 \mid \mathbf{y}) \text{ at a}\}.$$

Remark4.1: If r = 0 then the above estimator reduces to

$\delta^*(\mathbf{X}) = \frac{X_{1:N} + X_{n:N}}{2} - \frac{1}{a}\{cgf$ of $(\delta_0 \mid \mathbf{y})$ at a}, which is the one obtained under Linex Loss function of Progressive Type-II Right censored sample case. (Leo Alexander, 2000)

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