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Some Common Fixed Point Theorem for Cone Metric Space

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Research Article

Abstract: In this paper, we proof some fixed point and common fixed point theorem for Cone metric space.

Let X be a Real banach space and P a subset of X.P is called a cone if P satisfy followings conditions:

(1) P is closed, non empty and P≠0
(2) Ax + By Є P for all x,y Є P and non negative real numbers a,b
(3) P ∩ (-P) = { 0 }

Given a cone P ⊂ X, we define a partial ordering ≤ on X with respect to P by y-x Є P.

We shall write x << y if (y-x) Є int P, denoted by $\|.\|$ the norm on X. the cone P is called normal if there is a number k > 0 such that for all x,y Є X

$$0 \le x \le y \Rightarrow \|x\| \le k\|y\| \quad \text{.....................(A)}$$

The least positive number k satisfying the above condition (A) is called the normal constant of P.

The authors showed that there is no normal cones with normal constant M < 1 and for each k > 1

There are cone with normal constant M > k.

The cone P is called regular if every increasing sequence which is bounded from the above is convergent, that is if $\{x_n\}_{n\ge1}$ is a sequence such that $x_1 \le x_2 \le \ldots \le y$ for some y Є X,

Then there is x Є X $\lim_{n\to\infty}\|x_n - x\| = 0$.

The cone P is regular iff every decreasing sequence which is bounded from below is convergent.

Definition:1 Let X be a nonempty set and X is a real Banach Space, d is a mapping from X into itself such that, d satisfying following conditions,

.d_1 : d(x,y) ≥ 0 ∀ x,y Є X
.d_2 : d(x,y) = 0 ⇔ x=y
.d_3 : d(x,y) = d(y,x)
.d_4 : d(x,y) ≤ d(x,z) + d(z,y)

Then d is called a cone metric on X and (X,d) is called cone metric space.

Definition:2 Let A and S be two mapping of a cone metric space (X,d) then it is said to be compatible if $\lim_{n\to\infty} d(ASx_n, SAx_n) = 0$, whenever $\{x_n\}$ is a sequence in X such that $\lim_{n\to\infty} Ax_n = t$ and $\lim_{n\to\infty} Sx_n = t$

For some t Є X.

Let A and S be two self mapping of a cone metric space (X,d) then it is said to be weakly compatible, if they commute at coincidence point,that is Ax = Sx implies that, ASx = SAx for x Є X.

It is easy to see that compatible mapping commute at there coincidence points.It is note that a compatible maps are weakly compatible but converges need not be true.

Theorem:1.1 Let (X,d) be a complete cone metric space and P a normal cone with normal constant k. Suppose that the mapping T, from X into itself satisfy the condition,

$$d(Tx,Ty) \le \alpha d(x,y) + \beta[d(x,Tx)+d(y,Ty)] + \gamma[d(x,Ty)+d(y,Tx)] + \delta\frac{d(x,Tx)+d(y,Tx)}{1+d(x,Tx)d(y,Tx)}$$
$$+\eta\frac{d(y,Ty)+d(x,Ty)}{1+d(x,Tx)d(y,Tx)} \quad \text{.......................................(1)}$$

For all x,yЄX and $\alpha,\beta,\gamma,\delta,\eta \ge 0$ such that $0 \le \alpha + 2\beta + 2\gamma + \delta + 3\eta < 1$.Then T has unique fixed point in X.

Proof: For any arbitrary X_0, in X, we choose X_1, X_2Є X such that,

$Tx_0 = x_1$ and $Tx_1 = x_2$

In general we can define a sequence of elements of X such that,

$X_{2n+1} = TX_{2n}$ and $X_{2n+2} = TX_{2n+1}$

Now , from (1)

$d(X_{2n+1},X_{2n+2})=d(TX_{2n},TX_{2n+1})$

$$\leq \alpha d(X_{2n},X_{2n+1})+\beta[d(X_{2n},TX_{2n})+d(X_{2n+1},TX_{2n+1})]+\gamma[d(X_{2n},TX_{2n+1})+d(X_{2n+1},TX_{2n})]$$

$$+\delta\frac{d(X_{2n},TX_{2n})+d(X_{2n+1},TX_{2n})}{1+d(X_{2n},TX_{2n})+d(X_{2n+1},TX_{2n})}+\eta\frac{d(X_{2n+1},TX_{2n+1})+d(X_{2n},TX_{2n+1})}{1+d(X_{2n},TX_{2n})+d(X_{2n+1},TX_{2n})}$$

$$\leq \alpha d(X_{2n},X_{2n+1})+\beta[d(X_{2n},X_{2n+1})+d(X_{2n+1},X_{2n+2})]+\gamma[d(X_{2n},X_{2n+2})+d(X_{2n+1},X_{2n+1})]$$

$$+\delta\frac{d(X_{2n},X_{2n+1})+d(X_{2n+1},X_{2n+1})}{1+d(X_{2n},X_{2n+1})+d(X_{2n+1},X_{2n+1})}+\eta\frac{d(X_{2n+1},X_{2n+2})+d(X_{2n},X_{2n+2})}{1+d(X_{2n},X_{2n+1})+d(X_{2n+1},X_{2n+1})}$$

By using triangle inequality,we get

$$d(X_{2n+1},X_{2n+2})\leq [\frac{\alpha+\beta+\gamma+\delta+\eta}{1-\beta-\gamma-2\eta}]\, d(X_{2n},X_{2n+1})$$

Similarly we can show that,

$$d(X_{2n},X_{2n+1})\leq [\frac{\alpha+\beta+\gamma+\delta+\eta}{1-\beta-\gamma-2\eta}]\, d(X_{2n-1},X_{2n})$$

In general we can write,

$$d(X_{2n+1},X_{2n+2})\leq [\frac{\alpha+\beta+\gamma+\delta+\eta}{1-\beta-\gamma-2\eta}]^{2n+1}\, d(X_0,X_1)$$

On taking $[\frac{\alpha+\beta+\gamma+\delta+\eta}{1-\beta-\gamma-2\eta}]=\theta$

$$d(X_{2n+1},X_{2n+2})\leq \theta^{2n+1}\, d(X_0,X_1)$$

For n≤m, we have

$$d(X_{2n},X_{2m})\leq d(X_{2n},X_{2n+1})+d(X_{2n+1},X_{2n+2})\ldots\ldots\ldots\ldots+d(X_{2m-1},X_{2m})$$

$$d(X_{2n},X_{2m})\leq \{\theta^n+\theta^{n+1}+\ldots\ldots\ldots\ldots+\theta^m\}d(X_0,X_1)$$

$$d(X_{2n},X_{2m})\leq \frac{\theta^n}{1-\theta}d(X_0,X_1)$$

$$\|d(X_{2n},X_{2m})\|\leq \frac{\theta^n}{1-\theta}k\|d(X_0,X_1)\| \text{ as } n\to\infty$$

$$\lim_{n\to\infty}\|d(X_{2n},X_{2m})\|\to 0$$

In this way

$$\lim_{n\to\infty}\|d(X_{2n+1},X_{2n+2})\|\to 0 \quad \text{as } n\to\infty$$

Hence $\{x_n\}$ is a Cauchy sequence which converges to u∈X

Hence (X,d) is complete cone metric space.

Thus $x_n\to u$ as $n\to\infty$, $TX_{2n}\to u$ and $T_{2n+1}\to u$ as $n\to\infty$,

u is fixed point of T in X.

Uniqueness: Let us assume that , v is another fixed point of T in X different from v.then,

Tu=u andTv=v

From (1)

$d(u,v)=d(Tu,Tv)$

$$\leq \alpha d(u,v)+\beta[d(u,Tu)+d(v,Tv)]+\gamma[d(u,Tv)+d(v,Tu)]+\delta\frac{d(u,Tu)+d(v,Tu)}{1+d(u,Tu)d(v,Tu)}$$

$$+\eta\frac{d(v,Tv)+d(u,Tv)}{1+d(u,Tu)d(v,Tu)}$$

d(Tu,Tv)≤($\alpha + 2\gamma + \delta + \eta$)d(u,v)

Which contradiction,

u is unique fixed point of T in X.

Theorem:2 Let (X,d) be a complete cone metric space and P a normal cone with normal constant k. Suppose that S and T, the mapping from X into itself satisfies the condition,

$$d(Sx,Ty) \le \alpha d(x,y) + \beta[d(x,Sx)+d(y,Ty)] + \gamma[d(x,Ty)+d(y,Sx)] + \delta \frac{d(x,Sx)+d(y,Sx)}{1+d(x,Sx)d(y,Sx)}$$

$$+\eta \frac{d(y,Ty)+d(x,Ty)}{1+d(x,Sx)d(y,Sx)} \qquad \ldots\ldots(2)$$

For all x,y∈X and $\alpha,\beta,\gamma,\delta,\eta \ge 0$ such that $0 \le \alpha + 2\beta + 2\gamma + \delta + 3\eta < 1$.Then S and T has unique fixed point in X.Further more if, ST=TS then it have unique common fixed point in X.

Proof: For any arbitrary X_0, in X, we choose X_1, X_2∈ X such that,

$Sx_0=x_1$ and $Tx_1=x_2$

In general we can define a sequence of elements of X such that,

$X_{2n+1}=SX_{2n}$ and $X_{2n+2}=TX_{2n+1}$

Now , from (1)

$$d(X_{2n+1},X_{2n+2})=d(SX_{2n},TX_{2n+1})$$

$$\le \alpha d(X_{2n},X_{2n+1}) + \beta[d(X_{2n},SX_{2n})+d(X_{2n+1},TX_{2n+1})] + \gamma[d(X_{2n},TX_{2n+1})+d(X_{2n+1},SX_{2n})]$$

$$+\delta \frac{d(X_{2n},SX_{2n})+d(X_{2n+1},SX_{2n})}{1+d(X_{2n},SX_{2n})+d(X_{2n+1},SX_{2n})} + \eta \frac{d(X_{2n+1},TX_{2n+1})+d(X_{2n},TX_{2n+1})}{1+d(X_{2n},SX_{2n})+d(X_{2n+1},SX_{2n})}$$

$$\le \alpha d(X_{2n},X_{2n+1}) + \beta[d(X_{2n},X_{2n+1})+d(X_{2n+1},X_{2n+2})] + \gamma[d(X_{2n},X_{2n+2})+d(X_{2n+1},X_{2n+1})]$$

$$+\delta \frac{d(X_{2n},X_{2n+1})+d(X_{2n+1},X_{2n+1})}{1+d(X_{2n},X_{2n+1})+d(X_{2n+1},X_{2n+1})} + \eta \frac{d(X_{2n+1},X_{2n+2})+d(X_{2n},X_{2n+2})}{1+d(X_{2n},X_{2n+1})+d(X_{2n+1},X_{2n+1})}$$

By using triangle inequality,we get

$$d(X_{2n+1},X_{2n+2}) \le [\frac{\alpha+\beta+\gamma+\delta+\eta}{1-\beta-\gamma-2\eta}]\, d(X_{2n},X_{2n+1})$$

Similarly we can show that,

$$d(X_{2n},X_{2n+1}) \le [\frac{\alpha+\beta+\gamma+\delta+\eta}{1-\beta-\gamma-2\eta}]\, d(X_{2n-1},X_{2n})$$

In general we can write,

$$d(X_{2n+1},X_{2n+2}) \le [\frac{\alpha+\beta+\gamma+\delta+\eta}{1-\beta-\gamma-2\eta}]^{2n+1} d(X_0,X_1)$$

On taking $[\frac{\alpha+\beta+\gamma+\delta+\eta}{1-\beta-\gamma-2\eta}] = \theta$

$$d(X_{2n+1},X_{2n+2}) \le \theta^{2n+1} d(X_0,X_1)$$

For n≤m, we have

$$d(X_{2n},X_{2m}) \le d(X_{2n},X_{2n+1}) + d(X_{2n+1},X_{2n+2}) \ldots\ldots\ldots + d(X_{2m-1},X_{2m})$$

$$d(X_{2n},X_{2m}) \le \{\theta^n + \theta^{n+1} + \ldots\ldots\ldots + \theta^m\} d(X_0,X_1)$$

$$d(X_{2n},X_{2m}) \le \frac{\theta^n}{1-\theta} d(X_0,X_1)$$

$$\|d(X_{2n},X_{2m})\| \le \frac{\theta^n}{1-\theta} k \|d(X_0,X_1)\| \text{ as } n\to\infty$$

$$\lim_{n\to\infty}\left\|d(X_{2n},X_{2m})\right\|\to 0$$

In this way

$$\lim_{n\to\infty}\left\|d(X_{2n+1},X_{2n+2})\right\|\to 0 \quad \text{as } n\to\infty$$

Hence $\{x_n\}$ is a Cauchy sequence which converges to u∈X

Hence (X,d) is complete cone metric space.

Thus $x_n\to u$ as $n\to\infty$, $SX_{2n}\to u$ and $T_{2n+1}\to u$ as $n\to\infty$,

u is fixed point of S and T in X.

Since ST=TS this give,

u=Tu=TSu=STu=Su=u

u is common fixed point of S and T.

Uniqueness: Let us assume that , v is another fixed point of T in X different from v.then,

Tu=u andTv=v also Su=u and Sv=v

From (2)

d(u,v)=d(Su,Tv)

$$\le \alpha d(u,v)+\beta[d(u,Su)+d(v,Tv)]+\gamma[d(u,Tv)+d(v,Su)]+\delta\frac{d(u,Su)+d(v,Su)}{1+d(u,Su)d(v,Su)}$$

$$+\eta\frac{d(v,Tv)+d(u,Tv)}{1+d(u,Su)d(v,Su)}$$

d(Tu,Tv)≤($\alpha+2\gamma+\delta+\eta$)d(u,v)

Which contradiction,

u is unique fixed point of S and T in X.

Theorem:3 Let (X,d) be a complete cone metric space and P a normal cone with normal constant k. Suppose that S,R and T,be the mapping from X into itself satisfies the condition,

$$d(SRx,TRy)\le \alpha d(x,y)+\beta[d(x,SRx)+d(y,TRy)]+\gamma[d(x,TRy)+d(y,SRx)]+\delta\frac{d(x,SRx)+d(y,SRx)}{1+d(x,SRx)d(y,SRx)}$$

$$+\eta\frac{d(y,TRy)+d(x,TRy)}{1+d(x,SRx)d(y,SRx)} \quad \ldots\ldots\ldots\ldots(3)$$

For all x,y∈X and $\alpha,\beta,\gamma,\delta,\eta\ge 0$ such that $0\le\alpha+2\beta+2\gamma+\delta+3\eta<$ 1.Then S,R and T has unique fixed point in X.Further more either SR=RS or TR=RT then it have unique common fixed point in X.

Proof: For any arbitrary X_0, in X, we choose X_1,X_2∈ X such that,

$SRx_0=x_1$ and $TRx_1=x_2$

In general we can define a sequence of elements of X such that,

$X_{2n+1}=SRX_{2n}$ and $X_{2n+2}=TRX_{2n+1}$

Now ,

$d(X_{2n+1},X_{2n+2})=d(SRX_{2n},TRX_{2n+1})$

$$\le \alpha d\left(X_{2n},X_{2n+1}\right)+\beta[d\left(X_{2n},SRX_{2n}\right)+d\left(X_{2n+1},TRX_{2n+1}\right)]+\gamma[d\left(X_{2n},TRX_{2n+1}\right)+d\left(X_{2n+1},SRX_{2n}\right)]$$

$$+\delta\frac{d\left(X_{2n},SRX_{2n}\right)+d\left(X_{2n+1},SRX_{2n}\right)}{1+d\left(X_{2n},SRX_{2n}\right)+d\left(X_{2n+1},SRX_{2n}\right)}+\eta\frac{d\left(X_{2n+1},TRX_{2n+1}\right)+d\left(X_{2n},TRX_{2n+1}\right)}{1+d\left(X_{2n},SRX_{2n}\right)+d\left(X_{2n+1},SRX_{2n}\right)}$$

$$\le \alpha d(X_{2n},X_{2n+1})+\beta[d(X_{2n},X_{2n+1})+d(X_{2n+1},X_{2n+2})]+\gamma[d(X_{2n},X_{2n+2})+d(X_{2n+1},X_{2n+1})]$$

$$+\delta\frac{d(X_{2n},X_{2n+1})+d(X_{2n+1},X_{2n+1})}{1+d(X_{2n},X_{2n+1})+d(X_{2n+1},X_{2n+1})}+\eta\frac{d(X_{2n+1},X_{2n+2})+d(X_{2n},X_{2n+2})}{1+d(X_{2n},X_{2n+1})+d(X_{2n+1},X_{2n+1})}$$

By using triangle inequality,we get

$$d(X_{2n+1}, X_{2n+2}) \leq [\frac{\alpha+\beta+\gamma+\delta+\eta}{1-\beta-\gamma-2\eta}]\, d(X_{2n}, X_{2n+1})$$

Similarly we can show that,

$$d(X_{2n}, X_{2n+1}) \leq [\frac{\alpha+\beta+\gamma+\delta+\eta}{1-\beta-\gamma-2\eta}]\, d(X_{2n-1}, X_{2n})$$

In general we can write,

$$d(X_{2n+1}, X_{2n+2}) \leq [\frac{\alpha+\beta+\gamma+\delta+\eta}{1-\beta-\gamma-2\eta}]^{2n+1}\, d(X_0, X_1)$$

On taking $[\frac{\alpha+\beta+\gamma+\delta+\eta}{1-\beta-\gamma-2\eta}] = \theta$

$$d(X_{2n+1}, X_{2n+2}) \leq \theta^{2n+1}\, d(X_0, X_1)$$

For n≤m, we have

$$d(X_{2n}, X_{2m}) \leq d(X_{2n}, X_{2n+1}) + d(X_{2n+1}, X_{2n+2}) \ldots\ldots\ldots + d(X_{2m-1}, X_{2m})$$

$$d(X_{2n}, X_{2m}) \leq \{\theta^n + \theta^{n+1} + \ldots\ldots\ldots + \theta^m\} d(X_0, X_1)$$

$$d(X_{2n}, X_{2m}) \leq \frac{\theta^n}{1-\theta} d(X_0, X_1)$$

$$\|d(X_{2n}, X_{2m})\| \leq \frac{\theta^n}{1-\theta} k \|d(X_0, X_1)\| \text{ as } n\to\infty$$

$$\lim_{n\to\infty} \|d(X_{2n}, X_{2m})\| \to 0$$

In this way

$$\lim_{n\to\infty} \|d(X_{2n+1}, X_{2n+2})\| \to 0 \text{ as } n\to\infty$$

Hence $\{x_n\}$ is a Cauchy sequence which converges to u∈X

Hence (X,d) is complete cone metric space.

Thus $x_n \to u$ as $n\to\infty$, $SRX_{2n} \to u$ and $TR_{2n+1} \to u$ as $n\to\infty$,

u is fixed point of S and T in X.

Since ST=TS this give,

u=Tu=TSu=STu=Su=u

u is common fixed point of S and T.

Uniqueness: Let us assume that , v is another fixed point of T in X different from v.then,

Tu=u andTv=v also Su=u and Sv=v

From (3)

$$d(u,v)=d(Su,Tv)$$

$$\leq \alpha d(u,v) + \beta[d(u,Su)+d(v,Tv)] + \gamma[d(u,Tv)+d(v,Su)] + \delta \frac{d(u,Su)+d(v,Su)}{1+d(u,Su)d(v,Su)}$$

$$+\eta \frac{d(v,Tv)+d(u,Tv)}{1+d(u,Su)d(v,Su)}$$

$d(Tu,Tv) \leq (\alpha + 2\gamma + \delta + \eta)\, d(u,v)$

Which contradiction,

u is unique fixed point of S and T in X.

Theorem:4 Let (X,d) be a complete cone metric space and P a normal cone with normal constant k. Suppose that A,B S and T,be the mapping from X into itself satisfies the condition,

(1) $A(X) \subseteq T(X)$, $B(X) \subseteq S(X)$.

(2) {A,S} and {B,T} are weakly compatible.

(3) S or T is continuous.

(4)

$$d(Ax,By)\le \alpha d(Sx,Ty)+\beta[d(Sx,Ax)+d(Ty,By)]+\gamma[d(Sx,By)+d(Ty,Ax)]+\delta\frac{d(Sx,Ax)+d(Ty,Ax)}{1+d(Sx,Ax)d(Ty,Ax)}$$

$$+\eta\frac{d(Ty,By)+d(Sx,By)}{1+d(Sx,Ax)d(Ty,Ax)} \qquad \ldots\ldots\ldots\ldots(4)$$

For all x,yЄX and $\alpha,\beta,\gamma,\delta,\eta\ge 0$ such that $0\le\alpha+2\beta+2\gamma+\delta+3\eta<1$.Then A,B,S,R and T have unique fixed point in X.

Proof: For any arbitrary X_0, in X, we define the sequence $\{x_n\}$ and $\{y_n\}$ in X, such that,

$AX_{2n}=TX_{2n+1}=y_{2n}$ and $BX_{2n+1}=SX_{2n+2}=y_{2n+1}$ for all n=0,1,2,…….

Now ,

$$d(y_{2n},y_{2n+1})=d(AX_{2n},BX_{2n+1})$$

$$\le \alpha d(SX_{2n},TX_{2n+1})+\beta[d(SX_{2n},AX_{2n})+d(TX_{2n+1},BX_{2n+1})]+\gamma[d(SX_{2n},BX_{2n+1})+d(TX_{2n+1},AX_{2n})]$$

$$+\delta\frac{d(SX_{2n},AX_{2n})+d(TX_{2n+1},AX_{2n})}{1+d(SX_{2n},AX_{2n})+d(TX_{2n+1},AX_{2n})}+\eta\frac{d(TX_{2n+1},BX_{2n+1})+d(SX_{2n},BX_{2n+1})}{1+d(SX_{2n},AX_{2n})+d(TX_{2n+1},AX_{2n})}$$

$$\le \alpha d(y_{2n-1},y_{2n})+\beta[d(y_{2n-1},y_{2n})+d(y_{2n},y_{2n+1})]+\gamma[d(y_{2n-1},y_{2n+1})+d(y_{2n},y_{2n})]$$

$$+\delta\frac{d(y_{2n-1},y_{2n})+d(y_{2n},y_{2n})}{1+d(y_{2n-1},y_{2n})+d(y_{2n},y_{2n})}+\eta\frac{d(y_{2n},y_{2n+1})+d(y_{2n-1},y_{2n+1})}{1+d(y_{2n-1},y_{2n})+d(y_{2n},y_{2n})}$$

By using triangle inequality,we get

$$d(y_{2n},y_{2n+1})\le[\frac{\alpha+\beta+\gamma+\delta+\eta}{1-\beta-\gamma-2\eta}]\,d(y_{2n-1},y_{2n})$$

In general we can write,

$$d(y_{2n},y_{2n+1})\le[\frac{\alpha+\beta+\gamma+\delta+\eta}{1-\beta-\gamma-2\eta}]^{2n+1}d(y_0,y_1)$$

On taking $[\frac{\alpha+\beta+\gamma+\delta+\eta}{1-\beta-\gamma-2\eta}]=\theta$

$$d(y_{2n},y_{2n+1})\le\theta^{2n+1}d(y_0,y_1)$$

For n≤m, we have

$$d(y_{2n},y_{2m})\le\{\theta^n+\theta^{n+1}+\ldots\ldots\ldots\ldots+\theta^m\}d(y_0,y_1)$$

$$d(y_{2n},y_{2m})\le\frac{\theta^n}{1-\theta}d(y_0,y_1)$$

$$\|d(y_{2n},y_{2m})\|\le\frac{\theta^n}{1-\theta}k\|d(y_0,y_1)\| \text{ as } n\to\infty$$

$$\lim_{n\to\infty}\|d(y_{2n},y_{2m})\|\to 0$$

Hence $\{y_n\}$ is a Cauchy sequence which converges to uЄX,By the continuity of S and T,$\{x_n\}$ is also convergent sequence which converges to uЄX,

Hence (X,d) is complete cone metric space.

u is fixed point of A,B,S and T.

Since {A,S} and {B,T} are weakly compatible, implies that u is common fixed point of A,B,S and T

Uniqueness: Let us assume that , v is another fixed point of A,B,S and T in X different from v.then,

Au=u and Av=v also Bu=u and Bv=v

From (4)

d(u,v)=d(Au,Bv)

$$\leq \alpha d(Su,Tv) + \beta[d(Su,Au) + d(Tv,Bv)] + \gamma[d(Su,Bv) + d(Tv,Au)] + \delta \frac{d(Su,Au) + d(Tv,Au)}{1 + d(Su,Au)d(Tv,Au)}$$

$$+\eta \frac{d(Tv,Bv) + d(Su,Bv)}{1 + d(Su,Au)d(Tv,Au)}$$

d(Au,Bv)≤($\alpha + 2\gamma + \delta + \eta$)d(u,v)

Which contradiction,

u is unique fixed point of A,B, S and T in X.

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