Research Article

Cost-benefit analysis of two identical warm standby system subject to failure of the fuel pumps and fuel injectors caused by putting the wrong fuel in automobiles

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Abstract

In this paper we have taken failure of the fuel pumps and fuel injectors caused by putting the wrong fuel in automobiles. When the main unit fails then warm standby system becomes operative. Failure of fuel injectors in automobiles cannot occur simultaneously in both the units and after failure the unit undergoes Type-I or Type-II or Type-III repair facility immediately. Applying the regenerative point technique with renewal process theory the various reliability parameters MTSF, Availability, Busy period, Benefit-Function analysis have been evaluated.

Keywords: cost benefit analysis.

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INTRODUCTION

Why petrol is so bad for a diesel engine

By James Mills 3 February 2014 car could be disastrous for your engine as well as your pocket. Dr Geraint Owen from the University of Bath explains exactly what happens when you misfuel a car. He should know. As a senior lecturer in automotive engineering he helps nurture future generations of mechanical engineers and knows everything there is to know about engines.

Critical component 1: The fuel pump

There is a simple reason why petrol is not good for a diesel-powered car. Petrol acts as a solvent which means it prevents the lubricating action that diesel fuel delivers to the precious components of the engine, most significantly the fuel pump. Fill a diesel-powered vehicle with petrol and drive away until it shudders to a halt and you can do serious damage to the fuel pump. That sets off a chain reaction that is very bad news. Think of it like running an engine without oil: there is a high level of friction, the high-pressure fuel pump's high tolerance components are running without lubrication and grind together producing 'swarf'. These are tiny metal fragments, almost like shavings. They enter the fuel system and travel towards the fuel injectors with disastrous consequences.

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Critical component 2: The fuel lines

You've filled your diesel car with petrol, started the engine, driven down the road and suddenly the car conks out. In order to travel from the fuel tank (usually at the rear) to the engine (usually at the front), the petrol will use the fuel lines. These are now contaminated, as are their rubber seals and those of the fuel injectors. Petrol can eat away at some seals in a diesel engine so it needs to be flushed through with a cleaning agent and then assessed for potential damage. In a worst-case scenario, all contaminated parts must be replaced, which is a time-consuming and costly job.

Critical component 3: The fuel filter

A diesel fuel filter's job is to prevent any contaminants coming up from the fuel tank and getting into the engine. Like a racing driver wearing fireproof overalls and a crash helmet, it's a necessary preventative measure but once contaminated by petrol it has to be replaced. It's the simplest and cheapest component of the chain to fix, perhaps costing less than £100, but that's small consolation if the garage finds that serious damage has been done to other parts of the fuel system.

Critical component 4: The fuel injectors

Modern diesel engines operate common rail direct fuel injection systems. These inject diesel directly into the combustion chamber, and are incredibly efficient because they operate at high pressure and under precisely controlled timings. If there's a drawback to these high-tech systems, it's that they have incredibly fine tolerances, injecting fuel through very small holes. The production of metal swarf fragments from the failed fuel pump can have catastrophic consequences. The swarf blocks the holes in the injectors, preventing fuel from entering the engine to be burned which causes the engine to misfire and eventually stop running. Replacement of the fuel injectors and the 'common rail' – the pressurised storage chamber for the diesel fuel – together with the fuel pump is extremely expensive, as in thousands and thousands of pounds. In this paper we have taken failure of Fuel pump and fuel injectors in automobiles. When the main operative unit fails then warm standby system becomes operative. Failure of Fuel pump and fuel injectors in automobiles cannot occur simultaneously in both the units. After failure the unit undergoes repair facility of Type- I or Type- II by ordinary repairman, Type III or Type IV by multispecialty repairman immediately when failure of Fuel pump and fuel injectors in automobiles. The repair is done on the basis of first fail first repaired.

Assumptions

1. $\lambda_{1,}$ λ_2 λ_3 are constant failure rates when failure of warm standby, failure of Fuel pump and fuel injectors in automobiles respectively. The CDF of repair time distribution of Type I, Type II and multispecialty repairmen Type-III, IV are $G_1(t)$, $G_2(t)$ and $G_3(t)$, $G_4(t)$.
2. The failure of fuel injectors in automobiles is non-instantaneous and it cannot come simultaneously in both the units.
3. The repair starts immediately after failure of Fuel pump and fuel injectors in automobiles and works on the principle of first fail first repaired basis. The repair facility does no damage to the units and after repair units are as good as new.
4. The switches are perfect and instantaneous.
5. All random variables are mutually independent.
6. When both the units fail, we give priority to operative unit for repair.
7. Repairs are perfect and failure of a unit is detected immediately and perfectly.
8. The system is down when both the units are non-operative.

SYMBOLS FOR STATES OF THE SYSTEM

Superscripts O, WS, FPAF, FIAF,

Operative, Warm Standby, failure of Fuel pump and fuel injectors in automobiles respectively

Subscripts: nfpaf, fpaf, fiaf, ur, wr, uR

No failure of Fuel pump in automobiles, failure of Fuel pump in automobiles, failure of fuel injectors in automobiles, under repair, waiting for repair, under repair continued from previous state respectively

Up states: 0, 1, 2, 3, 10;

Down states: 4, 5, 6, 7, 8,9,11,

Regeneration point: 0, 1, 2, 3, 8, 9,10

States of the System

0 (O_{nfpaf}, WS_{nfpaf}) One unit is operative and the other unit is warm standby and there is no failure of Fuel pump in automobiles of both the units.

1 ($FPAF_{fpaf,\, urI}$, O_{nfpaf}) The operating unit – failure of Fuel pump in automobiles is under repair immediately of Type- I and standby unit starts operating with no failure of Fuel pump in automobiles

2 ($FIAF_{fiaf,\, urII}$, O_{nfpaf}) The operative unit failure of fuel injectors in automobiles and undergoes repair of Type II and the standby unit becomes operative with no failure of Fuel pump in automobiles
3 ($FIAF_{fiaf,\, urIII}$, O_{nfpaf}) The first unit failure of fuel injectors in automobiles and under Type-III multispecialty repairman and the other unit is operative with no failure of Fuel pump in automobiles
4 ($FPAF_{fpaf,uR1}$, $FPAF_{fpaf,wrI}$) The unit failed due to FPAF resulting from failure of Fuel pump in automobiles under repair of Type- I continued from state 1and the other unit failed due to FPAF resulting from failure of Fuel pump in automobiles is waiting for repair of Type-I.
5 ($FPAF_{fpaf,uR1}$, $FIAF_{fiaf,\, wrII}$) The unit failed due to FPAF resulting from failure of Fuel pump in automobiles is under repair of Type- I continued from state 1and the other unit failure of fuel injectors in automobiles is waiting for repair of Type- II.
6 ($FIAF_{fiaf,\, uRII}$, $FPAF_{fpaf\, ,wrI}$) The operative unit failure of fuel injectors in automobiles is under repair continues from state 2 of Type –II and the other unit failed due to FPAF resulting from failure of Fuel pump in automobiles is waiting under repair of Type-I.
7 ($FIAF_{fiaf\, ,uRII}$, $FPAF_{fpaf,wrII}$) The one unit failure of fuel injectors in automobiles is continued to be under repair of Type II and the other unit failure due to FPAF resulting from failure of Fuel pump in automobiles is waiting for repair of Type-II.
8 ($FPAF_{fpaf,urIII}$, $FIAF_{fiaf,\, wrII}$) The one unit failure of Fuel pump in automobiles is under multispecialty repair of Type-III and the other unit failure of fuel injectors in automobiles is waiting for repair of Type-II.
9 ($FPAF_{fpaf,urIII}$, $FIAF_{fiaf,\, wrI}$) The one unit failure of Fuel pump in automobiles is under multispecialty repair of Type-III and the other unit failure of fuel injectors in automobiles is waiting for repair of Type-I
10 (O_{nfpaf} , $FIAF_{fiaf,\, urIV}$) The one unit is operative with no failure of Fuel pump in automobiles and warm standby unit failure of fuel injectors in automobiles and undergoes repair of type IV.
11 (O_{nfpaf} , $FIAF_{fiaf,\, uRIV}$) The one unit is operative with failure of Fuel pump in automobiles and warm standby unit failure of fuel injectors in automobiles and repair of type IV continues from state 10.

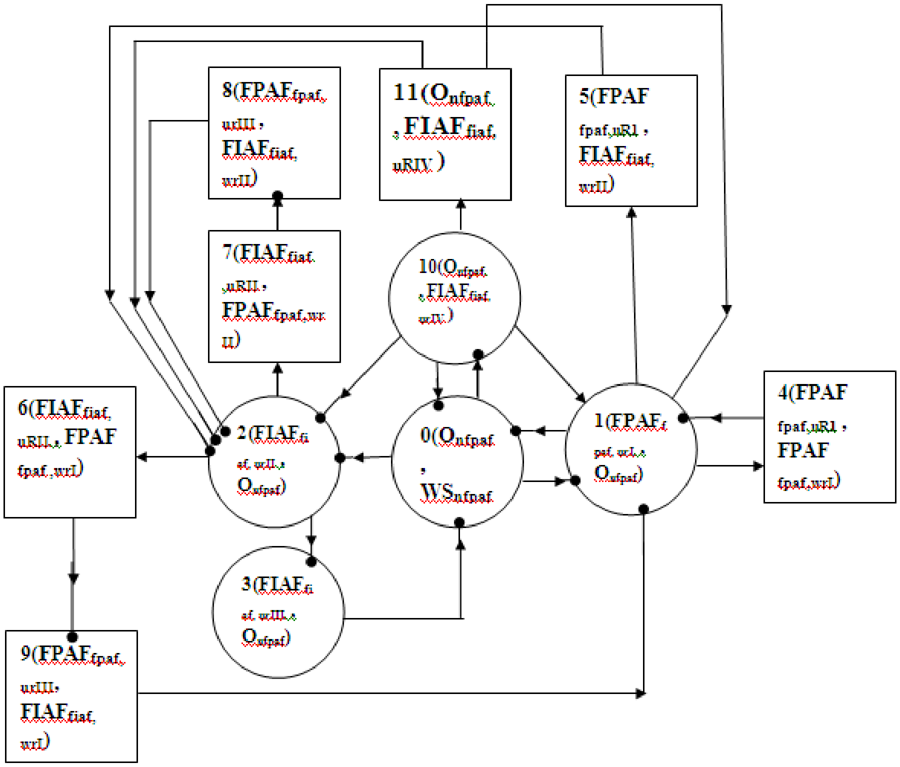

Figure 1: The State Transition Diagram

● Rregeneration point ○ Up State □ Down State

Transition Probabilities

Simple probabilistic considerations yield the following expressions:

$p_{01} = \lambda_1 / \lambda_1 + \lambda_2 + \lambda_3, p_{02} = \lambda_2 / \lambda_1 + \lambda_2 + \lambda_3$,
$p_{0,10} = \lambda_3 / \lambda_1 + \lambda_2 + \lambda_3$, $p_{10} = pG_1^*(\lambda_1) + q G_2^*(\lambda_2)$,
$p_{14} = p - pG_1^*(\lambda_1) = p_{11}^{(4)}$, $p_{15} = q - q G_1^*(\lambda_2) = p_{12}^{(5)}$,
$p_{23} = pG_2^*(\lambda_1) + q G_2^*(\lambda_2)$, $p_{26} = p - pG_2^*(\lambda_1) = p_{29}^{(6)}$,
$p_{27} = q - qG_2^*(\lambda_2) = p_{28}^{(7)}$, $p_{30} = p_{82} = p_{91} = 1$,
$p_{0,10} = pG_4^*(\lambda_1) + q G_4^*(\lambda_2)$, $p_{10,1} = p - pG_4^*(\lambda_1) = p_{10,1}^{(11)}$,
$p_{10,2} = q - q G_4^*(\lambda_2) = p_{10,2}^{(11)}$ (1)

We can easily verify that

$p_{01} + p_{02} + p_{03} = 1$,
$p_{10} + p_{14}\ (=p_{11}^{(4)}) + p_{15}\ (=p_{12}^{(5)}) = 1$,
$p_{23} + p_{26}\ (=p_{29}^{(6)}) + p_{27}\ (=p_{28}^{(7)}) = 1$
$p_{30} = p_{82} = p_{91} = 1$
$p_{10,0} + p_{10,1}^{(11)}\ (=p_{10,1}) + p_{10,2}^{(12)}\ (=p_{10,2}) = 1$ (2)

And mean sojourn time is

$\mu_0 = E(T) = \int_0^\infty P[T > t]\,dt$

MEAN TIME TO SYSTEM FAILURE

$Ø_0(t) = Q_{01}(t)[s]\, Ø_1(t) + Q_{02}(t)[s]\, Ø_2(t) + Q_{0,10}(t)[s]\, Ø_{10}(t)$
$Ø_1(t) = Q_{10}(t)[s]\, Ø_0(t) + Q_{14}(t) + Q_{15}(t)$
$Ø_2(t) = Q_{23}(t)[s]\, Ø_3(t) + Q_{26}(t) + Q_{27}(t)$
$Ø_3(t) = Q_{30}(t)[s]\, Ø_0(t)$
$Ø_{10}(t) = Q_{10,0}(t)[s]\, Ø_{10}(t) + Q_{10,1}(t)[s] Ø_1(t) + Q_{10,2}(t)[s]\, Ø_2(t)$ (3-6)

We can regard the failed state as absorbing
Taking Laplace-Stiljes transform of eq. (3-6) and solving for
$ø_0^*(s) = N_1(s) / D_1(s)$ (7)
where
$N_1(s) = \{Q_{01}^* + Q_{0,10}^*\, Q_{10,1}^*\}\, [\, Q_{14}^*(s) + Q_{15}^*(s)\,] + \{Q_{02}^* + Q_{0,10}^*\, Q_{10,2}^*\}\, [\, Q_{26}^*(s) + Q_{27}^*(s)\,]$
$D_1(s) = 1 - \{Q_{01}^* + Q_{0,10}^*\, Q_{10,1}^*\}\, Q_{10}^* - \{Q_{02}^* + Q_{0,10}^*\, Q_{10,2}^*\}\, Q_{23}^*\, Q_{30}^* - Q_{0,10}^*\, Q_{10,0}^*$
Making use of relations (1) and (2) it can be shown that $ø_0^*(0) = 1$, which implies that $ø_0(t)$ is a proper distribution.

$$\text{MTSF} = E[T] = \frac{d}{ds}\, ø_0^*(s) \Big|_{s=0} = (D_1'(0) - N_1'(0)) / D_1(0)$$

$= (\mu_0 + \mu_1 (p_{01} + p_{0,10}\, p_{10,1}) + (p_{02} + p_{0,10}\, p_{10,2})(\mu_2 + \mu_3) + \mu_{10}\, p_{0,10} / (1 - (p_{01} + p_{0,10}\, p_{10,1})\, p_{10} - (p_{02} + p_{0,10}\, p_{10,2})\, p_{23}) - p_{0,10}\, p_{10,0}$
where
$\mu_0 = \mu_{01} + \mu_{02} + \mu_{0,10}$, $\mu_1 = \mu_{10} + \mu_{11}^{(4)} + \mu_{12}^{(5)}$,
$\mu_2 = \mu_{23} + \mu_{28}^{(7)} + \mu_{29}^{(6)}$, $\mu_{10} = \mu_{10,0} + \mu_{10,1} + \mu_{10,2}$

AVAILABILITY ANALYSIS

Let $M_i(t)$ be the probability of the system having started from state i is up at time t without making any other regenerative state. By probabilistic arguments, we have
$M_0(t) = e^{-\lambda_1 t}\, e^{-\lambda_2 t}\, e^{-\lambda_3 t}$, $M_1(t) = p\, G_1(t)\, e^{-\lambda_1 t}$
$M_2(t) = q\, G_2(t)\, e^{-\lambda_2 t}$, $M_3(t) = G_3(t)$,
$M_{10}(t) = G_4(t)\, e^{-\lambda_3 t}$
The point wise availability $A_i(t)$ have the following recursive relations
$A_0(t) = M_0(t) + q_{01}(t)[c]A_1(t) + q_{02}(t)[c]A_2(t) + q_{0,10}(t)[c]A_{10}(t)$
$A_1(t) = M_1(t) + q_{10}(t)[c]A_0(t) + q_{12}^{(5)}(t)[c]A_2(t) + q_{11}^{(4)}(t)[c]A_1(t)$,
$A_2(t) = M_2(t) + q_{23}(t)[c]A_3(t) + q_{28}^{(7)}(t)[c]\, A_8(t) + q_{29}^{(6)}(t)]\, [c]A_9(t)$
$A_3(t) = M_3(t) + q_{30}(t)[c]A_0(t)$,
$A_8(t) = q_{82}(t)[c]A_2(t)$
$A_9(t) = q_{91}(t)[c]A_1(t)$,

$A_{10}(t) = M_{10}(t) + q_{10,0}(t)[c]A_0(t) + q_{10,1}{}^{(11)}(t)[c]A_1(t) + q_{10,2}{}^{(11)}(t)[c]A_2(t)$ (8-15)

Taking Laplace Transform of eq. (8-15) and solving for $\hat{A}_0(s)$

$$\hat{A}_0(s) = N_2(s) / D_2(s) \quad (16)$$

where

$N_2(s) = \{ \hat{q}_{0,10} \hat{M}_{10} + \hat{M}_0 \} [\{1 - \hat{q}_{11}{}^{(4)}\}\{1 - \hat{q}_{28}{}^{(7} \hat{q}_{82}\} - \hat{q}_{12}{}^{(5)} \hat{q}_{29}{}^{(6)} \hat{q}_{91}] + \{ \hat{q}_{01} +$
$\hat{q}_{0,10} \hat{q}_{10,1}{}^{(11)}\}[\hat{M}_1\{1 - \hat{q}_{28}{}^{(7)} \hat{q}_{82}\} + \hat{q}_{12}{}^{(5)} \hat{q}_{23} \hat{M}_3 + \hat{M}_2] + \{ \hat{q}_{02} + \hat{q}_{0,10} \hat{q}_{10,2}{}^{(11)}\} [\{ \hat{q}_{23} \hat{M}_3\}\{1 - \hat{q}_{11}{}^{(4)}\} +$
$\hat{q}_{29}{}^{(6)} \hat{q}_{91} \hat{M}_1]$

$D_2(s) = \{1 - \ddot{q}_{11}{}^{(4)}\}\{1 - \ddot{q}_{28}{}^{(7} \ddot{q}_{82}\} - \hat{q}_{12}{}^{(5)} \ddot{q}_{29}{}^{(6)} \ddot{q}_{91} - \{ \ddot{q}_{01} + \ddot{q}_{0,10} \ddot{q}_{10,1}{}^{(11)} \}[\ddot{q}_{10} \{1 - \ddot{q}_{28}{}^{(7)} \ddot{q}_{82}\} + \ddot{q}_{12}{}^{(5)} \ddot{q}_{23} \ddot{q}_{30}]$
$- \{ \hat{q}_{02} + \hat{q}_{0,10} \hat{q}_{10,2}{}^{(11)}\}\{[\hat{q}_{23} \hat{q}_{30} \{1 - \hat{q}_{11}{}^{(4)}\} +$
$\hat{q}_{29}{}^{(6)} \hat{q}_{91} \hat{q}_{10}]$

(Omitting the arguments s for brevity) The steady state availability

$$A_0 = \lim_{t\to\infty}[A_0(t)] = \lim_{s\to 0}[s\,\hat{A}_0(s)] = \lim_{s\to 0} \frac{s\,N_2(s)}{D_2(s)}$$

Using L' Hospitals rule, we get

$$A_0 = \lim_{s\to 0} \frac{N_2(s) + s\,N_2'(s)}{D_2'(s)} = \frac{N_2(0)}{D_2'(0)} \quad (17)$$

The expected up time of the system in (0, t] is $\lambda_u(t) = \int_0^{x} A_0(z)\,dz$ So that $\tilde{\lambda}_u(s) = \frac{\ddot{A}_0(s)}{s} = \frac{N_2(S)}{SD_2(S)}$ (18)

The expected down time of the system in (0, t] is $\lambda_d(t) = t - \lambda_u(t)$ So that $\tilde{\lambda}_d(s) = \frac{1}{s^2} - \tilde{\lambda}_u(s)$ (19)

Similarly, we can find out

1. The expected busy period of the server when there is failure of Fuel pump in automobiles, and failure of fuel injectors in automobiles in (0,t]-R_0
2. The expected number of visits by the repairman Type-I or Type-II for repairing the identical units in (0,t]-H_0
3. The expected number of visits by the multispecialty repairman Type-III or Type-IV for repairing the identical units in (0,t]-W_0, Y_0.

BENEFIT-FUNCTION

The Benefit-Function analysis of the system considering mean up-time, expected busy period of the system under failure of Fuel pump in automobiles, and failure of fuel injectors in automobiles, expected number of visits by the repairman for unit failure. The expected total Benefit-Function incurred in (0, t] is

$$C = \lim_{t\to\infty}(C(t)/t) = \lim_{s\to 0}(s^2 C(s)) = K_1A_0 - K_2R_0 - K_3H_0 - K_4W_0 - K_5Y_0$$

where

K_1: Revenue per unit up-time,
K_2: Cost per unit time for which the system is busy under repairing,
K_3: Cost per visit by the repairman type- I or type- II for units repair,
K_4: Cost per visit by the multispecialty repairman Type- III for units repair,
K_5: Cost per visit by the multispecialty repairman Type- IV for units repair

CONCLUSION

After studying the system, we have analyzed graphically that when the failure rate of failure of Fuel pump in automobiles and failure of fuel injectors in automobiles increases, the MTSF, steady state availability decreases and the Profit-function decreased as the failure increases.

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