Research Article

Quadratic type minimum risk equivariant estimation of an uniform location-scale using general progressive censored sample

T Leo Alexander

Associate Professor, Department of Statistics, Loyola College, Chennai, Tamil Nadu, INDIA.
Email: leoalex.62@gmail.com

Abstract In this paper, by assuming that a general progressive Type II right censored sample, we obtain Q_A-MRE estimators for the vector parameter of $(\xi, \tau)'$ based on the general Type II progressive right censored sample. Further MRE estimator of $(\xi, \tau)'$ is obtained with respect to Linex type loss function. These geralize the results of Chandrasekar et.al. (2002) for progressive Type II right censored sample. The paper is organized as follows: Section 3 deals with the problem of Q_A-MRE estimators for the vector parameters. In the last Section, we consider the problem of equivariant estimation of the vector parameter under Linex loss function(Varian,1975).
Keywords: Equivariant Estimation, General Progressive Censored sample, Linex Loss function, Q_A- MRE, Uniform Location- Scale model.

***Address for Correspondence:**
Dr. T. Leo Alexander, Associate Professor, Department of Statistics, Loyola College, Chennai, Tamil Nadu, INDIA.
Email: leoalex.62@gmail.com
Received Date: 15/03/2015 Accepted Date: 29/03/2015

Access this article online	
Quick Response Code:	Website: www.statperson.com
	DOI: 02 April 2015

INTRODUCTION

Progressive Type II right censored sampling is an important method of obtaining data in life-testing studies. As pointed out by Rita Aggarwala and Balakrishnan (1998), the scheme of progressive censoring enables us to use live units, removed early, in other tests. Balakrishnan and Sandhu (1996), by assuming a general progressive Type II right censored sample, derived the BLUE's for the parameters of one-and two-parameter exponential distributions. For the later, they also derived MLE's and shown that they are simply the BLUE's, adjusted for their bias. Let us consider the following general progressive Type II right censoring scheme (Balakrishnan and Sandhu, 1996) : Suppose N randomly selected units were placed on a life test; the failure times of the first r units to fail were not observed ; at the time of the $(r+1)^{th}$ failure, R_{r+1} number of surviving units are withdrawn from the test randomly, and so on; at the time of the $(r+i)^{th}$ failure, R_{r+i} number of surviving units are randomly withdrawn from the test ; finally, at the time of the n-th failure, the remaining $R_n = N - n - R_{r+1} - R_{r+2} - ... - R_{n-1}$ are withdrawn from the test. Suppose $X_{r+1:N} \le X_{r+2:N} \le .... \le X_{n:N}$ are the life-times of the completely observed units to fail, and $R_{r+1}, R_{r+2},... , R_n$ are the number of units withdrawn from the test at

How to site this article: K K Suresh, S Umamaheswari. Designing of bayesian skip lot sampling plan with multiple deferred sampling plan as reference plan. *International Journal of Statistika and Mathemtika* Feb. to Apr. 2015; 13(3): 79-83. http://www.statperson.com (accessed 04 April 2015).

these failure times, respectively. It follows that $N = n + \sum_{i=r+1}^{n} Ri$. If the failure times are from a continuous population with the pdf f and the distribution function F, then the joint pdf of ($X_{r+1:N}$, $X_{r+2:N}$,.... , $X_{n:N}$) is given by

$$g_\theta(x_{r+1},...,x_n)=c[\{F_\theta(x_{r+1})\}^r \times \prod_{i=r+1}^{n} f_\theta(x_i)\{1-F_\theta(x_i)\}^{R_i}] \qquad (1.1)$$

where $c = \binom{N}{r}(N-r)\prod_{j=r+2}^{n}\left(N-\sum_{i=r+1}^{j-1}R_i - j+1\right)$.

LOCATION- SCALE MODEL

In this case, the common pdf is taken to be

$$f_\theta(x) = \begin{cases} 1/\tau, \xi \le x \le \xi+1; \xi \in \Re, \tau > 0, \\ 0, otherwise \end{cases}$$

Note that $\theta = (\xi,\tau)'$. Then (1.1) reduces to

$$g_\theta(x_{r+1},...,x_n) = c[((x_{r+1}-\xi)/\tau)^r (1/\tau^{n-r-1}) \times \underset{i=r+1}{\overset{n}{\pi}} \{1-(x_i-\xi)/\tau\}^{R_i}], \quad ...(2.1)$$

$$\xi \le x_{r+1:N} \le ... \le x_{n:N} \le \xi + \tau; \xi \in \Re, \tau > 0.$$

Note that the above pdf belongs to a location – scale model. We are interested in deriving Q_A-MRE estimator as well as the MRE estimator based on Linex loss function for the vector parameter $(\xi,\tau)'$.

QUADRATIC TYPE LOSS FUNCTION

If the loss function is of the quadratic type, then by the simultaneous equivariant estimation approach of Edwin Prabakaran and Chandrasekar (1994), let us consider the problem of estimation of $(\xi,\tau)'$. Consider $\delta_0(X) = (\delta_{01}(X), \delta_{02}(X))'$,

where $\delta_{01}(X) = X_{r+1:N}$ and $\delta_{02}(X) = X_{n:N} - X_{r+1:N}$. Here $\delta_0(X)$ is an equivariant estimator and $(X_{r+1:N}, X_{n:N})'$ is a sufficient statistic but not complete. In order to find $(w_1^*, w_2^*)'$ as given by the following

$$w_1^* = \frac{E(\delta_{01}\delta_{02}|\mathbf{z})}{E(\delta_{02}^2|\mathbf{z})} \quad \text{and} \quad 1/w_2^* = \frac{E(\delta_{02}|\mathbf{z})}{E(\delta_{02}^2|\mathbf{z})},$$

consider the transformation

$Z_{r+1} = X_{r+1:N}$, $Z_{r+2} = X_{n:N} - X_{r+1:N}$

and $Z_i = \dfrac{X_{i-1:N} - X_{r+1:N}}{X_{n:N} - X_{r+1:N}}, i = r+3,...,n$,

so that $X_{r+1:N} = Z_{r+1}$, $X_{n:N} = Z_{r+1} + Z_{r+2}$

and $X_{i-1:N} = Z_{r+1} + Z_{r+2}Z_i, i = r+3,...,n.$

The Jacobian of the transformation is given by

$$J = Z_{r+2}^{n-r-2}.$$

Thus the joint pdf of $(Z_{r+1},...,Z_n)$ is given by

$$h\left(Z_{r+1},\ldots,Z_n\right)=c[z_{r+1}^{r}z_{r+2}^{n-r-2}\left(1-z_{r+1}^{r}\right)^{R_{r+1}}\times \{1-\left(z_{r+1}+z_{r+2}\right)\}^{R_n}\times \underset{i=r+2}{\overset{n-1}{\pi}}\{1-\left(z_{r+1}+z_{r+2}z_i\right)\}^{R_i}],\ldots(3.1)$$

$$0<z_{r+3}<\ldots<Z_n<1,$$

$$0<z_{r+1}+z_{r+2}<1.$$

Also the joint pdf of $\left(Z_{r+3},\ldots,Z_n\right)$ is given by

$$h_1\left(z_{r+3},\ldots,z_n\right)=c[\int_0^1\int_0^{1-z_{r+2}}z_{r+1}^{r}z_{r+2}^{n-r-2}\left(1-z_{r+1}\right)^{R_{r+1}}\times \{1-\left(z_{r+1}+z_{r+2}\right)\}^{R_n}\times \underset{i=r+2}{\overset{n-1}{\pi}}\{1-\left(z_{r+1}+z_{r+2}z_i\right)\}^{R_i}\,dz_{r+1}dz_{r+2}]$$

$$0<z_{r+3}<\ldots<z_n<1.\quad\ldots(3.2)$$

Thus the conditional pdf of $\left(Z_{r+1},Z_{r+2}\right)$ given $\left(Z_{r+3},\ldots,Z_n\right)$ is given by

$$h_2\left(\left(z_{r+1},z_{r+2}\right)\mid z_{r+3},\ldots,z_n\right)=\frac{h\left(z_{r+1},\ldots,z_n\right)}{h_1\left(z_{r+3},\ldots,z_n\right)},\qquad 0<z_{r+1}+z_{r+2}<1. \tag{3.3}$$

Where $h\left(z_{r+1},\ldots,z_n\right)$ and $h_1\left(z_{r+3},\ldots,z_n\right)$ are given in equations (3.1) and (3.2) respectively. Similarly

$$h_3\left(\left(z_{r+2}\right)\mid z_{r+3},\ldots,z_n\right)=\frac{\int_0^{1-z_{r+2}}h\left(z_{r+1},\ldots,z_n\right)dz_{r+1}}{h_1\left(z_{r+3},\ldots,z_n\right)}, \tag{3.4}$$

In view of (3.3).

Now $E\left(\delta_{01}\delta_{02}\mid \mathbf{z}\right)=E\left(Z_{r+1}Z_{r+2}\mid \mathbf{z}\right)$

$$=\frac{\int_0^1\int_0^{1-z_{r+2}}z_{r+1}z_{r+2}h\left(z_{r+1},\ldots,z_n\right)dz_{r+1}dz_{r+2}}{h_1\left(z_{r+3},\ldots,z_n\right)}, \tag{3.5}$$

In view of (3.3).

Also $E\left(\delta_{02}{}^{2}\mid \mathbf{z}\right)=E\left(Z_{r+2}^{2}\mid \mathbf{z}\right)$

$$=\frac{\int_0^1\int_0^{1-z_{r+2}}z_{r+2}^{2}h\left(z_{r+1},\ldots,z_n\right)dz_{r+1}dz_{r+2}}{h_1\left(z_{r+3},\ldots,z_n\right)}, \tag{3.6}$$

In view of (3.4).

Similarly, $E\left(\delta_{02}\mid \mathbf{z}\right)=E\left(Z_{r+2}\mid \mathbf{z}\right)$

$$= \frac{\int_0^1 \int_0^{1-z_{r+2}} z_{r+2} h(z_{r+1},\ldots,z_n) dz_{r+1} dz_{r+2}}{h_1(z_{r+3},\ldots,z_n)}, \tag{3.7}$$

In view of (3.4). Thus

$$w_1^* = \frac{E(\delta_{01}\delta_{02} \mid \mathbf{z})}{E(\delta_{02}{}^2 \mid \mathbf{z})},$$

where $E(\delta_{01}\delta_{02} \mid \mathbf{z})$ and $E(\delta_{02}{}^2 \mid \mathbf{z})$ are given in (3.6) and (3.7) respectively and

$$1/w_2{}^* = \frac{E(\delta_{02} \mid \mathbf{z})}{E(\delta_{02}{}^2 \mid \mathbf{z})},$$

where $E(\delta_{02} \mid \mathbf{z})$ and $E(\delta_{02}{}^2 \mid \mathbf{z})$ are given in (3.6) and (3.7) respectively.

Therefore the MRE estimator $\delta^* = (\delta_1{}^*, \delta_2{}^*)$ of $\theta = (\xi, \tau)'$ is given by

$$\delta_1{}^* = X_{r+1:N} - (X_{n:N} - X_{r+1:N}) w_1^*$$

and $$\delta_2^* = (X_{n:N} - X_{r+1:N}) / w_2{}^*.$$

Remark3.1: if r=0 and R_i=r_i then the above estimator reduces to the one in simultaneous Equivariant estimation of the parameters of a Uniform Location- Scale model based on progressive Type-II right censored sample case (Leo Alexander, 2000).

LINEX LOSS FUNCTION

Following Varian (1975), the MRE estimator of $(\xi, \tau)'$ under Linex loss function, is provided. We have

$$R(\delta \mid \mathbf{z}) = E[\{e^{a(\delta_{01} - g w_1)} - a(\delta_{01} - g w_1) - 1$$
$$+ e^{b(\delta_{02}/w_2 - 1)} - b(\delta_{02}/w_2 - 1) - 1\} \mid \mathbf{z}],$$

where $\delta_{01} = X_{r+1:N}$ and $\delta_{02} = X_{n::N} - X_{r+1:N}$. Consider

$$R(\delta \mid \mathbf{z}) = E[\{e^{a(\delta_{01} - a w_1 \delta_{02})} \mid \mathbf{z}]$$
$$- aE(\delta_{01} \mid \mathbf{z}) + a w_1 E(\delta_{02} \mid \mathbf{z}) - 1$$
$$+ e^{-b} E(e^{b/w_2 \delta_{02}} \mid \mathbf{z}) + b - b/w_2 E(\delta_{02} \mid \mathbf{z}) - 1$$

$$= \frac{\int_0^1 \int_0^{1-z_{r+2}} e^{a(z_{r+1} - w_1 z_{r+2})} h(z_{r+1},\ldots,z_n) dz_{r+1} dz_{r+2}}{h_1(z_{r+3},\ldots,z_n)} - a \frac{\int_0^1 \int_0^{1-z_{r+2}} z_{r+1} h(z_{r+1},\ldots,z_n) dz_{r+2} dz_{r+1}}{h_1(z_{r+3},\ldots,z_n)}$$

$$+ a w_1 \frac{\int_0^1 \int_0^{1-z_{r+2}} z_{r+2} h(z_{r+1},\ldots,z_n) dz_{r+1} dz_{r+2}}{h_1(z_{r+3},\ldots,z_n)} - 1 + e^{-b} \frac{\int_0^1 \int_0^{1-z_{r+2}} e^{b/w_2 z_{r+2}} h(z_{r+1},\ldots,z_n) dz_{r+1} dz_{r+2}}{h_1(z_{r+3},\ldots,z_n)} + b$$

$$- b/w_2 \frac{\int_0^1 \int_0^{1-z_{r+2}} z_{r+2} h(z_{r+1},\ldots,z_n) dz_{r+1} dz_{r+2}}{h_1(z_{r+3},\ldots,z_n)} - 1 \quad .$$

Thus $(w_1{}^*, w_2{}^*)'$ is to be obtained as the value of $(w_1, w_2)'$ by minimizing $R(\delta \mid \mathbf{z})$.

Therefore the MRE estimator of $(\xi,\tau)'$ is given by

$\delta_1^{*} = X_{r+1:N} - (X_{n:N} - X_{r+1:N})w_1^{*}$ and $\delta_2^{*} = (X_{n:N} - X_{r+1:N})/w_2^{*}$.

Remark 4.1: If r=0 and $R_i = r_i, i = 1,2,...,n,$ then the above estimators reduce to those obtained in (Leo Alexander, T, 2000).

ACKNOWLEDGEMENT

The author thank Dr. B. Chandrasekar, Department of Statistics, Loyola College, Chennai for his valuable guidance and suggestions.

REFERENCES

1. Aggarwala, R. and Balakrishnan, N. (1998). Some properties of progressive censored order statistics from arbitrary and uniform distributions with applications to inference and simulation, J. Statist. Planning. Infer. 70, 35-49.
2. Balakrishnan, N. and Sandhu, R.A. (1996). Best linear unbiased and maximum likelihood estimation for exponential distributions under general progressive Type II censored samples, Sankhya, Series B, 58, 1-9.
3. Chandrasekar, B., Leo Alexander, T. and Balakrishnan, N. (2002). Equivariant estimation for the parameters of an exponential model based on Type II progressively censored samples. Commun.Stat. Theory and Methods, 31(10), 1675-1686.
4. Edwin Prabakaran, T. and Chandrasekar, B. (1994). Simultaneous equivariant estimation for location-scale models, J. Statist. Planning Infer. 40, 51-59.
5. Leo Alexander, T.(2000).Contributions to simultaneous equivariant estimation based on censored samples, Ph.D.Thesis, University of Madras, India.
6. Varian, H.R. (1975). A Bayesian Approch to Real Estate Assessment In : Studies in BayesianEconometric and Statistics in Honor of Leonard J. Savage, eds. S.E. Frienberg and A.Zellner.North Holland, Amsterdam, 195-208.

Source of Support: None Declared
Conflict of Interest: None Declared