Research Article

Cost-Benefit Analysis of Two Similar Warm Standby Aircraft System subject to failure due to ultra light and effect of thunderstorm on aircraft

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Abstract In this paper we have taken failure due to ultra light, and effect of thunderstorm on aircraft. When the main unit fails then warm standby system becomes operative. Failure due to effect of thunderstorm on aircraft cannot occur simultaneously in both the units and after failure the unit undergoes Type-I or Type-II or Type-III repair facility immediately. Applying the regenerative point technique with renewal process theory the various reliability parameters MTSF, Availability, Busy period, Benefit-Function analysis have been evaluated.
Keywords: Warm Standby, failure due to ultra light, failure due to effect of thunderstorm on aircraft, first come first serve, MTSF, Availability, Busy period, Benefit -Function.

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INTRODUCTION

The pilot had been working on the ultra light aircraft for about an hour; he then took off, flew for about 10 minutes, and landed. About 10 minutes later, after he had donned a jacket, the pilot took off again far another local flight. Minutes later witnesses observed the ultra light in level flight and at about 100 feet above ground level (agl). They then heard a loud report and saw the aircraft descend rapidly and strike the ground. The aircraft was destroyed and the pilot was fatally injured. The wind was calm and the sky was clear at the time of the accident. Weather was not a factor in this occurrence. The aircraft had been kept in a barn on occasion. It was also reported to have spent long periods outside, unprotected from the elements and in direct sunlight, most recently during the several months prior to the accident. There was no evidence that the wings had been covered when the aircraft was kept outside. The quicksilver wing consists of an upper surface formed by a Dacron fabric wing sail stretched over the wing frame. The sail is composed of several Dacron panels, sewn together. On inspection, the upper surfaces of the sails were found to be severely faded when compared to the lower surfaces. Further, the dacron fabric was weak and tore easily when stressed. The dacron fabric sail was not maintained in accordance with the manufacturer's recommendations, and was severely weakened by exposure to the sun's ultraviolet rays. The dacron fabric tore when it was exposed to aerodynamic flight loads. Effect of thunderstorm and lightening on aircraft instruments. The lightning discharges emit radio waves – atmospherics or *'sferics* – at the low end of the AM broadcast band and at TV band 1, which are the basis for airborne storm mapping instruments such as

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Stormscope and Strikefinder. The NDB/ADF navigation aids also operate near the low end of the AM band so that the tremendous radio frequency energy of the storm will divert the radio compass needle. *Weather radars map storms from the associated precipitation.*

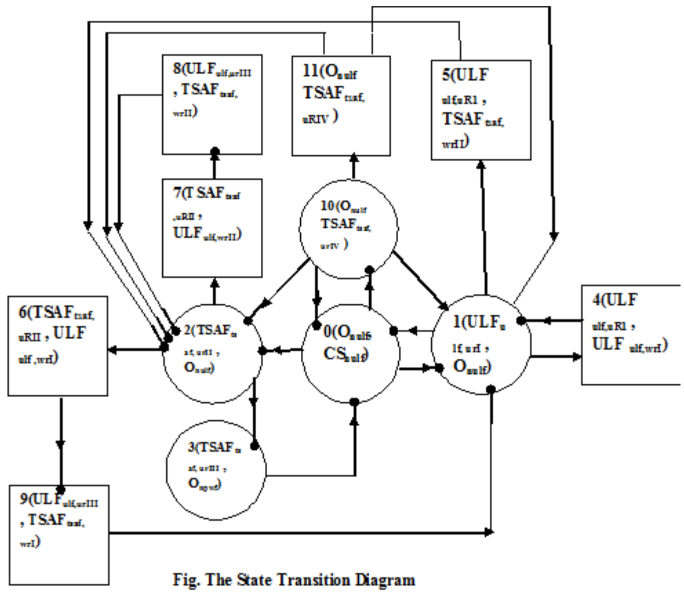

Fig. The State Transition Diagram

Up-State ◯ Down-State □

• regeneration point

STRIKE EFFECT ON AIRCRAFT

When most aeroplanes, excluding ultralights, are struck by lightning the streamer attaches initially to an extremity, such as the nose or wing tip then re-attaches itself to the fuselage at other locations as the aircraft moves through the channel. The current is conducted through the electrically bonded aluminium skin and structures of the aircraft and exits from an extremity, such as the tail. If an ultralight is struck by lightning the consequences cannot be determined but are likely to be very unpleasant. Although a basic level of protection is provided in most light aeroplanes for the airframe, fuel system and engines, damage to wing tips, propellers and navigation lights may occur and the current has the potential to induce transients into electrical cables or electronic equipment. The other main area of concern is the fuel tanks, lines, vents, filler caps and their supporting structure, where extra design precautions prevent sparking or burn through. In heavier aircraft radomes, being constructed of non-conductive material, are at risk.

Hail

Hail can cause considerable damage to aircraft and is usually encountered between 10,000 and 30,000 feet. At times it can also be found in clear air near thunderstorms.

Icing

High humidity and low winter freezing levels provide likely conditions for icing at low levels. Hopefully it is unlikely that a VFR GA pilot would venture into possible icing conditions but pilots may be tempted to fly through freezing rain or drizzle. Aircraft cruising in VMC above the freezing level and then descending through a cloud layer may pick up ice.

In this paper we have taken failure due to ultra light, and effect of thunderstorm on aircraft. When the main operative unit fails then warm standby system becomes operative. Failure due to effect of thunderstorm on aircraft cannot occur simultaneously in both the units. After failure the unit undergoes repair facility of Type- I or Type- II by ordinary repairman, Type III or Type IV by multispecialty repairman immediately when failure due to ultra light and effect of thunderstorm on aircraft. The repair is done on the basis of first fail first repaired.

Assumptions

1. λ_1, λ_2 λ_3 are constant failure rates when failure due to ultra light, failure due to effect of thunderstorm on aircraft respectively. The CDF of repair time distribution of Type I, Type II and multispecialty repairmen Type-III, IV are $G_1(t)$, $G_2(t)$ and $G_3(t)$, $G_4(t)$.
2. The failure due to effect of thunderstorm on aircraft is non-instantaneous and it cannot come simultaneously in both the units.

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3. The repair starts immediately after failure due to ultra light and failure due to effect of thunderstorm on aircraft and works on the principle of first fail first repaired basis. The repair facility does no damage to the units and after repair units are as good as new.
4. The switches are perfect and instantaneous.
5. All random variables are mutually independent.
6. When both the units fail, we give priority to operative unit for repair.
7. Repairs are perfect and failure of a unit is detected immediately and perfectly.
8. The system is down when both the units are non-operative.

SYMBOLS FOR STATES OF THE SYSTEM

Superscripts O, CS, ULF, TSAF,

Operative, Warm Standby, failure due to ultra light, failure due to effect of thunderstorm on aircraft respectively

Subscripts nulf, ulf, tsaf, ur, wr, uR

No failure due to ultra light, failure due to ultra light, failure due to effect of thunderstorm on aircraft, under repair, waiting for repair, under repair continued from previous state respectively

Up states – 0, 1, 2, 3, 10 ; Down states – 4, 5, 6, 7,8,9,11, regeneration point – 0,1,2, 3, 8, 9,10

States of the System

0(O_{nulf}, CS_{nulf}) One unit is operative and the other unit is warm standby and there is no failure due to ultra light of both the units.

1($ULF_{ulf,\ urI}$, O_{nulf}) The operating unit failure due to ultra light is under repair immediately of Type- I and standby unit starts operating with no failure due to ultra light

2($TSAF_{TSAF,\ urII}$, O_{nulf}) The operative unit failure due to effect of thunderstorm on aircraft and undergoes repair of Type II and the standby unit becomes operative with no failure due to ultra light

3($TSAF_{TSAF,\ urIII}$, O_{nulf}) The first unit failure due to effect of thunderstorm on aircraft and under Type-III multispecialty repairman and the other unit is operative with no failure due to ultra light

4($ULF_{ulf,uR1}$, $ULF_{ulf,wrI}$) The unit failed due to ULF resulting from failure due to ultra light under repair of Type- I continued from state 1and the other unit failed due to ULF resulting from failure due to ultra light is waiting for repair of Type-I.

5($ULF_{ulf,uR1}$, $TSAF_{TSAF,\ wrII}$) The unit failed due to ULF resulting from failure due to ultra light is under repair of Type-I continued from state 1and the other unit fails due to effect of thunderstorm on aircraft is waiting for repair of Type- II.

6($TSAF_{tsaf,\ uRII}$, $ULF_{ulf\ ,wrI}$) The operative unit failed due to effect of thunderstorm on aircraft is under repair continues from state 2 of Type –II and the other unit failed due to ULF resulting from failure due to ultra light is waiting under repair of Type-I.

7($TSAF_{tsaf,uRII}$, $ULF_{ulf,wrII}$) The one unit failed due to effect of thunderstorm on aircraft is continued to be under repair of Type II and the other unit failed due to ULF resulting from failure due to ultra light is waiting for repair of Type-II.

8($ULF_{ulf,urIII}$, $TSAF_{tsaf,\ wrII}$) The one unit failure due to ultra light is under multispecialty repair of Type-III and the other unit failed due to effect of thunderstorm on aircraft is waiting for repair of Type-II.

9($ULF_{ulf,urIII}$, $TSAF_{tsaf,\ wrI}$) The one unit failure due to ultra light is under multispecialty repair of Type-III and the other unit failed due to effect of thunderstorm on aircraft is waiting for repair of Type-I

10(O_{nulf} $TSAF_{tsaf,\ urIV}$) The one unit is operative with no failure due to ultra light and warm standby unit fails due to effect of thunderstorm on aircraft and undergoes repair of type IV.

11(O_{nulf} $TSAF_{tsaf,\ uRIV}$) The one unit is operative with no failure due to ultra light and warm standby unit fails due to effect of thunderstorm on aircraft and repair of type IV continues from state 10.

Transition Probabilities

Simple probabilistic considerations yield the following expressions:

$p_{01} = \lambda_1 / \lambda_1 + \lambda_2 + \lambda_3$, $p_{02} = \lambda_2 / \lambda_1 + \lambda_2 + \lambda_3$, $p_{0,10} = \lambda_3 / \lambda_1 + \lambda_2 + \lambda_3$

$p_{10} = pG_1^*(\lambda_1) + q\, G_2^*(\lambda_2)$, $p_{14} = p - pG_1^*(\lambda_1) = p_{11}^{(4)}$,

$p_{15} = q - q\, G_1^*(\lambda_2) = p_{12}^{(5)}$, $p_{23} = pG_2^*(\lambda_1) + q\, G_2^*(\lambda_2)$,

$p_{26} = p - pG_2^*(\lambda_1) = p_{29}^{(6)}$,$p_{27} = q - qG_2^*(\lambda_2) = p_{28}^{(7)}$,

$p_{30} = p_{82} = p_{91} = 1$, $p_{0,10} = pG_4^*(\lambda_1) + q\, G_4^*(\lambda_2)$,

$p_{10,1} = p - pG_4^*(\lambda_1) = p_{10,1}^{(11)}$,$p_{10,2} = q - q\, G_4^*(\lambda_2) = p_{10,2}^{(11)}$

(1)

We can easily verify that
$p_{01} + p_{02} + p_{03} = 1,\ p_{10} + p_{14} (=p_{11}^{(4)}) + p_{15} (=p_{12}^{(5)}) = 1,$
$p_{23} + p_{26} (=p_{29}^{(6)}) + p_{27} (=p_{28}^{(7)}) = 1\ p_{30} = p_{82} = p_{91} = 1$
$p_{10,0} + p_{10,1}^{(11)} (=p_{10,1}) + p_{10,2}^{(12)} (=p_{10,2}) = 1$
(2)
And mean sojourn time is
$\mu_0 = E(T) = \int_0^\infty P[T > t]dt$

Mean Time To System Failure

$Ø_0(t) = Q_{01}(t)[s]\ Ø_1(t) + Q_{02}(t)[s]\ Ø_2(t) + Q_{0,10}(t)[s]\ Ø_{10}(t)$
$Ø_1(t) = Q_{10}(t)[s]\ Ø_0(t) + Q_{14}(t) + Q_{15}(t)$
$Ø_2(t) = Q_{23}(t)[s]\ Ø_3(t) + Q_{26}(t) + Q_{27}(t)\ ,\ Ø_3(t) = Q_{30}(t)[s]\ Ø_0(t)\ ,$
$Ø_{10}(t) = Q_{10,0}(t)[s]\ Ø_{10}(t) + Q_{10,1}(t)[s]Ø_1(t) + Q_{10,2}(t)[s]\ Ø_2(t)$
(3-6)
We can regard the failed state as absorbing
Taking Laplace-Stiljes transform of eq. (3-6) and solving for
$ø_0^*(s) = N_1(s) / D_1(s)$ (7)
where
$N_1(s) = \{Q_{01}^* + Q_{0,10}^*\ Q_{10,1}^*\}\ [\ Q_{14}^*(s) + Q_{15}^*(s)\ ] + \{Q_{02}^* + Q_{0,10}^*\ Q_{10,2}^*\}\ [\ Q_{26}^*(s) + Q_{27}^*(s)\ ]$
$D_1(s) = 1 - \{Q_{01}^* + Q_{0,10}^*\ Q_{10,1}^*\}\ Q_{10}^* - \{Q_{02}^* + Q_{0,10}^*\ Q_{10,2}^*\}\ Q_{23}^*\ Q_{30}^* - Q_{0,10}^*\ Q_{10,0}^*$
Making use of relations (1) & (2) it can be shown that $ø_0^*(0) = 1$, which implies that $ø_0(t)$ is a proper distribution.

$$\text{MTSF} = E[T] = \left.\frac{d}{ds}\, ø_0^*(s)\right|_{s=0}$$
$$= (D_1'(0) - N_1'(0))\ /\ D_1(0)$$
$= (\ \mu_0 + \mu_1\ (p_{01} + p_{0,10}\ p_{10,1}) + (p_{02} + p_{0,10}\ p_{10,2})(\ \mu_2 + \mu_3) + \mu_{10}\ p_{0,10}\ /\ (1 - (p_{01} + p_{0,10}\ p_{10,1})\ p_{10} - (p_{02} + p_{0,10}\ p_{10,2})\ p_{23}\ ) - p_{0,10}\ p_{10,0}$
where
$\mu_0 = \mu_{01} + \mu_{02} + \mu_{0,10}\ ,\ \mu_1 = \mu_{10} + \mu_{11}^{(4)} + \mu_{12}^{(5)},$
$\mu_2 = \mu_{23} + \mu_{28}^{(7)} + \mu_{29}^{(6)},\ \mu_{10} = \mu_{10,0} + \mu_{10,1} + \mu_{10,2}$

AVAILABILITY ANALYSIS

Let $M_i(t)$ be the probability of the system having started from state i is up at time t without making any other regenerative state. By probabilistic arguments, we have
$M_0(t) = e^{-\lambda_1 t} e^{-\lambda_2 t} e^{-\lambda_3 t},\ ,\ M_1(t) = p\ G_1(t)\ \overline{\ }\ e^{-\lambda_1 t}$
$M_2(t) = q\ \overline{G_2}(t)\ e^{-\lambda_2 t},\quad M_3(t) = G_3(t),\ \overline{M}_{10}(t) = G_4(t)\ e^{-\lambda_3 t}$
The point wise availability $A_i(t)$ have the following recursive relations
$A_0(t) = M_0(t) + q_{01}(t)[c]A_1(t) + q_{02}(t)[c]A_2(t) + q_{0,10}(t)[c]A_{10}(t)$
$A_1(t) = M_1(t) + q_{10}(t)[c]A_0(t) + q_{12}^{(5)}(t)[c]A_2(t) + q_{11}^{(4)}(t)[c]A_1(t)\ ,$
$A_2(t) = M_2(t) + q_{23}(t)[c]A_3(t) + q_{28}^{(7)}(t)[c]\ A_8(t) + q_{29}^{(6)}(t)]\ [c]A_9(t)$
$A_3(t) = M_3(t) + q_{30}(t)[c]A_0(t)\ , A_8(t) = q_{82}(t)[c]A_2(t)$
$A_9(t) = q_{91}(t)[c]A_1(t),\ A_{10}(t) = M_{10}(t) + q_{10,0}(t)[c]A_0(t) + q_{10,1}^{(11)}(t)[c]A_1(t) +$
$q_{10,2}^{(11)}(t)[c]A_2(t)$
(8-15)
Taking Laplace Transform of eq. (8-15) and solving for $\hat{A}_0(s)$
$\hat{A}_0(s) = N_2(s)/D_2(s)$
(16)
where
$N_2(s) = \{\ \hat{q}_{0,10}\ \hat{M}_{10} + \hat{M}_0\ \}\ [\{1 - \hat{q}_{11}^{(4)}\}\{1 - \hat{q}_{28}^{(7}\ \hat{q}_{82}\} - \hat{q}_{12}^{(5)}\ \hat{q}_{29}^{(6)}\ \hat{q}_{91}\ ] + \{\ \hat{q}_{01} + \hat{q}_{0,10}\ \hat{q}_{10,1}^{(11)}\}[\ \hat{M}_1\{1 - \hat{q}_{28}^{(7)}\ \hat{q}_{82}\} + \hat{q}_{12}^{(5)}\ \hat{q}_{23}\ \hat{M}_3 + \hat{M}_2] + \{\ \hat{q}_{02} + \hat{q}_{0,10}\ \hat{q}_{10,2}^{(11)}\}\ [\{\ \hat{q}_{23}\ \hat{M}_3\}\{1 - \hat{q}_{11}^{(4)}\} + \hat{q}_{29}^{(6)}\ \hat{q}_{91}\ \hat{M}_1]$

$D_2(s) = \{1 - \hat{q}_{11}^{(4)}\}\{1- \hat{q}_{28}^{(7} \hat{q}_{82}\}- \hat{q}_{12}^{(5)} \hat{q}_{29}^{(6)} \hat{q}_{91} -\{ \hat{q}_{01}+ \hat{q}_{0,10} \hat{q}_{10,1}^{(11)}\}[\hat{q}_{10} \{1 - \hat{q}_{28}^{(7)} \hat{q}_{82}\} +\hat{q}_{12}^{(5)} \hat{q}_{23} \hat{q}_{30}] - \{ \hat{q}_{02}+\hat{q}_{0,10} \hat{q}_{10,2}^{(11)}\}\{[\hat{q}_{23} \hat{q}_{30} \{1 - \hat{q}_{11}^{(4)}\}+ \hat{q}_{29}^{(6)} \hat{q}_{91} \hat{q}_{10}]$

(Omitting the arguments s for brevity)

The steady state availability

$A_0 = \lim_{t\to\infty}[A_0(t)] = \lim_{s\to 0}[s\, \ddot{A}_0(s)] = \lim_{s\to 0} \frac{s\, N_2(s)}{D_2(s)}$

Using L' Hospitals rule, we get

$A_0 = \lim_{s\to 0} \frac{N_2(s)+s\ N_2'(s)}{D_2'(s)}$

$= \frac{N_2(0)}{D_2'(0)}$ (17)

The expected up time of the system in (0,t] is

$\lambda_u(t) = \int_0^{\infty} A_0(z)\,dz$ So that $\overline{\lambda_u}(s) = \frac{\hat{A}_0(s)}{s} = \frac{N_2(S)}{SD_2(S)}$

(18)

The expected down time of the system in (0,t] is

$\lambda_d(t) = t - \lambda_u(t)$ So that $\overline{\lambda_d}(s) = \frac{1}{s^2} - \overline{\lambda_u}(s)$ (19)

The expected busy period of the server when there is failure due to ultra light, and failure due to effect of thunderstorm on aircraft in (0,t]-R_0

$R_0(t) = q_{01}(t)[c]R_1(t) + q_{02}(t)[c]R_2(t) + q_{0,10}(t)[c]R_{10}(t)$

$R_1(t) = S_1(t) + q_{10}(t)[c]R_0(t) + q_{12}^{(5)}(t)[c]R_2(t) + q_{11}^{(4)}(t)[c]R_1(t)$

$R_2(t) = S_2(t) + q_{23}(t)[c]R_3(t) + q_{28}^{(7)}(t) R_8(t) +q_{29}^{(6)}(t)][c]R_9(t)$

$R_3(t) = S_3(t) + q_{30}(t)[c]R_0(t)$

$R_8(t) = S_8(t) + q_{82}(t)[c]R_2(t)$

$R_9(t) = S_9(t) + q_{91}(t)[c]R_1(t)$

$R_{10}(t) = S_{10}(t) + q_{10,0}(t)[c]R_0(t) + q_{10,1}^{(11)}(t)[c]R_1(t)+ q_{10,2}^{(11)}(t)[c]R_2(t)$

(19-25)

where

$S_1(t) = p\, G_1\overline{(t)}\ e^{-\lambda_1 t}$, $S_2(t) = q\, \underline{G_2}(t)\ e^{-\lambda_2 t}$

$S_3(t) = S_8(t) = S_9(t) = G_3(t)$

$S_{10}(t) = G_4(\overline{t})$

(26)

Taking Laplace Transform of eq. (19-25) and solving for $\overline{R_0}(s)$

$\overline{R_0}(s) = N_3(s) / D_2(s)$

(27)

where

$N_3(s) = \{\hat{q}_{01} +\hat{q}_{0,10} \hat{q}_{10,1}^{(11)}\}[\hat{S}_1(1 - \hat{q}_{28}^{(7)} \hat{q}_{82}\} + \hat{q}_{12}^{(5)}[\hat{S}_2 + \hat{q}_{23}\hat{S}3+ \hat{q}_{28}^{(7)} \hat{S}8+\hat{q}_{29}^{(6)} \hat{S}9)]]+ \{\hat{q}_{02} +\hat{q}_{0,10} \hat{q}_{10,2}^{(11)}\} [\{ \hat{S}_2+ \hat{q}_{23}\hat{S}_3 +\hat{q}_{28}^{(7)} \hat{S}_8 + \hat{S}_9 \hat{q}_{29}^{(6)})(1- \hat{q}_{11}^{(4)})+ \hat{S}_1 \hat{q}_{29}^{(6)} \hat{q}_{91}] + \hat{q}_{0,10} \hat{S}_{10} [\{1-\hat{q}_{28}^{(7)}\hat{q}_{82}\}\{1- \hat{q}_{11}^{(4)}\}-\hat{q}_{29}^{(6)}\hat{q}_{91} \hat{q}_{12}^{(5)}]$

and $D_2(s)$ is already defined.

(Omitting the arguments s for brevity)

In the long run, $R_0 = \frac{N_3(0)}{D_2'(0)}$

(28)

Where

$N_3(0) = \{p_{01} +p_{0,10}\, p_{10,1}^{(11)}\}[\hat{S}_1(1 - p_{28}^{(7)}\} +p_{12}^{(5)}[\hat{S}_2 +p_{23}\hat{S}3+p_{28}^{(7)} \hat{S}8+p_{29}^{(6)} \hat{S}9)]]+ \{p_{02} +p_{0,10}\, p_{10,2}^{(11)}\} [\{ \hat{S}_2+ p_{23}\hat{S}_3 +p_{28}^{(7)} \hat{S}_8 + \hat{S}_9\ p_{29}^{(6)})(1- p_{11}^{(4)})+ \hat{S}_1 p_{29}^{(6)}] + p_{0,10}\ \hat{S}_{10} [\{1-p_{28}^{(7)}\}\{1- p_{11}^{(4)}\}- p_{29}^{(6)} p_{12}^{(5)}]$

and $D_2(0)$ is already defined.

The expected busy period of the server when there is failure due to ultra light and failure due to effect of thunderstorm on aircraft in (0,t]

$\lambda_{rv}(t) = \int_0^{\infty} R_0\,(z)dz$ So that $\widetilde{\lambda_{rv}}\,(s) = \frac{\hat{R}_0\,(s)}{s}$

The expected number of visits by the repairman Type-I or Type-II for repairing the identical units in (0,t]-H_0

$H_0(t) = Q_{01}(t)[s][1+ H_1(t)] + Q_{02}(t)[s][1+H_2(t)]+Q_{0,10}(t)[s]\ H_{10}(t)]$
$H_1(t) = Q_{10}(t)[s]H_0(t)] + Q_{12}^{(5)}(t)[s]\ H_2(t) + Q_{11}^{(4)}(t)]\ [s]H_1(t)$,
$H_2(t) = Q_{23}(t)[s]H_3(t) + Q_{28}^{(7)}(t)\ [s]\ H_8(t) +Q_{29}^{(6)}(t)]\ [c]H_9(t)$
$H_3(t) = Q_{30}(t)[s]H_0(t)$
$H_8(t) = Q_{82}(t)[s]H_2(t)$
$H_9(t) = Q_{91}(t)[s]H_1(t)$
$H_{10}(t) = Q_{10,0}(t)[s]H_{10}(t)] + Q_{10,1}^{(11)}(t)[s]H_1(t)]+Q_{10,2}^{(11)}(t)[s]\ H_2(t)]$
(29-35)

Taking Laplace Transform of eq. (29-35) and solving for $H_0^*(s)$

$H_0^*(s) = N_4(s) / D_3(s)$
(36)

$N_4(s) = \{ Q_{01}^* + Q_{02}^*\}[\{ 1 – Q_{11}^{(4)*}\}\{1\text{-}Q_{28}^{(7)*}\ Q_{82}^*\} – Q_{12}^{(5)*}\ Q_{29}^{(6)*}\ Q_{91}^*\ ]$

And

$D_3(s) = \{1 – Q_{11}^{(4)*}\}\ \{ 1\text{-}\ Q_{28}^{(7)*}\ Q_{82}^*\} – Q_{12}^{(5)*}\ Q_{29}^{(6)*}\ Q_{91}^*](1\text{-}\ Q_{0,10}^*\ Q_{10,0}^*)\text{-}\{ Q_{01}^*+ Q_{0,10}^*\ Q_{10,1}^{(11)*}\}[Q_{10}^*\{ 1 – Q_{28}^{(7)*}\ Q_{82}^*\}+ Q_{12}^{(5)*}\ Q_{23}^*\ Q_{30}^*] – \{Q_{02}^* + Q_{0,10}^*\ Q_{10,2}^{(11)*}\}[Q_{23}^*\ Q_{30}^*\{1 – Q_{11}^{(4)*}\}+ Q_{29}^{(6)*}\ Q_{91}^*\ Q_{10}^*]$

(Omitting the arguments s for brevity)

In the long run,

$H_0 = N_4(0) / D_3'(0)$ (37)

where

$N_4(0) =\{1 – p_{0,10}\}[\{1 – p_{11}^{(4)}\}\ \{ 1\text{-}\ p_{28}^{(7)}\} – p_{12}^{(5)}\ p_{29}^{(6)}]$

The expected number of visits by the multispecialty repairman Type-III for repairing the identical units in (0,t]-W_0

$W_0(t)=Q_{01}(t)[s]W_1(t)+ Q_{02}(t)[s]\ W_2(t) + Q_{10,0}(t)[s]\ W_{10}(t)$
$W_1(t) = Q_{10}(t)[s]W_0(t)] + Q_{12}^{(5)}(t)[s]\ W_2(t) + Q_{11}^{(4)}(t)]\ [s]W_1(t)$,
$W_2(t) = Q_{23}(t)[s]W_3(t) + Q_{28}^{(7)}(t)\ [s]\ W_8(t) +Q_{29}^{(6)}(t)]\ [c]W_9(t)$
$W_3(t) = Q_{30}(t)[s][1+W_0(t)\ ]$
$W_8(t) = Q_{82}(t)[s][1+W_2(t)\ ]$
$W_9(t) = Q_{91}(t)[s][1+W_1(t)\ ]$
$W_{10}(t)=Q_{10,0}(t)[s]W_0(t)+ Q_{10,1}^{(11)}(t)[s]\ W_1(t) + Q_{10,2}^{(12)}(t)[s]\ W_2(t)$
(38-44)

Taking Laplace Transform of eq. (33-39) and solving for $H_0^*(s)$

$H_0^*(s) = N_5(s) / D_3(s)$
(45)

$N_5(s) = \{Q_{01}^*+ Q_{0,10}^*Q_{0,10}^{(11)*}\}[Q_{12}^{(5)*}\ [Q_{23}^*\ Q_{30}^* + Q_{28}^{(5)*}\ Q_{82}^* + Q_{29}^{(6)*}\ Q_{91}^*\ ] + \{Q_{02}^* + Q_{0,10}^*Q_{10,2}^{(11)*}\}[[Q_{23}^*\ Q_{30}^* + Q_{28}^{(5)*}\ Q_{82}^* + Q_{29}^{(6)*}\ Q_{91}^*\ \{1 – Q_{11}^{(4)*}\}]$

(Omitting the arguments s for brevity)

In the long run,

$W_0 = N_5(0) / D_3'(0)$ (46)

where $N_5(0) = \{p_{01}+ p_{0,10}\ p_{10,1}^{(11)}\}p_{12}^{(5)} + \{ p_{02}+ p_{0,10}\ p_{10,2}^{(11)}\}\ \{1 – p_{11}^{(4)}\}]$

The expected number of visits by the multispecialty repairman Type-IV for repairing the identical units in (0,t]-Y_0

$Y_0(t)=Q_{01}(t)[s]Y_1(t)+ Q_{02}(t)[s]\ Y_2(t) + Q_{0,10}(t)[s]\ [1+Y_{10}(t)]$
$Y_1(t) = Q_{10}(t)[s]Y_0(t) + Q_{12}^{(5)}(t)[s]Y_2(t) + Q_{11}^{(4)}(t)]\ [s]Y_1(t)$,
$Y_2(t) = Q_{23}(t)[s]Y_3(t) + Q_{28}^{(7)}(t)\ [s]Y_8(t) +Q_{29}^{(6)}(t)]\ [c]Y_9(t)$
$Y_3(t) = Q_{30}(t)[s][1+Y_0(t)\ ]$
$Y_8(t) = Q_{82}(t)[s]Y_2(t)$

$Y_9(t) = Q_{91}(t)[s]Y_1(t)$
$Y_{10}(t)=Q_{10,0}(t)[s]Y_0(t)+ Q_{10,1}^{(11)}(t)[s]\ Y_1(t) + Q_{10,2}^{(12)}(t)[s]\ Y_2(t)$
(47-53)
Taking Laplace Transform of eq. (47-53) and solving for $Y_0^*(s)$,we get
$Y_0^*(s) = N_6(s) / D_3(s)$
(54)
$N_6(s) = Q_{0,10}^* [\{1 - Q_{11}^{(4)*}\}(1- Q_{28}^{(5)*} Q_{82}^* \} - Q_{12}^{(5)*}Q_{29}^{(6)*} Q_{91}^*\{1- Q_{0,10}^*Q_{,10,0}^* \}+\{Q_{02}^* + Q_{0,10}^*Q_{10,2}^{(11)*}\}[[Q_{23}^* Q_{30}^* \{1 - Q_{11}^{(4)*}\}+ Q_{10}^* Q_{29}^{(6)*} Q_{91}^*]$
(Omitting the arguments s for brevity)
In the long run,
$W_0 = N_6(0) / D_3'(0)$ (55)
where $N_6(0) = p_{0,10}[\{1-p_{11}^{(4)}\}\{1- p_{28}^{(7)}\}- p_{12}^{(5)}\ p_{29}^{(6)}]$
$p_{12}^{(5)} + \{ p_{02}+ p_{0,10}\, p_{10,2}^{(11)}\} \{1 - p_{11}^{(4)}\}]$

BENEFIT-FUNCTION

The Benefit-Function analysis of the system considering mean up-time, expected busy period of the system under failure due to ultra light and failure due to effect of thunderstorm on aircraft, expected number of visits by the repairman for unit failure. The expected total Benefit-Function incurred in (0,t] is

$C = \lim_{t\to\infty}(C(t)/t) = \lim_{s\to 0}(s^2 C(s))$
$= K_1A_0 - K_2R_0 - K_3H_0 - K_4W_0 - K_5Y_0$

where
K_1 - revenue per unit up-time,
K_2 - cost per unit time for which the system is busy under repairing,
K_3 - cost per visit by the repairman type- I or type- II for units repair,
K_4 - cost per visit by the multispecialty repairman Type- III for units repair,
K_5 - cost per visit by the multispecialty repairman Type- IV for units repair

CONCLUSION

After studying the system, we have analyzed graphically that when the failure rate due to ultra light and due to effect of thunderstorm on aircraft increases, the MTSF, steady state availability decreases and the Profit-function decreased as the failure increases.

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