Research Article

Stochastic analysis of two non-identical unit standby system model with different modes of failure available in the system

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Abstract The paper deals with the study of a two non identical unit standby system $N_{10}$and$N_{20}$with N_{10} as priority unit and N_{20} as non priority unit .The operative unit has three modes (normal, partial failure and total failure) and standby unit with two modes (normal and total failure).Whenever the operative unit fails partially or completely, the standby unit goes for activation and during activation period, failed unit waits for repair. When the activated unit starts operating, the failed unit is repaired as per the priority. The failure and activation rates are constant and assume to follow an exponential distribution while the distribution of repair time is assumed to be general. By regenerative point technique, the various important measures of system effectiveness have been obtained.
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INTRODUCTION

A two non identical unit cold standby systems have widely studied in literature of reliability theory. In the past reliability researchers analysed the reliability performance of redundant repairable system. Repair maintenance is one of the most important measures which plays an important role in enhancing system reliability. Many authors have studied various system models under different repair policies. Goel, Rakesh Gupta and Tyagi[3] analyzed two- unit standby system with preparation time and correlated failures and repairs. While Gupta, Jaiswal and L.R. Goel[2] discussed reliability analysis of a two-unit cold standby redundant system with two operating modes. Gupta, S.M. and Goel (1983)[1] analyzed stochastic behavior of a standby redundant system with three modes. Tuteja, Aroraand Taneja[5], analyzed of two-unit system with partial failures and three types of repair. Vikas Sharma, J P Singh Joorel, Ankush Bharti and Rakesh Chib[6] discussed the concept of stochastic behavior of a two unit system with partial failure and fault detection. Further Malik, S.C.[4] discussed cost-benefit analysis of reliability models with different types of repair and inspection policies. In most of the cases the failed unit immediately goes for repair and after completing the repair it becomes as good as new. But in real life situations, the failed unit takes considerable time before entering into repair. Taking this fact into consideration, in this paper we investigate a two non identical cold stand by unit system model with different modes of failure under the

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concept of patience time of repair with activation time of standby unit. It is assumed that failed unit requires significant time before entering into repair. The system is analyzed by making use of regenerative point technique. The various measures of system effectiveness are obtained:
Using regenerative point technique the following important reliability characteristics of interest are obtained:

(i) Transition probabilities in transient and steady state
(ii) Mean sojourn time
(iii) Mean time to system failure (MTSF).
(iv) Point wise and steady-state availabilities of the system.
(v) Expected busy period of the repairman during (0, t].
(vi) Expected number of visits for the repair facility.
(vii) Graphical study of the system model.

ASSUMPTIONS AND SYSTEM DESCRIPTION

The system consists of two non-identical (dissimilar) units—unit-1 and unit- 2.The unit-1 is called priority (p-unit) and unit-2 as the non-priority unit (o-unit). The operation of only one unit is required to run the system. Initially unit-1 is operative and unit-2 is kept as standby. The priority unit has three modes normal (N), partial failure (P) and total failure (F). When both the units are in N-mode, the p-unit gets preference in operation/ repair over the o-unit while the non-priority unit (o-unit) has two modes—Normal (N) and total failure (F). Upon failure of priority (p-unit) unit, non-priority unit (o-unit) goes for activation. During activation the repair of failed unit is not permissible. On activation of non-priority unit (o-unit), the failure unit will go for repair. A single repair facility is used to repair of both units in which unit-1 gets priority over repair. After repair a unit becomes as good as new. The failure time distribution as well as activation time distribution of each unit is taken as exponential while repair time is taken as general.

NOTATION AND STATES OF THE SYSTEM

A) Notations :

α_1 : Constant failure rate of priority unit in partial failure.
α_2 : Constant failure rate of priority unit in complete failure.
α_3 : Constant failure rate of priority unit in partial to complete failure.
μ : Activation time of non-priority unit.
λ : Failure rate of non-priority unit.
$\beta(x)$: Repair rate of Priority and non priority unit.
μ_i : Mean sojourn time in state S_i.
$M_i(t)$: Probability that the system sojourns in state S_i upto time t.
$*$: Symbol for Laplace transforms i.e. $f^*(s) = \int_0^\infty e^{-st} f(t)dt$
$\sim$: Symbol for Laplace -Stieltjes transforms i.e. $\tilde{F}(s) = \int_0^\infty e^{-st}\, dF(t)$
$\pi_i(t)$: cdf of time to system failure where starting from up state S_0
$A_i(t)$: P [system is up at epoch t]
$B_i(t)$: P [repairman is busy in repair at an epoch t]
© : symbol for Laplace convolution
$P_j(t)$: Probability that the system is in state S_j at time t.

B) Symbols for the states of the system:

N_{1O}/ N_{2O} : p-unit/o-unit is operative and in N-mode.
N_{2S}/ N_{2a} : unit2 is standby in N-mode / under cold standby activation.
F_{1r}/ F_{1W} : unit1 in F-mode and under repair/waiting for repair.
F2r / F2w : unit2 in F-mode and under repair/waiting for repair
P_{1Or}/P_{1OW} : unit1 is in partially operative mode and under repair/waiting for repair.

With the help of above symbols and keeping in view the assumptions, the possible states S_0 to S_7 of the system along with the transitions between the states and transition rates are shown in transition diagram as given below:

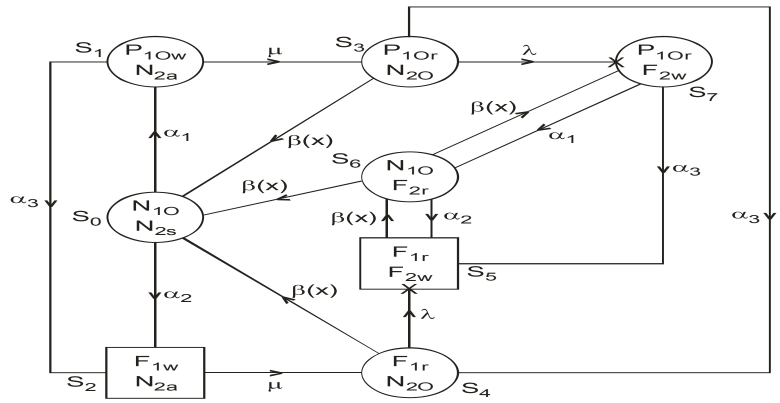

Figure 1: Transition Diagram

○ : Up State □ : Down State × : Non-regenerative Point

C) States of the system

The possible states of the system are:

$S_0 = [N_{1O}, N_{2s}]$ $S_1 = [P_{1OW}, N_{2a}]$

$S_2 = [F_{1w} N_{2a}]$ $S_3 = [P_{1Or}, N_{2O}]$

$S_4 = [F_{1r}, N_{2O}]$ $S_5 = [N_{2O} N_{1O}]$

$S_6 = [N_{1O}, F_{2r}]$ $S_7 = [P_{1Or}, F_{2w}]$

The states S_O, S_1, S_3, S_4, S_6 *and* S_7 are up states while S_2 *and*S_5 are down states. Further, states S_5*and*S_7 are non-regenerative states and all other states are regenerative states.

TRANSITION PROBABILITIES

A) Steady state transition probabilities

The steady state transition probabilities are defined as

$P_{01} = Q_{ij}(\infty) \int_0^\infty Q_{ij}(t)$

The following expressions for the non-zero elements are obtained

$P_{01} = \frac{\alpha_1}{\alpha_1+\alpha_2}$ $\quad P_{02} = \frac{\alpha_2}{\alpha_1+\alpha_2}$

$P_{12} = \frac{\alpha_3}{\alpha_3+\mu}$ $\quad P_{13} = \frac{\mu}{\alpha_3+\mu}$

$P_{24} = 1$ $P_{30} = \tilde{\beta}(\lambda + \alpha_3)$

$P_{34} = \frac{\alpha_3}{\lambda+\alpha_3}[1 - \tilde{\beta}(\lambda + \alpha_3)]$

$P_{36}^{(7)} = [1 - \tilde{\beta}(\alpha_3)] - \frac{\alpha_3}{\lambda+\alpha_3}[1 - \tilde{\beta}(\lambda + \alpha_3)]$

$P_{35}^{(7)} = \tilde{\beta}(\alpha_3) - \tilde{\beta}(\lambda + \alpha_3)$

$P_{40} = \tilde{\beta}(\lambda)$

$P_{46}^{(5)} = [1 - \tilde{\beta}(\lambda)]$ $P_{56} = 1$

$P_{60} = \tilde{\beta}(\alpha_1 + \alpha_2)]$

$P_{65} = \frac{\alpha_2}{\alpha_1+\alpha_2}[1 - \tilde{\beta}(\alpha_1 + \alpha_2)]$

$P_{67} = \frac{\alpha_1}{\alpha_1+\alpha_2}[1 - \tilde{\beta}(\alpha_1 + \alpha_2)]$

$P_{75} = [1 - \tilde{\beta}(\alpha_3)]$

$P_{76} = \tilde{\beta}(\alpha_3)$

Here it can easily be verified that $\sum_j P_{ij} = 1; for\ all\ possible\ values\ of\ i.$

MEAN SOJOURN TIME

The mean sojourn time in state S_i , denoted by μ_i, is defined as the expected time taken by the system in state S_i before transiting to any other state. To obtain mean sojourn timeμ_i, in stateS_i, we observe that as long as the system is in stateS_i, there is no transition from S_i to any other state. If T_i denotes the sojourn time in state S_i then mean sojourn time μ_i in state S_i is defined as

$\mu_i = E[T_i] = \int_0^\infty P(T_i > t)dt$

Thus

$\mu_0 = \int_0^\infty e^{-(\alpha_1+\alpha_2)t}dt = \frac{1}{(\alpha_1+\alpha_2)}$

$\mu_1 = \int_0^\infty e^{-(\alpha_3+\mu)t}dt = \frac{1}{(\alpha_3+\mu)}$

$\mu_2 = \int_0^\infty e^{-\mu t}dt = \frac{1}{\mu}$

$\mu_3 = \int_0^\infty e^{-(\lambda+\alpha_3)t}\bar{\beta}(t)dt = \frac{1}{(\lambda+\alpha_3)}[1 - \tilde{\beta}(\lambda+\alpha_3)]$

$\mu_4 = \int_0^\infty e^{-(\lambda)t}\bar{\beta}(t)dt = \frac{1}{\lambda}[1 - \tilde{\beta}(\lambda)]$

$\mu_5 = \int_0^\infty \bar{\beta}(t)\,dt$

$\mu_6 = \int_0^\infty e^{-(\alpha_1+\alpha_2)t}\bar{\beta}(t)dt = \frac{1}{(\alpha_1+\alpha_2)}[1 - \tilde{\beta}(\alpha_1+\alpha_2)]$

$\mu_7 = \int_0^\infty e^{-(\alpha_3)t}\bar{\beta}(t)dt = \frac{1}{(\alpha_3)}[1 - \tilde{\beta}(\alpha_3)]$

MEAN TIME TO SYSTEM FAILURE

Let the random variable T_i denotes the time to system failure when $E_o = E_i \in E$and $\pi_i(t)$ is the c.d.f. of the time to system failure for the first time when the system starts operation from state S_i. To obtain the expressions of $\pi_i(t)$ for different values of i, the arguments of regenerative point processes has been used. Taking the Laplace transform and solving the resultant set of equations for$A_0^*(s)$, we have

$\tilde{\pi}_0(s) = \frac{N_1(s)}{D_1(s)}$

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where

$N_1(s) = \tilde{Q}_{01}(s)\left[\left(1 - \tilde{Q}_{67}(s)\tilde{Q}_{76}(s)\right)\left(\tilde{Q}_{12}(s) + \tilde{Q}_{13}(s)\tilde{Q}_{34}(s)\tilde{Q}_{45}(s) + \tilde{Q}_{13}(s)\tilde{Q}_{37}(s)\tilde{Q}_{75}(s)\right) + \tilde{Q}_{13}\tilde{Q}_{37}(s)\tilde{Q}_{76}(s)(\tilde{Q}_{65}(s) + \tilde{Q}_{67}(s)\tilde{Q}_{75}(s))\right] + \tilde{Q}_{02}(s)\left[1 - \tilde{Q}_{67}(s)\tilde{Q}_{76}(s)\right]$

and

$D_1(s) = \left(1 - \tilde{Q}_{67}(s)\tilde{Q}_{76}(s)\right)\left[1 - \tilde{Q}_{01}(\tilde{Q}_{13}(s)\tilde{Q}_{30}(s) + \tilde{Q}_{13}(s)\tilde{Q}_{34}(s)\tilde{Q}_{40}(s))\right] - \widetilde{(Q}_{01}\tilde{Q}_{13}(s)\tilde{Q}_{37}(s)\tilde{Q}_{76}(s)\tilde{Q}_{60}(s)\tilde{Q}_{76})$

On taking $s \to 0$ and using the relation $\check{Q}_{ij}(s) \to P_{ij}$, we have

$$\tilde{\pi}_0(0) = \frac{N_1(0)}{D_1(0)} = 1$$

Thus $N_1(0) = D_1(0)$ showing that $\tilde{\pi}_0(0) = 1$.Hence$\pi_0(t)$ is a proper cdf.

Therefore, mean time to system failure when the initial state is S_0,is given by

$E(T) = -\frac{d\tilde{\pi}_0(s)}{ds}\Big|_{s=0} = \frac{D_1'(0) - N_1'(0)}{D_1(0)}$

where,

$$N_1(0) = (1 - P_{67}P_{76})[1 - P_{01}(P_{13}P_{30} + P_{13}P_{34}P_{40}) - (P_{01}P_{13}P_{37}P_{76}P_{60})$$
$$D_1(0) = (1 - P_{67}P_{76})[1 - P_{01}(P_{13}P_{30} + P_{13}P_{34}P_{40})] - (P_{01}P_{13}P_{37}P_{76}P_{60})$$

Now put value of μ_i's and P_{ij}'s in above equation, we get

$D'_1(0) - N'_1(0) = \alpha_3[\{(\alpha_1+\alpha_2) - \alpha_1[1 - \tilde{\beta}(\alpha_1+\alpha_2)\tilde{\beta}(\alpha_3)\}\{\lambda(\mu+\alpha_3)(\alpha_3+\lambda) + \alpha_1\lambda(\lambda+\alpha_3) + \lambda\mu\alpha_1[1 - \tilde{\beta}(\lambda+\alpha_3)] + \alpha_1\alpha_3\mu[1 - \tilde{\beta}(\alpha_3+\lambda)]$
$[1 - \tilde{\beta}(\lambda)]\}] + \lambda(\alpha_3+\lambda)[\{\alpha_1\mu[\tilde{\beta}(\alpha_3) - \tilde{\beta}(\lambda+\alpha_3)]\}\{\alpha_3\tilde{\beta}(\alpha_3)$
$[1 - \tilde{\beta}(\alpha_1+\alpha_2)] + (\alpha_1+\alpha_2)[1 - \tilde{\beta}(\alpha_3)]\}]$

$D_1(0) = (\alpha_1+\alpha_2)\alpha_3\lambda[\{(\alpha_1+\alpha_2) - \alpha_1[1 - \tilde{\beta}(\alpha_1+\alpha_2)\tilde{\beta}(\alpha_3)\}\{\lambda(\alpha_1+\alpha_2)(\alpha_3+\mu)$
$(\alpha_3+\lambda) - \alpha_1\mu(\alpha_3+\mu)\tilde{\beta}(\lambda+\alpha_3) - \alpha_3\mu(\alpha_1+\alpha_2)\tilde{\beta}(\lambda)[1 - \tilde{\beta}(\lambda+\alpha_3)]\} -$
$(\alpha_1+\alpha_2)(\alpha_3+\lambda)\{\alpha_1\mu[\tilde{\beta}(\alpha_3) - \tilde{\beta}(\lambda+\alpha_3)]\tilde{\beta}(\alpha_3)\tilde{\beta}(\alpha_1+\alpha_2)\}]$

AVAILABILITY ANALYSIS

We define $A_i(t)$ as the probability that the system is up at epoch 't' when it initially starts from regenerative state S_i. It is also called pointwise availability of the system. To obtain recurrence relations among different point wise availabilities $A_i(t)$, we use the simple probabilistic arguments. Taking the Laplace transform and solving the resultant set of equations for $A_0^*(s)$, we have:

$$A_0^*(s) = \frac{N_2(s)}{D_2(s)}$$

where

$$N_2(s) = [1 - q_{65}^* q_{56}^* - q_{67}^* q_{76-}^* q_{67}^* q_{75}^* q_{56}^*][M_0^* + q_{01}^* M_1^* + q_{01}^* q_{13}^* M_3^* + (q_{01}^* q_{12}^* q_{24}^* + q_{02}^* q_{24}^* + q_{01}^* q_{13}^* q_{34}^*) M_4^*] + \left[q_{01}^* q_{12}^* q_{24}^* q_{46}^{*(5)} + q_{02}^* q_{24}^* q_{46}^{*(5)} + q_{01}^* q_{13}^* q_{34}^* q_{46}^{*(5)} + q_{01}^* q_{13}^* q_{56}^* q_{35}^{*(7)} + q_{01}^* q_{13}^* q_{56}^* q_{36}^{*(7)}\right][M_6^* + q_{67}^* M_7^*] \quad \textbf{(1)}$$

$$D_2(s) = [1 - q_{65}^* q_{56}^* - q_{67}^* q_{76-}^* q_{67}^* q_{75}^* q_{56}^*][1 - \{q_{01}^* q_{12}^* q_{24}^* q_{40}^* + q_{02}^* q_{24}^* q_{40}^* + q_{01}^* q_{13}^* q_{34}^* q_{40}^* + q_{01}^* q_{13}^* q_{30}^*\}] - \left\{q_{01}^* q_{12}^* q_{24}^* q_{46}^{*(5)} + q_{02}^* q_{24}^* q_{46}^{*(5)} + q_{01}^* q_{13}^* q_{34}^* q_{46}^{*(5)} + q_{01}^* q_{13}^* q_{56}^* q_{35}^{*(7)} + q_{01}^* q_{13}^* q_{56}^* q_{36}^{*(7)}\right\} q_{60}^* \quad \textbf{(2)}$$

The steady state Availability will be given by

$$\mathrm{A}_0 = \lim_{t\to\infty} \mathrm{A}_0(\mathrm{t}) = \lim_{s\to 0} \mathrm{s}\, \mathrm{A}_0^*(\mathrm{s}) = N_2(0)/D_2'(0)$$

where,

$$N_2(0) = \mu\, \alpha_3 \{[(\alpha_1+\alpha_2) - \alpha_2[1-\tilde\beta(\alpha_1+\alpha_2)] - \alpha_1\tilde\beta(\alpha_3)[1-\tilde\beta(\alpha_1+\alpha_2)] - \alpha_1\tilde\beta(\alpha_3)[1-\tilde\beta(\alpha_1+\alpha_2)] - \alpha_1[1-\tilde\beta(\alpha_3)][1-\tilde\beta(\alpha_1+\alpha_2)]][\lambda(\alpha_3+\mu)(\alpha_3+\lambda) + \alpha_1\lambda(\alpha_3+\lambda) + [\lambda\alpha_1\mu[1-\tilde\beta(\lambda+\alpha_3)]] + \{\alpha_1\alpha_3(\alpha_3+\lambda) + \alpha_2(\alpha_3+\mu)(\alpha_3+\lambda) + \alpha_1\alpha_3\mu[1-\tilde\beta(\lambda+\alpha_3)][1-\tilde\beta(\lambda)]\} + \lambda\{[\alpha_1\alpha_3(\alpha_3+\lambda)[1-\tilde\beta(\lambda)] + \alpha_2(\alpha_3+\mu)(\alpha_3+\lambda)[1-\tilde\beta(\lambda)] + \alpha_1\alpha_3\mu[1-\tilde\beta(\lambda+\alpha_3)][1-\tilde\beta(\lambda)\} + \alpha_1\mu\{[1-\tilde\beta(\alpha_3)] - \alpha_3[1-\tilde\beta(\lambda+\alpha_3)]\} + \alpha_1\mu(\lambda+\alpha_3)\{\tilde\beta(\alpha_3) - \tilde\beta(\lambda+\alpha_3)\}][\alpha_3[1-\tilde\beta(\alpha_1+\alpha_2)] + \alpha_1[1-\tilde\beta(\alpha_1+\alpha_2)][1-\tilde\beta(\alpha_3)]]\} \quad \textbf{(3)}$$

$$D_2'(0) = \alpha_3(\alpha_1+\alpha_2)\tilde\beta(\alpha_1+\alpha_2)[\lambda\mu(\alpha_3+\lambda)[(\alpha_3+\mu) + \mu\alpha_1\lambda(\alpha_3+\lambda) + \lambda(\alpha_3+\lambda)\{(\alpha_1+\alpha_2)(\alpha_3+\mu) + \alpha_1\mu\} + \lambda\mu(\alpha_1+\alpha_2)(\alpha_3+\mu)[1-\tilde\beta(\lambda+\alpha_3)]\tilde\beta(\alpha_1+\alpha_2) + \mu[1-\tilde\beta(\lambda)]\{\alpha_1\alpha_3(\alpha_3+\lambda) + \alpha_2(\alpha_3+\lambda)(\alpha_3+\mu) + \alpha_1\alpha_3\mu[1-\tilde\beta(\lambda+\alpha_3)]\}] + \lambda\mu[\{(\alpha_1+\alpha_2)[(\alpha_3+\mu)(\alpha_3+\lambda) - \alpha_1\alpha_3(\alpha_3+\lambda)\tilde\beta(\lambda) - \alpha_2(\alpha_3+\mu)(\alpha_3+\lambda)\tilde\beta(\lambda) - \alpha_1\alpha_3\mu\tilde\beta(\lambda)[1-\tilde\beta(\lambda+\alpha_3)] - \alpha_1\mu(\lambda+\alpha_3)\tilde\beta(\lambda+\alpha_3)\}\{\alpha_3\int_0^\infty \bar\beta(u)\,du\{\alpha_2[1-\tilde\beta(\alpha_1+\alpha_2)] + \alpha_1[1-\tilde\beta(\alpha_1+\alpha_2)]1-\tilde\beta(\alpha_3)] + \alpha_3[1-\tilde\beta(\alpha_1+\alpha_2)]\} + \alpha_1[1-\tilde\beta(\alpha_3)][1-\tilde\beta(\alpha_1+\alpha_2)]\}] + \alpha_3(\alpha_1+\alpha_2)(\lambda+\alpha_3)\{\int_0^\infty \bar\beta(u)du[\alpha_1\mu\{\tilde\beta(\alpha_3) - \tilde\beta(\lambda+\alpha_3)\}\tilde\beta(\alpha_1+\alpha_2)\}] \quad \textbf{(4)}$$

BUSY PERIOD ANALYSIS FOR REPAIRMAN

We define $B_i(t)$ as the probability that the regular repairman is busy in the repair of the failed unit when the system initially starts from state $S_i \epsilon E$. Using probabilistic arguments, taking the Laplace transform and solving the resultant set of equations for $B_0^*(s)$, we have

$$B_0^*(s) = N_3(s)/D_2(s)$$

$$N_3(s) = [1 - q_{65}^* q_{56}^* - q_{67}^* q_{76-}^* q_{67}^* q_{75}^* q_{56}^*][q_{01}^* q_{13}^* Z_3^* + (q_{01}^* q_{12}^* q_{24}^* + q_{02}^* q_{24}^* + q_{01}^* q_{13}^* q_{34}^*) Z_4^* + q_{01}^* q_{13}^* q_{35}^{*(7)} Z_5^*] + [q_{01}^* q_{12}^* q_{24}^* q_{46}^{*(5)} + q_{02}^* q_{24}^* q_{46}^{*(5)} + q_{01}^* q_{13}^* q_{34}^* q_{46}^{*(5)} + q_{01}^* q_{13}^* q_{56}^* q_{35}^{*(7)} + q_{13}^* q_{36}^{*(7)}][Z_6^* + q_{67}^* Z_7^* + q_{65}^* Z_5^*]$$

In the steady state, the probability that the regular repairman will be busy is given by

$$B_0 = \lim_{t\to\infty} B_0(t) = \lim_{s\to 0} sB_0^*(s) = N_3(0)/D_2'(0)$$

Again substituting values of $P_{ij}' sand \mu_i' sinaboveequation, weget,$

$$N_3(0) = \alpha_3\{[(\alpha_1+\alpha_2) - \alpha_2[1-\tilde\beta(\alpha_1+\alpha_2)] - \alpha_1\tilde\beta(\alpha_3)[1-\tilde\beta(\alpha_1+\alpha_2)]\} - \alpha_1[1-\tilde\beta(\alpha_3)]$$

$[[1-\tilde{\beta}(\alpha\_1+\alpha\_2)][\lambda\alpha_1\alpha_3(\alpha_3+\lambda)+\{\alpha_1\alpha_3(\alpha_3+\lambda)+\alpha_2(\alpha_3+\mu)(\alpha_3+\lambda)+\alpha_1\alpha_3\mu\}$
$[1-\tilde{\beta}(\lambda+\alpha_3)]\}[1-\tilde{\beta}(\lambda)\}+\lambda\alpha_1\mu(\lambda+\alpha_3)\{\tilde{\beta}(\alpha_3)-\tilde{\beta}(\lambda+\alpha_3)\}[\int_0^\infty\bar{\beta}(u)du\}+\lambda\{[\alpha_1\alpha_3$
$(\alpha_3+\lambda)[1-\tilde{\beta}(\lambda)](\alpha_3+\mu)(\alpha_3+\lambda)[1-\tilde{\beta}(\lambda)]+\alpha_1\alpha_3\mu[1-\tilde{\beta}(\lambda+\alpha_3)][1-\tilde{\beta}(\lambda)\}+$
$\alpha_1\mu\{[1-\tilde{\beta}(\alpha_3)]-\alpha_3[1-\tilde{\beta}(\lambda+\alpha_3)]\}+\alpha_1\mu(\lambda+\alpha_3)\{\tilde{\beta}(\alpha_3)-\tilde{\beta}(\lambda+\alpha_3)\}]$
$[\alpha_3[1-\tilde{\beta}(\alpha_1+\alpha_2)]+\alpha_1[1-\tilde{\beta}(\alpha_1+\alpha_2)][1-\tilde{\beta}(\alpha_3)]\}+\alpha_1\alpha_3[1-\tilde{\beta}(\alpha_1+\alpha_2)]$
$\int_0^\infty\bar{\beta}(u)du$
and$D_3'(0)=D_2'(0)$ is same as in the case of availability given by equation (4).

EXPECTED NUMBER OF VISITS BY REPAIRMAN

Let us define $V_i(t)$ as the expected number of visits by regular repairman during the time interval $(0,t]$ when the system initially starts from regenerative state S_i. Using probabilistic arguments, taking the Laplace transform and solving the resultant set of equations for$\tilde{V}_0(s)$, we get

$\tilde{V}_0(s)=\frac{N_4(s)}{D_2(s)}$

where

$N_4(s)=[1-q_{65}^*q_{56}^*-q_{67}^*q_{76-}^*q_{67}^*q_{75}^*q_{56}^*][q_{01}^*q_{13}^*+q_{01}^*q_{12}^*q_{24}^*+q_{02}^*q_{24}^*]$

In steady state, number of visits per unit time is given by

$V_0(0)=\lim_{t\to\infty}\frac{V_0(t)}{t}=\frac{N_4(0)}{D_2'(0)}$

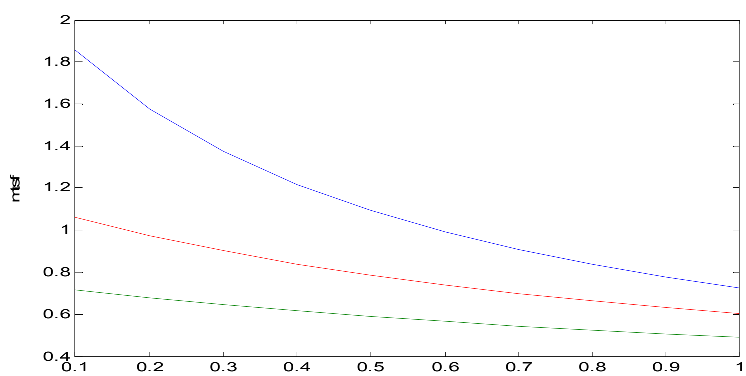
$N_4(0)=\{[(\alpha_1+\alpha_2)-\alpha_2[1-\tilde{\beta}(\alpha_1+\alpha_2)]-\alpha_1\tilde{\beta}(\alpha_3)[1-\tilde{\beta}(\alpha_1+\alpha_2)]\}-\alpha_1[1-\tilde{\beta}(\alpha_3)]$

$and D_4'(0)=D_2'(0)$is same as availability analysis given in equation (4).

GRAPHICAL STUDY OF THE SYSTEM MODEL

The behavior of MTSF and availability of the system is studied graphically in this section and to plot their graphs, the repair time distributions are also assumed to be distributed exponentially. The graphs of MTSF and that of availability are depicted with respect to the different parameters. It is observed that the MTSF decreases uniformly as the failure rates of the system increases irrespective of the other fixed parameters. However, we note that MTSF increases with increasing repair rates. Thus, we can conclude that the expected life of the system can be increased by increasing repair rate of the unit. Further, it is observed that the availability of the system gradually decreases with increasing failure rates irrespective of type of failures and increases with increasing repair rate of unit

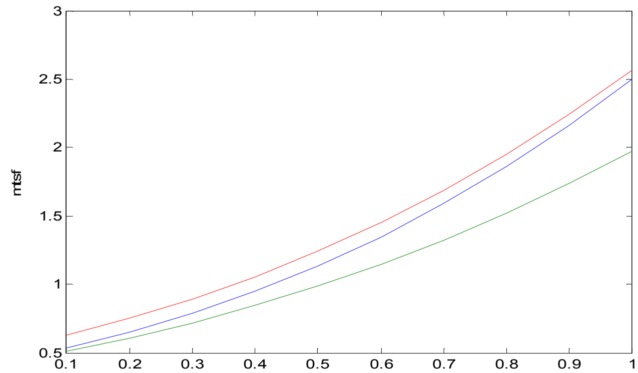
BEHAVIOUR OF MTSF w.r.t.α_1FOR DIFFERENT VALUES OF $\alpha_2,\alpha_{3,}\lambda,\mu,\beta$



α_1

Figure 2:

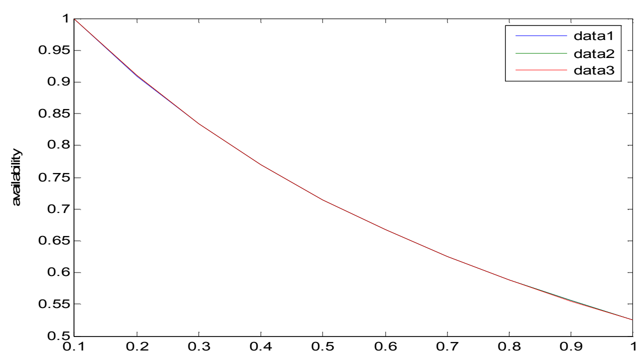
In fig 2, we plot MTSF w.r.t. α_1 and fixed values of parameter $\alpha_2,\alpha_{3,}\lambda,\mu,\beta$. It is observed that MTSF of the system decreases w.r.t. α_1 irrespective of the other parameters so we conclude that expected life of the system increases with decreasing failure rate of unit. BEHAVIOUR OF MTSF W.R.T.βFOR DIFFERENT VALUES OF $\alpha_1,\alpha_2,\alpha_{3,}\lambda,\mu,$



β

Figure 3:

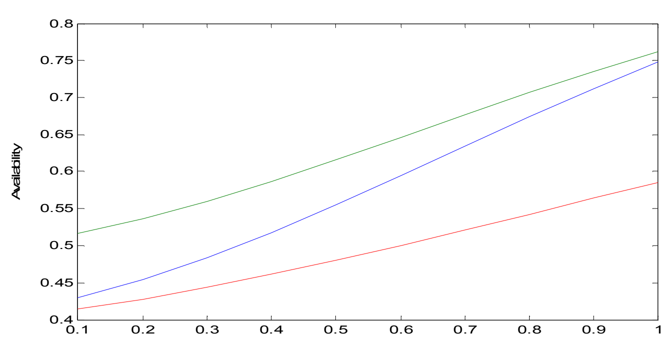
In fig 3, we plot MTSF β w.r.t. and fixed values of parameter $\alpha_1, \alpha_2, \alpha_3, \lambda, \mu$,. It is quiet clear that MTSF of the system increases w.r.t.β irrespective of the other parameters so we conclude that expected life of the system increases with increasing repair rate of unit in repair mode (β). BEHAVIOUR OF AVAILABILITY W.R.T. α_1 FOR DIFFERENT VALUES OF α_2, α_3, μ, λ, β



α_1

Figure 4:

In the fig 4, we plot Availability w.r.t.α_1,and fixed values of parameter. It is observed that Availability of the system decreases w.r.t. α_1 irrespective of the other$\alpha_2, \alpha_3, \lambda, \mu, \beta$ parameters. Therefore, we conclude that expected life of the system increases with decreasing failure rate of unit in failure mode (α_1). BEHAVIOUR OF AVAILABILITY W.R.T. β FOR DIFFERENT VALUES OF $\alpha_1, \alpha_2, \lambda, \alpha_3$, μ



β

Figure 5:

In Fig. 5, represents the behaviour of Availability w.r.t.β and fixed values of parameter$\alpha_1, \alpha_2, \lambda, \gamma_2, n_1, n_2$. It is observed that Availability of the system increases w.r.t. β irrespective of the other parameters. Hence, we conclude that expected life of the system increases with increasing repair rate of unit in repair mode.

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