Original Research Article

A simple derivation of the identity between double h vector class and three stage least square estimator

Manoj Kumar

Department of Statistics, Panjab University, Chandigarh, INDIA.
Email: mantiwa@gmail.com

Abstract A simple derivation of the identity connecting DhVC and 3 SLS estimators is presented in this article which will be used for studying the properties of other estimators.
Key Words: Three Stage Least Squares (3 SLS), double h – vector class (DhVC), h vector class.

***Address for Correspondence:**
Dr. Manoj Kumar,Department of Statistics, Panjab University, Chandigarh, INDIA.
Email: mantiwa@gmail.com
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INTRODUCTION

Schraf (1976, 1980) presented a full-information relationship between three stage least squares (3SLS) and double h-vector class (DhVC) estimators. The DhVC is full-information generalization of well known h-Class estimators. Following Srivastava and Tiwari (1977), a simple derivation of the identity connecting DhVC and 3 SLS estimators is presented in this article which will useful for the study of the properties the other estimators.

DOUBLE h-VECTOR CLASS AND THREE-STAGE LEAST SQUARES ESTIMATORS

Consider a complete system of T-structural equations in N jointly dependent and Λ predetermined variables.
Let the ith structural equation be

$$y_i = \gamma_i Y_i + \beta_i X_i + u_i \quad (1)$$

or

$$y_i = \delta_i z_i + u_i;\ Z_i = (Y_i X_i),\ \delta_i = (\gamma_i' \beta_i')' \quad (2)$$

Where, y_i is a column vector of T-observations on the jointly dependent variables to be explained, Y_i and X_i are the matrices of T-observations on $n_i$explanatory jointly dependent and $l_i$explanatory predetermined variables respectively, γ_i and β_i are the associated coefficient vector and u_i is the column vector of T-structural disturbances. These disturbances are assumed to be temporally independent and generated by a stationary multivariate stochastic process with

$$E(u_i) = 0$$
$$E(u_i u_j) = \sigma_{ij} I;\ (\mathrm{i,j} = 1,2,\ldots,\mathrm{N}) \quad (3)$$

Where, I is the identity matrix of order T. Similarly, the predetermined variables are also assumed to be generated by a multivariate stochastic process, independently of the process generating disturbances, with a non-singular moment's matrix. We assume that all the structural equations of the system are identifiable so that the system has been solved initially to eliminate all identities. Finally, we assume that the system can be solved for the jointly dependent variable. Let

$$y_i = X\Pi_i + V_i \quad (4)$$

bethe reduced form corresponding to the explanatory jointly dependent variable of (1), where X is a $T \times \Lambda$matrix, assumed to be of full column rank, of the values taken by all Λ- predetermined variables and Π_i is a $\Lambda \times n_\mathrm{i}$matrix of parent reduced form coefficients. Let us define the following matrices.

$$\hat{\hat{V}}_i = (\hat{V}_i\, O) = (MY_1, O) \quad (5)$$

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whereO is a $T \times l_i$null matrices, $l_i$being the number of predetermined variables in (1) and
$M = I_T - X(X'X)^{-1}X'$ (6)
writing$\delta' = (\delta_1', \delta_2', \dots, \delta_N')'$, the double h-vector class estimators of the parameter vector δ is given by

$$\hat{\delta}(h_1,h_2) = \left[\sigma^{ik}\left\{Z_i'Z_k - (1-h_{i1}h_{k1})\hat{\hat{V}}_i'\hat{\hat{V}}_k\right\}\right]^{-1}\left[\sum_{k=1}^{N}\sigma^{ik}\left\{Z_i' - (1-h_{i1}(1+h_{k2}))\hat{\hat{V}}_i'\right\}y_k\right] \qquad (7)$$

where$h_1 = (h_{11}, h_{21}, \dots, h_{N1})'$ and $h_2 = (h_{12}, h_{22}, \dots, h_{N2})'$ and are $N \times 1$ vectors which fix DhVC estimator.
If $h_1 = 0$ i.e. $h_{i1} = 0, \forall\, i = 1,2, \dots, N$.
We get 3SLS estimator given by

$$\hat{\delta}_{3SLS} = \left[\sigma^{ik}\left\{Z_i'Z_k - \hat{\hat{V}}_i'\hat{\hat{V}}_k\right\}\right]_{i,k}^{-1}\left[\sum_{k=1}^{N}\sigma^{ik}\left\{Z_i' - \hat{\hat{V}}_i'\right\}y_k\right]_{i,1} \qquad (8)$$

SIMPLE DERIVATION OF IDENTITY BETWEEN DHVC AND 3SLS ESTIMATORS

Scharf's identity between DhVC and 3SLS estimators is given by

$$\begin{aligned}\hat{\delta}(h_1,h_2) &= \hat{\delta}_{3SLS} \\ &+ \left[\sigma^{ik} Z_i'\left\{I - (1-h_{i1}h_{k1})M\right\}Z_k\right]_{i,k}^{-1}\left[\left\{\sum_{k=1}^{N}\sigma^{ik}h_{i1}(1+h_{k2})Z_i'M\,y_k\right\}_{i,1} - (\sigma^{ik}h_{i1}h_{k1}Z_i'MZ_k)_{i,k}\,\hat{\delta}_{3SLS}\right] \qquad (9)\end{aligned}$$

we present a simple derivation of this identity.
Noticing that for any two matrices Q_1 and Q_2

$$(Q_1 + Q_2)^{-1} = [I - (Q_1 + Q_2)^{-1}Q_2]Q_1^{-1} \qquad (10)$$

provided Q_1 is invertible. Consider

$$\begin{aligned}&\left[\sigma^{ik}\left\{Z_i'Z_k - (1-h_{i1}h_{k1})\hat{\hat{V}}_i'\hat{\hat{V}}_k\right\}\right]_{i,k}^{-1} \\ &= \left[\sigma^{ik}\left\{Z_i'Z_k - \hat{\hat{V}}_i'\hat{\hat{V}}_k\right\} + \sigma^{ik}h_{i1}h_{k1}\hat{\hat{V}}_i'\hat{\hat{V}}_k\right]_{i,k}^{-1} \qquad (11) \\ &= \left[I - \left[\sigma^{ik}\left\{Z_i'Z_k - (1-h_{i1}h_{k1})\hat{\hat{V}}_i'\hat{\hat{V}}_k\right\}\right]_{i,k}^{-1}(\sigma^{ik}h_{i1}h_{k1}\hat{\hat{V}}_i'\hat{\hat{V}}_k)_{i,k}\right]\left\{\sigma^{ik}(Z_i'Z_k - \hat{\hat{V}}_i'\hat{\hat{V}}_k)\right\}_{i,k}^{-1}\end{aligned}$$

So that

$$\begin{aligned}&\left[\sigma^{ik}\left\{Z_i'Z_k - (1-h_{i1}h_{k1})\hat{\hat{V}}_i'\hat{\hat{V}}_k\right\}\right]_{i,k}^{-1}\left\{\sum_{k=1}^{N}\sigma^{ik}(Z_i' - \hat{\hat{V}}_i')y_k\right\}_{i,1} \\ &= \left[I - \left[\sigma^{ik}\left\{Z_i'Z_k - (1-h_{i1}h_{k1})\hat{\hat{V}}_i'\hat{\hat{V}}_k\right\}\right]_{i,k}^{-1}(\sigma^{ik}h_{i1}h_{k1}\hat{\hat{V}}_i'\hat{\hat{V}}_k)_{i,k}\right]\hat{\delta}_{3SLS} \qquad (12) \\ &= \hat{\delta}_{3SLS} - \left[\sigma^{ik}\left\{Z_i'Z_k - (1-h_{i1}h_{k1})\hat{\hat{V}}_i'\hat{\hat{V}}_k\right\}\right]_{i,k}^{-1}(\sigma^{ik}h_{i1}h_{k1}\hat{\hat{V}}_i'\hat{\hat{V}}_k)_{i,k}\;\hat{\delta}_{3SLS}\end{aligned}$$

expressing

$$\left[\sum_{k=1}^{N}\sigma^{ik}\left\{Z_i' - (1-h_{i1}(1+h_{k2})\hat{\hat{V}}_i'\right\}y_k\right]_{i,1} = \left[\sum_{k=1}^{N}\sigma^{ik}(Z_i' - \hat{\hat{V}}_i')y_k\right]_{i,1} + \left\{\sum_{k=1}^{N}\sigma^{ik}h_{i1}(1+h_{k2})\hat{\hat{V}}_1'y_k\right\}_{i,1} \qquad (13)$$

and using (13), we find from (7)

$$\hat{\delta}(h_1,h_2)=\hat{\delta}_{3SLS}-\left[\sigma^{ik}\left\{Z_i^{'}Z_k-(1-h_{i1}h_{k1})\hat{\hat{V}}_i^{'}\hat{\hat{V}}_k\right\}\right]_{i,k}^{-1}(\sigma^{ik}h_{i1}h_{k1}\hat{\hat{V}}_i^{'}\hat{\hat{V}}_k)_{i,k}\ \hat{\delta}_{3SLS}$$

$$+\left[\sigma^{ik}\left\{Z_i^{'}Z_k-(1-h_{i1}h_{k1})\hat{\hat{V}}_i^{'}\hat{\hat{V}}_k\right\}\right]_{i,k}^{-1}\left[\sum_{k=1}^{N}\sigma^{ik}h_{i1}(1+h_{K2})\hat{\hat{V}}_i^{'}y_k\right]_{i,1}$$

$$=\hat{\delta}_{3SLS}+\left[\sigma^{ik}\left\{Z_i^{'}Z_k-(1-h_{i1}h_{k1})\hat{\hat{V}}_i^{'}\hat{\hat{V}}_k\right\}\right]_{i,k}^{-1}\left[\left\{\sum_{k=1}^{N}\sigma^{ik}h_{i1}(1+h_{K2})\hat{\hat{V}}_i^{'}y_k\right\}_{i,1}\ \hat{\delta}_{3SLS}\right](14)$$

$$-(\sigma^{ik}h_{i1}h_{k1}\hat{\hat{V}}_i^{'}\hat{\hat{V}}_k)_{i,k}\ \hat{\delta}_{3SLS}$$

$$=\hat{\delta}_{3SLS}+\left[\sigma^{ik}Z_i^{'}\left\{I-(1-h_{i1}h_{k1})M\right\}Z_k\right]_{i,k}\left[\sum_{k=1}^{N}\sigma^{ik}h_{i1}(1+h_{K2})Z_i^{'}My_k\right]_{i,1}$$

$$-(\sigma^{ik}h_{i1}h_{k1}Z_i^{'}MZ_k)_{i,k}\ \hat{\delta}_{3SLS}$$

Which is same as (9).

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