Research Article

Designing of bayesian skip lot sampling plan with multiple deferred sampling plan as reference plan

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Abstract In this paper, a optimal designing methodology is proposed to determine the parameters for skip-lot sampling plan of type SkSP-2 with multiple deferred sampling plan as reference plan. The approach of two points on the operating characteristic curve is used to find the optimal parameters for the proposed plan. Producer's risk and Consumer's risk has been minimized by minimizing the tangent angle passing through (AQL, 1-α) and (LQL, β). Designing methodologies are provided to the solution of sampling procedures. The plan parameters are determined to displays the discriminatory power of the sampling plan and to satisfying both producer and consumer risks simultaneously at the certain ponits on acceptable and limiting quality levels.
Keywords: Skip Lot Sampling plan, Multiple Deferred Sampling Plan, Bayesian Analysis, Minimum Angle Method, Gamma-Poisson Distribution.

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INTRODUCTION

Acceptance sampling plan is used to either accept or reject a lot based on the sampling inspection. The primary objective of sampling inspection is to reduce the cost of inspection, at the same time assuring the customer full satisfaction at adequate level of quality products being inspected. Inspection of raw materials, semi-finished products, or a finished product is an important part of quality assurance. When inspection is done for the purpose of acceptance or rejection of a product, and it is based on adherence to a standard type of inspection procedure employed, such procedure is usually called acceptance sampling. Sampling is widely used in government sector and industry for controlling the quality of shipment of components, supplies and final products. In this paper a new procedure is proposed towards minimizing risks with skip-lot sampling plan of type-2 with multiple deferred sampling plan as reference plan through bayesian methodology. According to acceptance sampling methodology which is one of the statistical tool used to make decisions concerning whether or not a lot of products should be relaesed for consumer use. An acceptance sampling plan is a statement regarding that the required sample size of product inspection is associated with acceptance or rejection criteria for specification of individual lots. The results of measuring performance of sampling plan is based on the operating characteristic curve which implies the both risks such as producers and consumers risk.

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Any time substantial evidence of process variation exists, proper assessment of sampling risks dictates the use of Bayesian Acceptance Sampling (BAS). Suppose a product is being supplied in a series of lots, due to random fluctuations these lots will differ in quality even though the process is stable and control. These fluctuations can be separated into within lot (sampling) variation of individual units and between lot (sampling and process) variation. If these two sources of variations are equal, which implies zero process variation and dispersion of the process average is zero, and each lot can be considered a random sample drawn from process with constant probability towards producing non-conforming unit. This is the existing basis for conventional acceptance sampling. While studies have indicated that Bayesian Schemes may be quite robust to errors in prior distribution and loss function, they nevertheless assume the prior discussion to be stationary in long-term sense (i.e. Process in control). Bayesian plans are quite application-specific, requiring extension information, updating proper application and variable sampling, they are applied to one characteristic at a time. The Skip lot Sampling Plan initially designed by Dodge (1955), which is based on the principle of Continuous Sampling Plan (CSP-1). The CSP-1 deals with series of units whereas the SkSP-1 type deals with series of lots. It is proposed that when a quality is good or rather accepted, then only a fraction of the submitted lot requires to be inspected. On the other hand, when a defective unit is found during sampling phase, then it becomes necessary to revert to 100% inspection again.The extended type concept of CSP-1 is used to individual lots, under the conditions were a single determination on analysis is made for each of the specified quality characteristic subject to the inspection. Single determination on analysis of means the ascertainment of acceptability or non-acceptability of lots. The next procedure is to examine the case where each lot to be inspected is sampled according to some given inspected plan. Perry (1970) has developed a system of sampling inspection plan known as SkSP-2. This plan involves inspection of only some fraction ‘f’ of the submitted lots when quality of submitted product is good as demonstrated by the quality of the product. These plans are applicable to products produced or furnished in successive lots or batches. This study is concerned with the development and evaluation of a system of skip-lot sampling plans for lot inspection designated type SkSP-2, where provision is made for skipping inspection of some of the lots when the quality of the submitted product is good. This system of plans uses a given lot inspection, known as “reference plan” and calls for inspecting every lot according to the reference plan, but for inspecting only a fraction “f” of the lots when “i” consecutive lots have been accepted and returning to every lot inspection when a lot is rejected. The values of “f” and “i” called the skipping parameters together with the reference plan completely characterize a type SkSP-2 plan. Parker and Kessler (1981) have modified the existing SkSP-2 plan under which at least one unit is always sampled from a lot. The expression for the probability of acceptance is derived and compared with standard skip lot plans. Calvin (1984) has distributed procedures and tables for employing Bayesian sampling plans. A set of tables presented by Oliver and Springer (1972) which is based on assumption of Beta prior distribution with specific posterior risk. Hald (1981) has also provided an excellent comparison of classical and Bayesian theory and methodology for attributes acceptance sampling. Case and Keats (1982) examined the relationship between defectives in the sample and defectives in the remaining lot for each of the five prior distributions. Wetherill & Chiu (1975) also point out that the economic schemes based on Bayesian theory is more precise and scientific, leaving much less to judgement than those based on classical theory. Peach and Littauer (1946), Grubbs (1949) and Cameron (1952) have given the procedures for selection of single sampling plan based on the unity (np) value, while its OC curves passes through two points namely (p_1, 1-α) and (p_2, β). Suresh and Srivenkataramana (1996) have designed procedure for the selection of single sampling plan using producer and consumer quality levels. Suresh (1993) has given procedures for the selection of Skip-lot Sapling Plan of type SkSP-2 with reference plans SSP(c=0), SSP (c≠0) and DSP (0, 1) using consumer and producer quality levels. Singaravelu (1993) has developed the angle between two points on the OC curve for single and double attribute sampling plans. Soundararajan and Lilly Christina (1997) have studied minimum angle for single sampling by variables plan with known standard deviation. Suresh and Latha (2001) have studied average probability of acceptance function for single sampling plan with Gamma prior distribution. Formula of inflection point and tangent at the inflection point are also derived. A selection procedure for Bayesian single sampling attributes plan when the acceptance number is fixed and when the sample size is fixed. Jayalakshmi (2009) has presented a procedure for designing skip lot sampling plan of type SkSP-2 with STDS as reference plan involving minimum angle method between the lines formed by the points (AQL, 1-α), (AQL, β) and (AQL, 1-α), (LQL, β). Kalaichelvi (2012) has studied the selection of skip-lot sampling plans for given p_1, p_2, α, and β involving producer and consumer risks with various reference plans. Suresh and Kavithamani (2013) have proposed the minimum angle approach between two points on the OC curve using the attribute sampling plan of SkSP-V with MRGS plan as reference plan.

2. OPERATING PROCEDURE FOR SkSP-2

A SkSP-2 plan is one that uses a given lot inspection plan by the method of attributes (Single, Double Sampling, Multiple Sampling, Chain Sampling...etc) called the 'reference plan' together with a procedure that calls for normal inspection of every lot, but for inspecting only a fraction *'f'* of the lots when the quality is good. The plan includes specific rules based on the record of lot acceptance and rejections for switch back and forth between 'normal inspections' (inspecting every lot) and 'skipping inspections' (inspecting only a fraction *'f'* of the lot).

The operating procedure is given below:

1. Start with normal inspection, using the reference plan
2. When *'i'* consecutive lots are accepted on normal inspection, switch to skipping inspection towards inspecting a fraction *'f'* of the lots.
3. When a lot is rejected on skipping inspection, switch to normal inspection.
4. Screen each rejected lot and correct or replace all defective units which are found.

These rules completely define the basic skip-lot plan of type SkSP-2, one that use a single level (single f value) of skipping.

Associated with SkSP-2, a given reference plan and the parameters *'i'* and *'f'*. In general $0 < f < 1$ and '*i*' is a positive integer. When $f = 1$ the plan degenerates into the original reference plan

2.1. Operating Characteristic Function for SkSP-2

The operating characteristics function is associated with a SkSP-2 under type B situation, based on probabilities of sampling from an infinite universe or process. The conditions associated with sampling from an infinite universe are based on the notion of process producing a theoretically continuous infinite product flow.

The OC function for a SkSP-2 plan is obtained through

$$P_a(f,i) = \frac{fP + (1-f)P^i}{f + (1-f)P^i} \qquad (2.1.1)$$

Where P is the OC function for reference plan, i is clearing interval and f is sampling inspection. Single sampling attributes plan is commonly used attribute type plan. An attempt has been made to study the operating characteristic behavior for SkSP-2 plan when MDS (0, 1) has been used as a reference plan.

3. OPERATING PROCEDURE FOR BMDS (0, 1) PLAN

Wortham and Baker (1976) have developed the Multiple Deferred State Sampling plan with r = 0 and b = 1 is operated as follows:

1. From each lot, take a random sample of n units and observe the non-conforming units d.
2. If d = 0, accept the lot; if d > 1 reject the lot. If d = 1 accept the lot provided the forthcoming m lots in succession are all accepted (previous m lots in case of multiple dependent state sampling).

The Average probability of acceptance based on Gamma-Poisson distribution is given as

$$\overline{P} = \frac{s^s}{(s+n\mu)^s} + \frac{n\mu(s)^{s+1}}{(s+n\mu+mn\mu)^{s+1}} \qquad (3.1)$$

4. SELECTION PROCEDURE FOR BAYESIAN SKSMDSP FOR SPECIFIED AQL AND LQL USING TRIGNOMETRICAL RATIO

The operating characteristics function is influenced by the plan parameters such as sample size (n), acceptance number (c) and the parameters of prior distribution is p. Analysis of OC function for different values of these parameters can determine range of the parameters in which the bayesian acceptance sampling provides protection to both producer and consumer.

Norman Bush, et.al (1953) have used different techniques to describe the direction of OC curve. The methods involve comparison of some portion of OC curve to be evaluated with the corresponding portion of the ideal OC curve. They have chord length, the line joining AQL and P_a of 0.5 as $CL = \sqrt{2025 + (P_1 - P_2)}^2$

$$n\tan\theta = \frac{n\mu_2 - n\mu_1}{P_a(\mu_1) - P_a(\mu_2)} \qquad (4.1)$$

The smaller value of tanθ, closer to the angle θ approaching zero and the chord AB approaching AC, the ideal condition through (*AQL, 1-α*). This criterion minimizes simultaneously to the consumers and producers. Using this formula, the angle θ is minimized for the given $n\mu_1$ and $n\mu_2$ values. Hence, for two given points on the OC curve, the minimum values of $tan\theta$ can be calculated, as tabulated in Table 1.

The operating procedure for Trignometrical ratio of BSkSMDSP

1. Compute the operating ratio μ_2/μ_1.
2. With the computed values of μ_2/μ_1 enter the value from table headed by μ_2/μ_1, this is equal to or just greater than the computed ratio.
3. The sample size is obtained as $n = n\mu_1/\mu_1$.
4. The minimum angle can be found as $\{\theta = tan^{-1}(ntan\theta / n)\}$.
5. Select the parameter of the sampling plan corresponding to smallest value of θ.

4.1. Selection of plan for given *f, i, s, m,* μ_1 and μ_2

To select a plan for given *f, i, s, m,* μ_1 and μ_2, first calculate the operating ratio μ_2/μ_1. Select and the table corresponding to the given *f, i, s* when *m*= 1 and locate the value or in the row headed with OR which is very close to the desired ratio. The parameter $n\mu_1$ and $ntan\theta$ are can obtained from the selected table corresponding to given *f, i, s* when *m*= 1 along with producers and consumers risk. The sample size thus obtained as $n = n\mu_1/\mu_1$ and the minimum angle $\theta = tan^{-1}\{(ntan\theta)/n\}$.

Example 4.1.1: For given $m = 1$, $\mu_1 = 0.312$, $\mu_2 = 3.515$ one can compute (OR) $\mu_2/\mu_1 = 3.515/0.312 = 11.26 \approx 11$, $\alpha = 0.05$, $\beta = 0.10$ from Table 1, one can observe the minimum angle of θ from Table. Thus the required sample plan has parameters, $f = 1/3$, $i=5$, $s=5$, $m=1$.

1. $\theta = tan^{-1}\{(n\ tan\theta)/n\}$
   $= tan^{-1}\{(4.358824)/11.26\} = 21.161$
2. $\theta = tan^{-1}\{(n\ tan\theta)/n\}$
   $= tan^{-1}\{(3.941489)/11.26\} = 19.319$
3. $\theta = tan^{-1}\{(n\ tan\theta)/n\}$
   $= tan^{-1}\{(4.162921)/11.26\} = 20.289$
4. $\theta = tan^{-1}\{(n\ tan\theta)/n\}$
   $= tan^{-1}\{(3.780612)/11.26\} = 18.559$

Now the minimum value of angle based on four choices is θ=18.559. Hence the selected parameters for BSkSMDSP is $\mu_1 = 0.312$, $\mu_2 = 3.515$, $\alpha = 0.05$, $\beta = 0.10$, $f = 1/5$, $i = 5$ and s=5 with minimum angle θ =18.559.

5. CONSTRUCTION OF TABLES

The expression for APA function of Bayesian Multiple Deferred Sampling Plan (0, 1) is given in equation,

$$\overline{P} = \int_0^\infty P(n,i/p)w(p)dp \qquad (5.1)$$

The probability density function for the Gamma distribution with parameters α and β is

$$\Gamma(p/\alpha,\beta) = \begin{cases} \dfrac{e^{-p\beta}p^{\alpha-1}\beta^{\alpha}}{\Gamma\alpha}, & p \ge 0, \alpha \ge 0, \beta \ge 0 \\ 0 \end{cases} \qquad (5.2)$$

Suppose that the defects per unit in the submitted lots p can be modeled with Gamma distribution having parameters α and β.

Let p has a prior distribution with density function given as

$$w(p) = \frac{e^{-pt}p^{s-1}t^s}{\Gamma(s)}, s,t > 0 \; and \; p > 0$$

Where μ = s/t is the mean value of the product quality p.

The Average Probability of Acceptance (APA) for BMDS Plan is given by

$$\overline{P} = \frac{s^s}{(s+n\mu)^s} + \frac{n\mu(s)^{s+1}}{(s+n\mu+mn\mu)^{s+1}} \qquad (5.3)$$

The operating characteristics function for BSkSP-2 with reference plan is given by

$$P(\mu) = P_a(f,i) = \frac{fP+(1-f)P^i}{f+(1-f)P^i} \qquad (5.4)$$

The unity value approach is used to obtain the optimum value for *m* and *s* which minimizes the tangent angle for certain specific values of $n\mu_1$ and $n\mu_2$ by keeping the producer risk below 5% and consumer risk below 10%. Computer program is used to search for optimum parametric values.

Table 1: Minimum Angle value of BSkSP–2 with MDS plan for given $n\mu_2$ and $n\mu_1$ with fixed value of m=1

f	I	s	$n\mu_1$	$n\mu_2$	OR	ntanθ α = 0.05 β = 0.10	Ntanθ α = 0.05 β = 0.01	ntanθ α = 0.01 β = 0.10	ntanθ α = 0.01 β = 0.01
1/2	1	1	0.250001	22.7350	90.93964	26.45294	23.92021	25.26404	22.94388
1/3	1	1	0.34562	34.3356	15.20717	4.429412	4.005319	4.230337	3.841837
1/4	1	1	0.41565	45.2356	13.13196	3.582353	3.239362	3.421348	3.107143
1/5	1	1	0.48593	56.4959	12.54076	3.405882	3.079787	3.252809	2.954082
2/3	1	1	0.20507	17.0351	11.84612	3.194118	2.888298	3.050562	2.770408
1/2	3	3	0.265007	4.030007	99.3450	39.98824	36.15957	38.19101	34.68367
1/3	3	3	0.31505	4.05005	12.85526	4.394118	3.973404	4.196629	3.811224
1/4	3	3	0.36092	4.06592	11.02272	3.529412	3.191489	3.370787	3.061224
1/5	3	3	0.41009	4.08509	10.53337	3.352941	3.031915	3.202247	2.908163
2/3	3	3	0.220052	4.015052	10.5014	3.158824	2.856383	3.016854	2.739796
1/2	5	5	0.25099	3.29599	108.8311	52.72941	47.68085	50.35955	45.73469
1/3	5	5	0.29932	3.29932	11.26543	4.358824	3.941489	4.162921	3.780612
1/4	5	5	0.34492	3.29992	9.567204	3.476471	3.143617	3.320225	3.015306
1/5	5	5	0.36415	3.30415	9.556085	3.317647	3.00000	3.168539	2.877551
2/3	5	5	0.22919	3.30419	9.494641	3.123529	2.824468	2.983146	2.709184
1/2	6	6	0.25085	3.14585	116.2635	65.89412	59.58511	62.93258	57.15306
1/3	6	6	0.29895	3.14895	9.961447	4.323529	3.909574	4.129213	3.750000
1/4	6	6	0.32959	3.14959	9.073596	3.458824	3.12766	3.303371	3.00000
1/5	6	6	0.36323	3.13823	8.639787	3.264706	2.952128	3.117978	2.831633
2/3	6	6	0.22853	3.13853	8.925246	3.088235	2.792553	2.949438	2.678571
1/2	8	8	0.25032	2.96532	83.06954	19.80000	17.90426	18.91011	17.17347
1/3	8	8	0.28259	2.96759	18.24592	4.464706	4.037234	4.264045	3.872449
1/4	8	8	0.31255	2.96755	14.41682	3.617647	3.271277	3.455056	3.137755
1/5	8	8	0.33122	2.95622	13.73356	3.423529	3.095745	3.269663	2.969388
2/3	8	8	0.22631	2.95631	13.06310	3.211765	2.904255	3.067416	2.785714

6. CONCLUSION

Traditional sampling plan have wide potential applicability in industries to ensure a higher standard of quality attainment and increased customer satisfaction. Here, an attempt is made to apply the concept of Bayesian SkSP-2 with MDS based on prior distributions. Applications of acceptance sampling in industries through the operation of sampling plan yields quality assurance. In acceptance sampling the producer and consumer plays a dominant role and hence one allows certain level of risk for both producer and consumer, namely α=0.05, β=0.10. This ability of the plan is to discriminate between good and bad quality. Thus these plans overcome the difficulty for fixing the acceptance number. Tables are provided here which are tailor made, handy and ready- made uses to the industrial shop-floor conditions.

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