Occasionally Weakly Compatible Mappings and Fixed point Theorem In Fuzzy Metric Spaces Satisfying Integral Type Inequality

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Research Article

Abstract: The aim of this paper is to present common fixed point theorem in fuzzy metric spaces for occasionally weakly compatible mappings with integral type inequality by reducing its minimum value.

Keywords: *Fuzzy metric space, occasionally weakly compatible (owc) mappings, common fixed point.*

1. Introduction:

Fuzzy set was defined by Zadeh [26]. Kramosil and Michalek [14] introduced fuzzy metric space, many authors extend their views, Grorge and Veermani [6] modified the notion of fuzzy metric spaces with the help of continuous t-norms Grabiec[7], Subramanyam[28],Vasuki[25], Pant and Jha,[20] obtained some analogous results proved by Balasubramaniam et al. Subsequently, it was developed extensively by many authors and used in various fields, Jungck [10] introduced the notion of compatible maps for a pair of self maps. Several papers have come up involving compatible mapping proving the existence of common fixed points both in the classical and fuzzy metric spaces.

The theory of fixed point equations is one of the basic tools to handle various physical formulations. Fixed point theorems in fuzzy mathematics has got a direction of vigorous hope and vital trust with the study of Kramosil and Michalek [14], who introduced the concept of fuzzy metric space. Later on this concept of fuzzy metric space was modified by George and Veermani [6] Sessa [27] initiated the tradition of improving commutative condition in fixed point theorems by introducing the notion of weak commuting property. Further Juncgk [10] gave a more generalized condition defined as compatibility in metric spaces.

Jungck and Rhoades [11] introduced the concept of weakly compatible maps which were found to be more generalized than compatible maps.

Grabiec [7] obtained fuzzy version of Banach contraction principle. Singh and M.S. Chauhan [29] brought forward the concept of compatibility in fuzzy metric space. Pant [18, 19, 20] introduced the new concept reciprocally continuous mappings and established some common fixed point theorems. Balasubramaniam et al. [4], have shown that Rhoades [22] open problem on the existence of contractive definition which generates a fixed point but does not force the mappings to be continuous at the fixed point, posses an affirmative answer. Recent literature in fixed point in fuzzy metric space can be viewed in [1, 2, 9, 16, 24].

This paper offers the fixed point theorems on fuzzy metric spaces which generalize extend and fuzzify several known fixed point theorems for occasionally compatible maps on metric space by making use of integral type inequality.

2. Preliminary Notes:

Definition 2.1 [26] A fuzzy set A in X is a function with domain X and values in [0,1].

Definition 2.2 [23] A binary operation $*$: $[0, 1] \times [0, 1] \rightarrow [0, 1]$ is a continuous t-norms if it satisfies the following conditions:

(i) $*$ is associative and commutative

(ii) $*$ is continuous;

(iii) $a*1 = a$ for all $a \in [0,1]$;

$a*b \leq c*d$ whenever $a \leq c$ and $b \leq d$, and $a, b, c, d \in [0,1]$.

Definition 2.3 [6] A 3-tuples $(X,M, *)$ is said to be a fuzzy metric space (shortly FM Space) if X is an

arbitrary set, * is a continuous t-norm and M is a fuzzy set on
$X^2 \times [0, \infty)$ satisfying the following conditions, for all x, y, z ∈ X and s, t > 0 ;
(FM 1): M(x,y,t) > 0
(FM 2): M(x, y, t) = 1 for all t > 0 if and only if x = y
(FM 3): M(x, y, t) = M(y, x, t)
(FM 4): M(x, y, t) $*$ M(y, z, s) ≤ M(x, z, t + s)
(FM 5): M(x, y, .) : [0,∞) → (0,1] is left continuous.
(X, M,*) denotes a fuzzy metric space, (x,y,t) can be thought of as
degree of nearness between x and y with respect to t. We identify x = y with M(x, y, t) = 1
for all t > 0 . In the following example every metric induces a fuzzy metric.

Example 2.4 Let X = [0,1], t-norm defined by a*b = min{a,b} where a,b ∈ [0,1] and M is the fuzzy set on $X^2 \times (0, \infty)$ defined by M(x,y,t) = $[\exp\{\frac{|x-y|}{t}\}]^{-1}$for all x,y ∈ X, t > 0. Then (X,M, ∗) is a fuzzy metric space.

Example 2.5 (Induced fuzzy metric [6]) Let (X, d) be a metric space, denote a * b = a.b & for all a,b ∈ [0,1] and let M_d be fuzzy sets on $X^2 \times (0, \infty)$ defined as follows

$$M_d(x,y,t) = \frac{t}{t+d(x,y)}$$

Then (X,M, ∗) is a fuzzy metric space. We call this fuzzy metric induced by a metric **d** as the standard intuitionistic fuzzy metric.

Definition 2.6 [11] Two self mappings f and g of a fuzzy metric space (X,M,*) are called compatible if $\lim_{n\to\infty} M(fg\,x_n, gf\,x_n, t) = 1$ wherever $\{x_n\}$ is sequence in X such that
$\lim_{n\to\infty} f\,x_n = \lim_{n\to\infty} g\,x_n = x$ for some x in X

Definition 2.7 [5] Two self maps f and g of a fuzzy metric space (X, M,*) are called reciprocally continuous on X if $\lim_{n\to\infty} f\,x_n = fx$ and $\lim_{n\to\infty} gf\,x_n = gx$ wherever $\{x_n\}$ is sequence in X such that
$\lim_{n\to\infty} f\,x_n = \lim_{n\to\infty} g\,x_n = x$ for some x in X.

Definition 2.8 [6] : Let (X,M, ∗) be a fuzzy metric space. Then

(a) A sequence $\{x_n\}$in X is said to converges to x in X if for each ε > 0 and each t > 0, there exist $n_o \in N$ such that M (x_n , x ,t) > 1 – ε for all $n \geq n_o$..
(b) A sequence $\{x_n\}$ in X is said to be Cauchy if for each ε > 0 and each t > 0, there exist $n_o \in N$ such that M (x_n , x_m ,t) > 1 – ε for all n, $m \geq n_o$..
(c) A fuzzy metric space in which every Cauchy sequence is convergent is said to be complete.

Definition 2.9 Two self maps f and g of a set X are occasionally weakly compatible (owc) iff there is a point x in X which is a coincidence point of f and g at which f and g commute.

A. Al-Thagafi and Naseer Shahzad [3] shown that occasionally weakly is weakly
compatible but converse is not true.

Definition 2.10 Let (X,d) be a compatible metric space, α ∈ [0,1], f:X→X a mapping such that for each x,y ∈ X
$\int_0^{d(fx,fy)} \varphi(t)dt \leq \alpha \int_0^{d(x,y)} \varphi(t)dt$ where $\varphi : R^+ \to R$ is lebesgue integral mapping which is summable,

$$\varepsilon > 0, \int_0^{\varepsilon} \varphi(t)dt > 0$$

nonnegative and such that, for each. Then f has a unique common fixed z ∈ X such that for each x ∈ X, $\lim_{n\to\infty} f^n x = z$
Rhodes[30], extended this resul by replacing the above condition by the following

$$\int_0^{d(fx,fy)} \varphi(t)dt \leq \alpha \int_0^{\max\{d(x,y),d(x,fx),d(y,fy),\frac{1}{2}[d(x,fy)+d(x,fx)\}} \varphi(t)dt$$

Ojha et al.(2010). Let (X,d) be a metric space and let $f: X \to X$, $F: X \to CB(X)$ be single and a multi valued map respectively, suppose that f and Fare occasionally weakly commutative (owc) and satisfy the inequality

$$\int_0^{d(Fx,Fy)^p} \varphi(t)dt \leq \int_0^{\substack{\max\{ad(fx,fy)d^{p-1}(fx,Fx),ad(fx,fy)d^{p-1}(fy,Fy),\\ ad(fx,Fx)d^{p-1}(fy,Fy),cd^{p-1}(fx,Fy)d(fy,Fx)\}}} \varphi(t)dt$$

For all x,y in X, where p ≥ 2 is an integer a ≥ o and $0 < c < 1$ then f and F have unique common fixed point in X.

Example 2.11 [3] Let R be the usual metric space. Define S, T: R → R by Sx = 2x and Tx = x^2 for all x ∈ R. Then Sx = Tx for x = 0, 2 but ST0 = TS0, and ST2 ≠ TS2. Hence S and T are occasionally weakly compatible self maps but not weakly compatible

Lemma 2.12 [12] Let X be a set, f, g owc self maps of X. If f and g have a unique point of coincidence, w = fx = gx, then w is the unique common fixed point of f and g.

Lemma 2.13 Let (X,M, ∗) be a fuzzy metric space. If there exist q ∈ (0, 1) such that
M (x, y, qt) ≥ M (x, y, t) for all x, y ∈ X & t > 0 then x = y

3. Main Results:

Theorem 3.1 Let (X,M, ∗) be a complete fuzzy metric space and let F, G, S and T are self–mapping of X. Let the pairs {F,S} and {G ,T} be owc . If there exists q ∈ (0,1) such that

$$\int_0^{M(Fx,Gy,qt)} \varphi(t)d(t) \geq \int_0^{\min\{M(Sx,Ty,t),M(Sx,Fx,t).M(Gy,Ty,t),M(Fx,Ty,t),M(Gy,Sx,t)\}} \varphi(t)d(t) \quad (3.1)$$

for all x, y ∈ X and for all t > 0, then there exists a unique point w ∈ X such that Fw = Sw = w and a unique point z ∈ X such that Gz = Tz = z Moreover , z = w , so that there is a unique common fixed point of F,G,S and T.

Proof: Let the pairs { F,S} and { G ,T} be owc so there are points x,y ∈ X such that Fx = Sx and Gy = Ty. We claim that Fx = Gy. If not by inequality (3.1)

$$\int_0^{M(Fx,Gy,qt)} \varphi(t)d(t) \geq \int_0^{\min\{M(Sx,Ty,t),M(Sx,Fx,t).M(Gy,Ty,t),M(Fx,Ty,t),M(Gy,Sx,t)\}} \varphi(t)d(t)$$

$$= \int_0^{\min\{M(Fx,Gy,t),M(Fx,Fx,t).M(Gy,Gy,t),M(Fx,Gy,t),M(Gy,Fx,t)\}} \varphi(t)d(t)$$

$$= \int_0^{M(Fx,Gy,t)} \varphi(t)\, d(t)$$

Therefore Fx = Gy , i.e. Fx = Sx = Gy =Ty. Suppose that z such that Fz = Sz then by (1) we have Fz = Sz = Gy = Ty so Fx = Fz and w = Fx = Sx is the unique point of coincidence of F and S.

Similarly there is a unique point z ∈ X such that z = Gz = Tz.

Assume that w ≠ z .We have

$$\int_0^{M(w,z,qt)} \varphi(t)d(t) = \int_0^{M(Fw,Gz,qt)} \varphi(t)d(t) \geq \int_0^{\min\{M(Sw,Tz,t),M(Sw,Fz,t).M(Gz,Tz,t),M(Fw,Tz,t),M(Gz,Sw,t)\}} \varphi(t)\, d(t)$$

$$= \int_0^{\min\{M(w,z,t),M(w,z,t).M(z,z,t),M(w,z,t),M(z,w,t)\}} \varphi(t)\, d(t)$$

$$= \int_0^{M(w,z,t)} \varphi(t)\, d(t)$$

Therefore we have z = w by Lemma 2.14 and z is a common fixed point of F, G, S and T. The uniqueness of fixed point holds from (3.1)

Theorem3.2.1: Let (X,M, ∗) be complete fuzzy metric space and let F,G,S and T be self mappings of X .let the pairs {F,S} and{G,T} be owc . If there exists q ∈ (0,1) such that

$$\int_0^{M(Fx,Gy,qt)} \varphi(t)d(t) \geq \int_0^{\emptyset[\min\{M(Sx,Ty,t),M(Sx,Fx,t).M(Gy,Ty,t),M(Fx,Ty,t),M(Gy,Sx,t)\}]} \varphi(t)d(t) \quad (3.2)$$

for all x , y ∈ X and Ø:[0,1] ⟶ [0,1] such that Ø(t) > t for all 0 < t < 1 ,then there exist a unique common fixed point of F, G,S and T

Proof: From equation (3.2)

$$\int_0^{M(Fx,Gy,qt)} \varphi(t)d(t) \geq \int_0^{\emptyset[\min\{M(Sx,Ty,t),M(Sx,Fx,t).M(Gy,Ty,t),M(Fx,Ty,t),M(Gy,Sx,t)\}]} \varphi(t)\, d(t)$$

$$\geq \int_0^{\emptyset[M(Fx,Gy,t)]} \varphi(t)d(t) \quad \text{from theorem 3.1}$$

Now proof fallows by (3.1)

Theorem3.2.2: Let (X,M, ∗) be a complete fuzzy metric space and let F,G,S and T are self mappings of X. Let the pairs {F,S}and {G,T} be owc . If there exists q ∈ (0,1) such that

$$\int_0^{M(Fx,Gy,qt)} \varphi(t)d(t) \geq \int_0^{\emptyset\{M(Sx,Ty,t),M(Sx,Fx,t).M(Gy,Ty,t),M(Fx,Ty,t),M(Gy,Sx,t)\}} \varphi(t)\, d(t) \quad (3.3)$$

for all x, y ∈ X and Ø: $[0,1]^4 \to [0,1]$ such that Ø (t,1,t,t) > t for all ,0 < t < 1 then there exists a unique common fixed point of F,G,S and T.

Proof: Let the pairs {F,S}and {G,T} are owc ,there are points x, y ∈ X such that Fx = Sx and Gy = Ty are Claim that Fx = Gy. By inequality (3.3) we have

$$\int_0^{M(Fx,Gy,qt)} \varphi(t)d(t)$$
$$\geq \int_0^{\emptyset\{M(Sx,Ty,t),M(Sx,Fx,t).M(Gy,Ty,t),M(Fx,Ty,t),M(Gy,Sx,t)\}} \varphi(t)d(t)$$
$$= \int_0^{\emptyset\{M(Fx,Gy,t),M(Fx,Fx,t).M(Gy,Gy,t),M(Fx,Gy,t),M(Gy,Fx,t)\}} \varphi(t)d(t)$$
$$= \int_0^{\emptyset\{M(Fx,Gy,t),1.1,M(Fx,Gy,t),M(Gy,Fx,t)\}} \varphi(t)\, d(t)$$
$$[\because\ M(Fx,Fx,t)=1, M(Gy,Gy,t)=1]$$
$$> \int_0^{M(Fx,Gy,t)} \varphi(t)d(t)$$

a contradiction , therefore Fx = Gy i.e. Fx = Sx = Gy = Ty

suppose that there is another point z such that Fz = Sz then by (3.3) we have

Fz = Sz = Gy = Ty so Fx = Fz and w = Fx = Tx is unique point of coincidence of F and T.

By Lemma 2.14 w is a unique common fixed point of F and S, similarly there is a unique point z ∈ X such that z = Gz = Tz..Thus z is common fixed point of F,G,S and T. The uniqueness of fixed point holds from (3.3)

Theorem3.2.3: Let (X,M,∗) be complete fuzzy metric space and let F,G,S and T be self mappings of X, let the pairs {F,S} and {G,T} are owc. If there exists a points q ∈ (0,1) for all x, y ∈ X and t > 0

$$\int_0^{M(Fx,Gy,qt)} \varphi(t)d(t) \geq \int_0^{\{M(Sx,Ty,t)*M(Fx,Sx,t).M(Gy,Ty,t)*M(Fx,Ty,t)\}} \varphi(t)d(t) \qquad (3.4)$$

then there exists a unique common fixed points of F,G ,S and T.

Proof: Let the points {F, S} and {G,T} are owc and there are points x , y ∈ X such that Fx = Sx and Gy = Ty and claim that Fx = Gy

By inequality (3.4)

We have

$$\int_0^{M(Fx,Gy,qt)} \varphi(t)d(t)$$
$$\geq \int_0^{\{M(Sx,Ty,t)*M(Fx,Sx,t).M(Gy,Ty,t)*M(Fx,Ty,t)\}} \varphi(t)d(t)$$
$$= \int_0^{M(Fx,Gy,t)*M(Fx,Fx,t).M(Gy,Gy,t)*M(Fx,Gy,t)} \varphi(t)d(t)$$
$$\geq \int_0^{M(Fx,Gy,t)*1.1*M(Fx,Gy,t)} \varphi(t)d(t)$$
$$\geq \int_0^{M(Fx,Gy,t)} \varphi(t)\, d(t)$$

Thus we have Fx = Gy i.e. Fx = Sx = Gy = Ty suppose that there is another point z such that Fz=Sz then by (3) we have

Fz = Sz = Gy = Ty so Fx = Fz and w = Fx = Sx is unique point of coincidence of F and S.

Similarly there is a unique point z ∈ X such that z = Gz = Tz. Thus w is a common fixed point of F,G ,S, and T.

Corollary3.2.4: Let (X,M,*) be a complete fuzzy metric space and let F,G,S and T be self mapping of X Let the pairs {F,S} and {G,T} are owc.

If there exists a point q ∈ (0,1) for all x, y ∈ X and t > 0

$$\int_0^{M(Fx,Gy,qt)} \varphi(t)d(t) \geq \int_0^{M(Sx,Ty,t)*M(Fx,Sx,t).M(Gy,Ty,t)*M(Gy,Sx,2t)*M(Fx,Ty,t)} \varphi(t)d(t) \qquad (3.5)$$

then there exists a unique common fixed point of F,G,S and T.

Proof: We have

$$\int_0^{M(Fx,Gy,qt)} \varphi(t)d(t) \geq \int_0^{M(Sx,Ty,t)*M(Fx,Sx,t).M(Gy,Ty,t)*M(Gy,Sx,2t)*M(Fx,Ty,t)} \varphi(t)d(t) \geq$$

$\int_0^{M(Sx,Ty,t)*M(Fx,Sx,t).M(Gy,Ty,t)*M(Sx,Ty,t)*M(Ty,Gy,t)*M(Fx,Ty,t)} \varphi(t)d(t) \geq$

$\int_0^{M(Sx,Ty,t)*M(Fx,Fx,t).M(Gy,Ty,t)*M(Sx,Ty,t)*M(Gy,Gy,t)*M(Fx,Ty,t)} \varphi(t)d(t)$

$\geq \int_0^{M(Sx,Ty,t)*1.1*M(Sx,Ty,t)*1*M(Fx,Ty,t)} \varphi(t)d(t)$

$\geq \int_0^{M(Sx,Ty,t)*M(Sx,Ty,t)*M(Fx,Ty,t)} \varphi(t)d(t)$

[∴ Fx = Sx and Gy = Ty]

and therefore from Theorem 3.2.3 , F, G, S and T have common fixed point.

Corollary3.2.5: Let (X,M,*) be complete fuzzy metric space and let F,G,S and T be self-mapping of X. Let the pairs {F,S} and {G,T} are owc. If there exist point $q \in (0,1)$ for all x, y $\in$ X and t > 0

$$\int_0^{M(Fx,Gy,qt)} \varphi(t)d(t) \geq \int_0^{M(Sx,Ty,t)} \varphi(t)d(t) \quad (3.6)$$

then there exists a unique common fixed point of F,G,S and T.

Proof: The proof follows from Corollary 3.2.4

Theorem 3.2.6: Let (X,M,*) be complete fuzzy metric space . Then continues self mappings S and T of X have a common fixed point in X if and only if there exists a self mapping F of X such that the following conditions are satisfied

(i) FX ⊂ TX ∩ SX
(ii) pairs { F,S} and {F,T} are weakly compatible,
(iii) there exists a point $q \in (0,1)$ such that for every x, y ∈ X and t > 0

$\int_0^{M(Fx,Gy,qt)} \varphi(t)d(t) \geq$

$\int_0^{M(Sx,Ty,t)*M(Fx,Sx,t).M(Fy,Ty,t)*M(Fx,Ty,t)} \varphi(t)d(t) \quad (3.7)$

then F, S and T have a unique common fixed point.

Proof: Since compatible implies owc, the result follows from (Theorem3.2.3)

Theorem 3.2.7: Let (X,M,*) be a complete fuzzy metric space and let F and G be self mapping of X .Let F and G are owc. If there exists a point $q \in (0,1)$ for all x, y ∈ X and t > 0

$\int_0^{M(Sx,Sy,qt)} \varphi(t)d(t) \geq$

$\int_0^{\alpha M(Fx,Fy,t)+\beta \min\{M(Fx,Fy,t),M(Sx,Fx,t).M(Sy,Fy,t)\}} \varphi(t)d(t) \quad (3.8)$

for all x,y ∈ X , where $\alpha, \beta > 0$, $\alpha+\beta > 1$. Then F and S have a unique common fixed point.

Proof: Let the pairs {F,S}be owc, so there is a point x ∈X such that Fx = Sx. Suppose that there exist another point y ∈ X for which Fy = Sy We claim that Sx = Sy by equation (8) we have

$\int_0^{M(Sx,Sy,qt)} \varphi(t)d(t) \geq$

$\int_0^{\alpha M(Fx,Fy,t)+\beta \min\{M(Fx,Fy,t),M(Sx,Fx,t).M(Sy,Fy,t)\}} \varphi(t)d(t)$

$= \int_0^{\alpha M(Sx,Sy,t)+\beta \min\{M(Sx,Sy,t),M(Sx,Sx,t).M(Sy,Sy,t)\}} \varphi(t)d(t)$

$= \int_0^{(\alpha+\beta)M(Sx,Sy,t)} \varphi(t)d(t)$

A contradiction, since $(\alpha+\beta) > 1$therefore Sx = Sy. Therefore Fx = Fy and Fx is unique.

From lemma 2.14, F and S have a unique fixed point.

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