Research Article

The product estimators for estimating population mean using auxiliary information on both occasions in successive sampling

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Abstract In this paper, we propose a product ratio estimator using one auxiliary variable for the estimation of the current population mean under successive sampling scheme. The estimator has been studied by Murthy (1964) using one auxiliary variable for estimation of the population mean. The bias and mean squared error are obtained upto the first order of approximation. We show theoretically that the proposed estimator is more efficient than the estimator proposed by Cochran[2] using no auxiliary variable and simple mean per unit estimator. Optimum replacement strategy is also discussed. Results have been justified through empirical interpretation.
Keywords: Auxiliary variable, Bias, Mean square error, Product Estimator, Optimum replacement, Successive sampling.

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INTRODUCTION

Now a days, it is often seen that sample surveys are not limited to one time inquiries. A survey carried out on a finite population is subject to change overtime if the value of study character of a finite population is subject to change (dynamic) overtime. A survey carried out on a single occasion will provide information about the characteristics of the surveyed population for the given occasion only and cannot give any information on the nature or the rate of change of the characteristics over different occasions and the average value of the characteristics over all occasions or most recent occasion. A part of the sample is retained being replaced for the next occasion (or sampling on successive occasions, which is also called successive sampling or rotation sampling). The successive method of sampling consists of selecting sample units on different occasions such that some units are common with samples selected on previous occasions. If sampling on successive occasions is done according to a specific rule, with replacement of sampling units, it is known as successive sampling. Replacement policy was examined by Jessen[5] who examined the problem of sampling on two occasions, without or with replacement of part of the sample in which what fraction of the sample on the first occasion should be replaced in order that the estimator of $\bar{Y}$ may have maximum precision. Yates[15] extended Jessen's scheme to the situation where the population mean of a character is estimated on each of $(h > 2)$ occasions from a rotation sample design. These results were generalized by Patterson[8] and Narain[7], among others. Rao and Mudhdkar[9] and

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Das[3], used the information collected on the previous occasions for improving the current estimate. Data regarding changing properties of the population of cities or counties and unemployment statistics are collected regularly on a sample basis to estimate the changes from one occasion to the next or to estimate the average over a certain period. An important aspect of continuous surveys is the structure of the sample on each occasion. To meet these requirements, successive sampling provides a strong tool for generating the reliable estimates at different occasions. Sen[10] developed estimators for the population mean on the current occasion using information on two auxiliary variables available on previous occasion. Sen[11] extended his work for several auxiliary variables. Singh {\it{et.al}}[12] and Singh and Singh[14] used the auxiliary information on current occasion for estimating the current population mean in two occasions successive sampling and Singh[13] extended his work for $h-$occasions successive sampling. Feng and Zou[4] and Biradar and Singh[1] used the auxiliary information on both the occasions for estimating the current mean in successive sampling. In many situations, information on an auxiliary variate may be readily available on the first as well as the second occasions; for example, tonnage (or seat capacity) of each vehicle or ship is known in survey sampling of transportation and number of beds in hospital surveys. Most of the studies related to product estimators have been developed by Murthy[6]. He considered, the relationship between the response y and the subsidiary variate x, is linear through the origin and variance of y is proportional to x. It is assumed that X is known. Motivated with the above argument and utilizing the information on an addition auxiliary variable is available on the both occasions, the product estimator for estimating the population mean on current occasion in successive sampling has been proposed. It has been assumed that the additional auxiliary variable over two occasions. The paper is spread over nine sections. Sample structure and notations have been discussed in section 2 and section 3 respectively. In section 4, the proposed estimators have been formulated and properties of proposed including mean square error are derived. In section 5, optimum replacement policy is discussed. Section 6 contains comparison of the proposed estimator with Cochran[3] and simple mean per unit when there is no matching from the previous occasion and the estimator when no additional auxiliary information has been used. In Section 7, the theoretical results are supported by a numerical interpretation and we give conclusion in Section 8.

Selection of the sample

Consider a finite population U = (U1, U2…UN) which has been sampled over two occasions. Let x and y be the study variables on the first and second occasions respectively. We further assumed that the information on the auxiliary variable z, whose population mean is known which is closely related (negatively related) to x and y on the first and second occasions respectively available on the first as well as on the second occasion. For convenience, it is assumed that the population under consideration is large enough. Allowing SRSWOR (Simple Random Sampling without Replacement) design in each occasion, the successive sampling scheme as follows is carried out:

- We have n units which constitutes the sample on the first occasion. A random sub sample of nm = nλ (0 < λ < 1) units is retained (matched) for use on the second occasion.
- In the second occasion nu = nμ (= n - nm) (0 < μ < 1) units are drawn from the remaining (N - n) units of the population.

Where μ is the fraction of fresh sample on the current occasion. So that the sample size on the second occasion is also n (= nλ + nμ).

Description of Notations

We use the following notations in this paper.

$\bar{X}$: The population mean of the study variable on the first occasion.

$\bar{Y}$: The population mean of the study variable on the second occasion.

$\bar{Z}$: The population mean of the auxiliary variable on both occasions.

S_y^2: Population mean square of y.

$\bar{z}_n$: The sample mean based on n units drawn on the first occasion.

$\bar{z}_{n_u}$: The sample mean based on n_u units drawn on the second occasion.

$\bar{x}_n$: The sample mean based on n units drawn on the first occasion.

$\bar{x}_{n_m}$ The sample mean based on n_m units observed on the second occasion and common with the first occasion.

$\bar{y}_{n_u}$: The sample mean based on n_u units drawn afresh on the second occasion.

$\bar{y}_{n_m}$: The sample mean based on $n_m$units common to both occasions and observed on the first occasion.

ρ_{yx}: The correlation coefficient between the variables y on x.

ρ_{xz}: The correlation coefficient between the variables x on z.

ρ_{yz}: The correlation coefficient between the variables y on z.

n_m: The sample units observed on the second occasion and common with the first occasion.
n_u: The sample size of the sample drawn afresh on the second occasion.
n: Total sample size.

PROPOSED PRODUCT ESTIMATORS IN SUCCESSIVE SAMPLING

In this section some product ratio estimators using one auxiliary variable have been proposed. To estimate the population mean $\bar{Y}$ on the second occasion, two different estimators are suggested. The first estimator is product ratio estimator based on sample of size nu (= nμ) drawn afresh on the second occasion and is given by:

$$t_{nu} = \bar{y}_{nu} \frac{\bar{z}_{nu}}{\bar{Z}} . \quad (4.1)$$

The second estimator is a chain product ratio estimator based on the sample of size nm (= nλ) common with both the occasions and is defined as,

$$t_{nm} = \bar{y}_{nm} \frac{\bar{x}_{nm}}{\bar{x}\bar{n}} \frac{\bar{z}_n}{\bar{Z}} , \quad (4.2)$$

combining the estimators t_{n_u} and t_{n_m} , we have the final estimator tpr as follows

$$\text{tpr}=\psi\ t_{n_u} + (1 - \psi)\ t_{n_m}, \quad (4.3)$$

where ψ is an unknown constant to be determined such that MSE (tpr) is minimum. We prove theoretically that the estimator is more efficient than the proposed estimator by (i) Cochran[3] when no auxiliary variables are used at any occasion. This classical difference estimator is a widely used estimator to estimate the population mean $\bar{Y}$, in successive sampling. It is given by

$$\bar{y}'_2 = {}_{\varphi}\bar{y}'_{2u} + (1- {}_{\varphi})\ \bar{y}'_{2m} ,$$

where $_{\varphi}$ is an unknown constant to be determined such that V $(\hat{\bar{Y}})_{opt}$ is minimum and $\bar{y}'_{2u} = \bar{y}_{2u}$ is the sample mean of the unmatched portion on the second occasion and $\bar{y}'_{2m} = \bar{y}_{2m}$ +b $(\bar{y}_1 - \bar{y}_{1m})$ is based on matched portion. The variance of this estimator is

$$V\ (\hat{\bar{Y}})_{opt} = [1 + \sqrt{(1 - \rho_{yx}^2)}\ ] \frac{S_y^2}{2n}.$$

Similarly, the variance of the mean per unit estimator is given by

$$V(\bar{y}) = \frac{S_y^2}{n}.$$

PROPERTIES OF TPR

Since t_{n_u} and t_{n_m} both are biased estimators of tpr, therefore, resulting estimator tpr is also a biased estimator. The bias and MSE up to the first order of approximation are derived as using large sample approximation given below:

$\bar{y}_{n_u} = \bar{Y}\ (1 + e_{\bar{y}_{n_u}}), \bar{y}_{n_m} = \bar{Y}\ (1 + e_{\bar{y}_{n_m}}),$
$\bar{x}_{n_m} = \bar{X}\ (1 + e_{\bar{x}_{n_m}}), \bar{x}_n = \bar{X}\ \left(1 + e_{\bar{x}_n}\right),$
$\bar{z}_n = \bar{Z}\ (1 + e_{\bar{z}_n}), \bar{z}_u = \bar{Z}\ (1 + e_{\bar{z}_u}),$

Where $e_{\bar{y}_{n_u}}, e_{\bar{y}_{n_m}}, e_{\bar{x}_{n_m}}, e_{\bar{x}_n}, e_{\bar{z}_u}$, and $e_{\bar{z}_n}$ are sampling errors and are of very small quantities. We assume that E $(e_{\bar{y}_{n_u}})$ = E $(e_{\bar{y}_{n_m}})$ = E $(e_{\bar{x}_{n_m}})$ = E $(e_{\bar{x}_n})$ = E $(e_{\bar{z}_u})$ = E $(e_{\bar{z}_n})$ = 0.

Then for simple random sampling without replacement for both first and second occasions, we write by using occasion wise operation of expectation as:

$E\ (e_{\bar{y}_{n_u}}^2) = (\frac{1}{n_u} - \frac{1}{N})\ S_y^2$, $E\ (e_{\bar{y}_{n_m}}^2) = \left(\frac{1}{n_m} - \frac{1}{N}\right) S_y^2$,
$E\ (e_{\bar{x}_{n_m}}^2) = (\frac{1}{n_m} - \frac{1}{n})\ S_x^2$, $E\ (e_{\bar{x}_n}^2) = \left(\frac{1}{n} - \frac{1}{N}\right) S_x^2$,
$E\ (e_{\bar{z}_n}^2) = (\frac{1}{n} - \frac{1}{N})\ S_z^2$,
$E\ (\ e_{\bar{y}_{n_u}} e_{\bar{z}_{n_u}}\ ) = (\frac{1}{n_u} - \frac{1}{N})\ S_{yz}$, $E\ (\ e_{\bar{y}_{n_m}} e_{\bar{x}_n}\ ) = (\frac{1}{n_m} - \frac{1}{n})\ S_{yx}$,
$E\ (\ e_{\bar{y}_{n_m}} e_{\bar{x}_{n_m}}\ ) = (\frac{1}{n_m} - \frac{1}{n})\ S_{yx}$, $E\ (\ e_{\bar{y}_{n_m}} e_{\bar{x}_n}\ ) = (\frac{1}{n} - \frac{1}{N})\ S_{yx}$,
$E\ (\ e_{\bar{y}_{n_m}} e_{\bar{z}_n}\ ) = (\frac{1}{n} - \frac{1}{N})\ S_{yz}$, $E\ (\ e_{\bar{x}_{n_m}} e_{\bar{x}_n}\ ) = (\frac{1}{n} - \frac{1}{N}) S_x^2$,
$E\ (\ e_{\bar{x}_{n_m}} e_{\bar{z}_n}\ ) = (\frac{1}{n} - \frac{1}{N})\ S_{xz}$, $E\ (\ e_{\bar{x}_n} e_{\bar{z}_n}\ ) = (\frac{1}{n} - \frac{1}{N})\ S_{xz}$,

We derive the bias of t_{n_u} and t_{n_m} in lemma 4.1. and lemma 4.2.

Lemma 4.1.: The bias of t_{n_u} denoted by B (t_{n_u}) is given by
B (t_{n_u}) = $\bar{Y}$ $(\frac{1}{n_u} - \frac{1}{N})$ $\rho_{yz} S_y S_z$.
Proof: Expressing (4.1) in terms of e's, we have
$t_{n_u} = \bar{Y}\,(1 + e_{\bar{y}_{n_u}})(1 + e_{\bar{z}_{n_u}})$
$= \bar{Y}\,(e_{\bar{y}_{n_u}} + e_{\bar{z}_{n_u}} + e_{\bar{y}_{n_u}} e_{\bar{z}_{n_u}})$,
Taking expectation on both sides and ignoring higher orders, we get
$t_{n_u} = \bar{Y}\,(\frac{1}{n_u} - \frac{1}{N})$ E ($e_{\bar{y}_{n_u}} e_{\bar{z}_{n_u}}$)
B (t_{n_u}) = $\bar{Y}$ $(\frac{1}{n_u} - \frac{1}{N})$ $\rho_{yz} S_y S_z$.
Lemma 4.2: The bias of t_{n_m} is denoted by B (t_{n_m})given by
B (t_{n_m}) = $\bar{Y}$ $(\frac{1}{n_m} - \frac{1}{n})$ $(\rho_{yx} S_y S_x$ - $S_x^2)$ + $(\frac{1}{n} - \frac{1}{N})$ $\rho_{yz} S_y S_z$.
Proof: Expressing (4.2) in terms of e's, we have
$t_{n_m} = \bar{Y}\,(1 + e_{\bar{y}_{n_m}})\,(1 + e_{\bar{x}_n})^{-1}\left(1 + e_{\bar{x}_{n_m}}\right)\left(1 + e_{\bar{z}_n}\right)$,
Expanding the right hand side and ignoring higher orders, we get
$t_{n_m} = \bar{Y}\,(1 + e_{\bar{y}_{n_m}})\,(1 - e_{\bar{x}_n} + {e_{\bar{x}_n}}^2)\left(1 + e_{\bar{x}_{n_m}}\right)\left(1 + e_{\bar{z}_n}\right)$
$= \bar{Y}\,[1 + e_{\bar{y}_{n_m}} + e_{\bar{x}_{n_m}} - e_{\bar{x}_n} + {e_{\bar{x}_n}}^2 + e_{\bar{z}_n} - e_{\bar{y}_{n_m}} e_{\bar{x}_{n_m}} - e_{\bar{x}_n} e_{\bar{z}_n} - e_{\bar{y}_{n_m}} e_{\bar{x}_n} +$
$e_{\bar{y}_{n_m}} e_{\bar{z}_n} - e_{\bar{x}_n} + e_{\bar{x}_n} e_{\bar{z}_n}]$, (4.5)
Taking expectation (4.5) on both sides, we get
B (t_{n_m}) = $\bar{Y}$ $(\frac{1}{n_m} - \frac{1}{n})$ $(\rho_{yx} S_y S_x$ - $S_x^2)$ + $(\frac{1}{n} - \frac{1}{N})$ $\rho_{yz} S_y S_z$. (4.6)
Using lemma 4.1. and 4.2., the bias of the estimator t_{pr} can be derived as follows;
Theorem4.4: Bias of the estimator t_{pr} to the first order approximation is,
B (t_{pr}) = ψ B (t_{n_m}) + (1- ψ) B (t_{n_m}), (4.7)
Where
B (t_{n_u}) = $\bar{Y}$ $(\frac{1}{n_u} - \frac{1}{N})$ $\rho_{yz} S_y S_z$,
And
B (t_{n_m}) = $\bar{Y}$ $(\frac{1}{n_m} - \frac{1}{n})$ $(\rho_{yx} S_y S_x$ - $S_x^2)$ + $(\frac{1}{n} - \frac{1}{N})$ $\rho_{yz} S_y S_z$.
Proof: The bias of the estimator $t_{pr}$is given by
B (t_{pr}) =E $(t_{pr} - \bar{Y})$
B (t_{pr}) = ψ E $(t_{n_u} - \bar{Y})$ + (1- ψ) E $(t_{n_m} - \bar{Y})$, (4.8)
Using lemma (4.1) and (4.2) into equation (4.8), we have the expression for the bias of the estimator t_{pr} as shown in (4.7).
We derive the MSE of t_{n_u} and t_{n_m} in lemma 4.1. and lemma 4.2. respectively.
Lemma 4.3: The mean square error of t_{n_u} denoted by M (t_{n_u}) is given by
M (t_{n_u}) = $\bar{Y}^2$ $(\frac{1}{n_u} - \frac{1}{N})$ $[S_y^2 + S_x^2 + 2\rho_{yx} S_y S_x]$.
Proof: Expressing (4.1) in terms of e's, we have
$t_{n_u} = \bar{Y}\,(1 + e_{\bar{y}_{n_u}})(1 + e_{\bar{z}_{n_u}})$ (4.9)
Expanding and squaring (4.9), the right hand side and ignoring the higher terms, we get
$t_{n_u} = \bar{Y}^2\,(1 + e_{\bar{y}_{n_u}} + e_{\bar{z}_{n_u}})^2$
$= \bar{Y}^2\,(1 + {e_{\bar{y}_{n_u}}}^2 + {e_{\bar{z}_{n_u}}}^2 + 2 e_{\bar{y}_{n_u}} e_{\bar{z}_{n_u}})$ (4.10)
Taking expectation (4.10) on both sides, we get M (t_{n_u})
M (t_{n_u}) = $\bar{Y}^2 E({e_{\bar{y}_{n_u}}}^2 + {e_{\bar{z}_{n_u}}}^2 + 2 e_{\bar{y}_{n_u}} e_{\bar{z}_{n_u}})$,
M (t_{n_u}) = $\bar{Y}^2$ $(\frac{1}{n_u} - \frac{1}{N})$ $[S_y^2 + S_x^2 + 2\rho_{yx} S_y S_x]$.
Lemma 4.4: The mean square error of t_{n_u} denoted by M (t_{n_u}) is given by

M (t_{n_m}) = $\bar{Y}^2 (\frac{1}{n_m} - \frac{1}{N}) S_y^2 + ((\frac{1}{n_m} - \frac{1}{n})[S_x^2 + 2\rho_{yx}S_yS_x] + (\frac{1}{n} - \frac{1}{N})[S_z^2 + 2\rho_{yz}S_yS_z]$.

Proof: Expressing (4.2) in terms of e's, we have

$t_{n_m} = \bar{Y}(1 + e_{\bar{y}_{n_m}})(1 + e_{\bar{x}_n})^{-1}\left(1 + e_{\bar{x}_{n_m}}\right)\left(1 + e_{\bar{z}_n}\right)$,

$\approx \bar{Y}(1 + e_{\bar{y}_{n_m}})(1 + e_{\bar{x}_n})\left(1 + e_{\bar{x}_{n_m}}\right)\left(1 + e_{\bar{z}_n}\right)$, (4.12)

Expanding and squaring on both sides (4.12) and taking expectation, we get MSE of the estimator t_{n_m} up to first order of approximation as,

M (t_{n_m}) =$\bar{Y}^2 E(e_{\bar{y}_{n_m}}{}^2 + e_{x_n}{}^2 - 2e_{\bar{y}_{n_m}}e_{\bar{x}_n} + e_{\bar{x}_{n_m}}{}^2 + e_{\bar{z}_n}{}^2 + 2e_{\bar{y}_{n_m}}e_{\bar{x}_{n_m}} + e_{\bar{y}_{n_m}}e_{\bar{z}_n} - 2e_{\bar{x}_{n_m}}e_{\bar{z}_n} + 2e_{\bar{x}_{n_m}}e_{\bar{z}_n} - 2e_{\bar{x}_n}e_{\bar{z}_n})$,

M (t_{n_m}) = $\bar{Y}^2 (\frac{1}{n_m} - \frac{1}{N}) S_y^2 + ((\frac{1}{n_m} - \frac{1}{n})[S_x^2 + 2\rho_{yx}S_yS_x] + (\frac{1}{n} - \frac{1}{N})[S_z^2 + 2\rho_{yz}S_yS_z]$.

Theorem 4.3: The mean square error of the estimator t_{pr} to the first order approximation is,

M (t_{pr}) = $\psi^2 M(t_{n_u}) + (1-\psi)^2 M(t_{n_m}) + 2\psi(1-\psi)Cov(t_{n_u}\, t_{n_m})$, (4.13)

Where

M (t_{n_u}) = $\bar{Y}^2 (\frac{1}{n_u} - \frac{1}{N})[S_y^2 + S_x^2 + 2\rho_{yx}S_yS_x]$,

M (t_{n_m}) = $\bar{Y}^2 (\frac{1}{n_m} - \frac{1}{N}) S_y^2 + ((\frac{1}{n_m} - \frac{1}{n})[S_x^2 + 2\rho_{yx}S_yS_x] + (\frac{1}{n} - \frac{1}{N})[S_z^2 + 2\rho_{yz}S_yS_z]$

And $Cov(t_{n_u}\, t_{n_m})$ = 0.

Proof: The mean square error of the estimator t_{pr} is given by

M (t_{pr}) $= E(t_{pr} - \bar{Y})^2$

M (t_{pr}) = $E[\psi(t_{n_u} - \bar{Y}) + (1-\psi)(t_{n_m} - \bar{Y})]^2$ (4.14)

M (t_{pr}) = $\psi^2 M(t_{n_u}) + (1-\psi)^2 M(t_{n_m}) + 2\psi(1-\psi)Cov(t_{n_u}\, t_{n_m})$,

Using lemma (4.3) and (4.4) into equation (4.14), we have the expression for the MSE of the estimator t_{pr} as shown in (4.13).

Remark 4.4: The estimators t_{n_u} *and* t_{n_m} are based on two independent samples of sizes n_u and n_m respectively, therefore the covariance term has been vanished.

MINIMUM MEAN SQUARE ERROR OF t_{pr}

To obtain the optimum value ofψ, we partially differentiate the expression (4.14) with respect to ψ, and put it equal to zero, we get

$\psi_{opt} = \frac{M(t_{n_m})}{M(t_{n_u}) + M(t_{n_m})}$, (5.1)

Substituting the values of $M(t_{n_u})$ *and* $M(t_{n_m})$ from (4.9) and (4.10) in (5.1), we get

$$\psi_{opt} = \frac{k_1 + \mu k_2}{k_1 + \mu^2 k_2}$$

$= \frac{\mu[\mu k_6 + k_2]}{k_2 + \mu k_1 - \mu^2 k_2}$,

Substitution of ψ_{opt} from (5.1) into (4.14) gives optimum value of MSE of t_{pr} as:

M ($t_{pr})_{opt}$= $\frac{M(t_{n_m})M(t_{n_u})}{M(t_{n_u}) + M(t_{n_m})}$ (5.2)

Substituting the values of $M(t_{n_u})$ *and* $M(t_{n_m})$ from (4.9) and (4.10) in (5.2), we get

M ($t_{pr})_{opt}$= $\frac{1}{n}[\frac{k_1{}^2 + \mu k_1 k_2}{k_1 + \mu^2 k_2}]$, (5.3)

Where $k_1 = 2 + 2\rho_{yz}$, $k_2 = 2(\rho_{yx} - \rho_{yz})$, here μ (= $\frac{u}{n}$) is the fraction of fresh sample drawn on the second occasion. Again M ($t_{pr})_{opt}$ derived in equation (5.3) is the function of μ. To estimate the population mean on each occasion the better choice of μ is 1 (no matching). However, to estimate the change in mean from one occasion to the other, μ should be 0 (compete matching).

Remark 5.1: Population size is sufficiently large (i.e., N → ∞), therefore finite population corrections (fpc) are ignored.

Remark 5.2: Population variance of X, Y, and Z are approximately equal.

REPLACEMENT POLICY

In order to estimate t_{pr} with maximum precision an optimum value of μ should be determined so as to know what fraction of the sample on the first occasion should be replaced. We minimize, M $(t_{pr})_{opt}$ in (5.3) with respect to μ, the optimum value of μ is obtained as,

$$\hat{\mu} = \frac{-k_1 \pm \sqrt{k_1^2+k_1k_2}}{k_2}, \tag{6.1}$$

Where $k_1 = 2 + 2\rho_{yz}$, $k_2 = 2$s obvious that for $\rho_{yz} \neq \rho_{yx}$ two values of $\hat{\mu}$ are possible, therefore to choose a value of$\hat{\mu}$, it should be remembered that $0 \leq \hat{\mu} \leq 1$. All other values of $\hat{\mu}$ are inadmissible. If both the real values of $\hat{\mu}$ are admissible, the lowest one will be the best choice as it reduces the total cost of the survey. Substituting the value of $\hat{\mu}$ from (6.1) in (5.3), we get

$$M\,(t_{pr})_{opt} = \frac{1}{n}\left[\frac{k_1^2+\hat{\mu}k_1k_2}{k_1+\hat{\mu}^2k_2}\right]. \tag{6.2}$$

EFFICIENCY COMPARISONS

In this section, to compare t_{pr} with respect to$\bar{y}$, (i) sample mean of y, when a sample units are selected at second occasion without any matched portion. (ii) difference estimator (Cochran[3]) when no auxiliary information is used at any occasion, have been obtained for known values of ρ_{yx} and ρ_{yz}. Since $\bar{y}$ and $\hat{\bar{Y}}$ are unbiased estimators of$\bar{Y}$, their variances for large N are respectively given by

$$V(\bar{y}) = \frac{S_y^2}{n}, \tag{7.1}$$

$$V\,(\hat{\bar{Y}})_{opt} = \left[1+\sqrt{(1-\rho_{yx}^2)}\,\right]\frac{S_y^2}{2n}. \tag{7.2}$$

For different values ρ_{yx} and ρ_{yz}, the below shows the optimum value of μ. That is$\hat{\mu}$. The percent relative efficiencies, R_1 *and* R_2 of $(t_{pr})_{opt}$ with respect to $\bar{y}$ and $\hat{\bar{Y}}$ respectively, where

$$R_1 = \frac{V(\bar{y})}{M\,(t_{pr})_{opt}} \text{ X } 100$$

And

$$R_2 = \frac{V\,(\hat{\bar{Y}})_{opt}}{M\,(t_{pr})_{opt}} \text{ X } 100.$$

The estimator t_{pr} (at optimal conditions) is also compared with respect to the estimators $V(\bar{y})$ *and* $V\,(\hat{\bar{Y}})_{opt}$, respectively. Where

$$M\,(t_{pr})_{opt} = \frac{1}{n}\left[\frac{k_1^2+\hat{\mu}k_1k_2}{k_1+\hat{\mu}^2k_2}\right]. \tag{7.3}$$

and

$$\hat{\mu} = \frac{-k_1 \pm \sqrt{k_1^2+k_1k_2}}{k_2}, \tag{7.4}$$

where $k_1 = 2 + 2\rho_{yz}$, $k_2 = 2(\rho_{yx} - \rho_{yz})$

Table 7.1: Optimum values of μ and percent relative efficiencies of tpr with respect to $\bar{y}$ and $\hat{\bar{Y}}$.

ρ_{yz}		ρ_{yx}			
		-0.2	**-0.4**	**-0.6**	**-0.8**
-0.3	$\hat{\mu}$	0.4833	0.5193	0.5695	0.6517
	R1	69.04	74.18	81.36	93.09
	R2	68.34	71.08	73.22	74.47
-0.5	$\hat{\mu}$	0.4415	0.4772	0.5279	0.6125
	R1	88.30	95.45	105.57	122.51
	R2	87.40	91.46	95.02	98.01
	$\hat{\mu}$	0.3797	0.4142	0.4641	0.5505
	R1	126.59	138.07	154.70	183.50
-0.7	R2	125.31	132.31	139.23	146.78

-0.9	$\bar{\hat{\mu}}$	0.2612	0.2898	0.3333	0.4142
	R1	261.12	290.08	333.33	414.21
	R2	258.53	277.98	299.99	331.35

CONCLUSION

Table 7.1 clearly indicates that the suggested estimators are more efficient than simple arithmetic mean and Cochran [3] estimators. The following conclusion can be made from Table 7.1. Fixedρ_{yx},ρ_{yz}, the values of R1 and R2 are increasing. This phenomenon indicates that smaller fresh sample at current occasion is required, if a highly negatively correlated auxiliary characters is available. That is the performance of precision of the estimates as well as reduces the cost of the survey.

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