On Strongly $\lambda_{(m, n)}$ - J - continuous functions

D.Sasikala[1], I.Arockiarani[2]

[1]Department of Mathematics with Computer Applications, Nirmala College for Women, Coimbatore, (TN) INDIA

[2]Department of Mathematics, Nirmala College for Women,Coimbatore, (TN)INDIA

Corresponding Address:

d.sasikala@rocketmail.com

Research Article

Abstract: In this paper, we introduce the concept of $\lambda_{(m, n)}$ - J- closed sets in bigeneralized topological spaces and study some of their properties. The notion of $Jg_{(m, n)}$- continuous functions and strongly $\lambda_{(m, n)}$ - J- closed set is also defined on bigeneralized topological spaces and investigate some of their characterizations.

Mathematics Subject Classification: 54A05, 54A10

Keywords: bigeneralized topological spaces, $\lambda_{(m,n)}$-J-closed sets, $Jg_{(m,n)}$- continuous functions, $\lambda_{(m, n)} - J^s$ – closed set.

1. Introduction

Á.Császár [3] introduced the concepts of generalized neighborhood systems and he also introduced the concepts of continuous functions and associated interior and closure operators on generalized neighborhood systems and generalized topological spaces. In particular, he investigated characterizations for generalized continuous functions (= (g,g'))-continuous functions). In [4], he introduced and studied the notions of g- α - open sets, g - semi-open sets, g – pre open sets and g - β open sets in generalized topological spaces.

After the introduction of the concept of generalized closed set by Levine[9] in a topological spaces. Several other authors gave their ideas to the generalizations of various concepts in topology. Kelly[8] introduced the concept of bitopological spaces .Since many mathematicians generalized the topological concepts into bitopological setting. In 1986, Fukutake [7] generalized the notion to bitopological spaces and he defined a set ij-generalized closed set. Some new types of generalized closed sets in bitopological spaces were introduced. In this paper, we introduce the notion of $\lambda_{m, n}$- J - closed sets in bigeneralized topological spaces and study some of their properties. We also introduce the concept of strongly $\lambda_{m, n}$- J - closed sets and strongly $\lambda_{m, n}$- J - continuous functions and investigate some of their characterizations.

2. Preliminaries

Let X be a non – empty set and λ be a collection of subsets of X. Then λ is called a generalized topology (briefly GT) on X iff $\phi \in \lambda$ and $G_i \in \lambda$ for $i \in I \neq \phi$ implies $G = \bigcup_{i \in I} G_i \in \lambda$ .We call the pair (X,λ), a generalized topological space(briefly GTS)on X. The elements of λ are called λ- open sets and the complements are called λ- closed sets. The generalized closure of a subset S of X, denoted by $c_\lambda(S)$, is the intersection of generalized closed sets including S and the interior of S, denoted by $i_\lambda(S)$, is the union of generalized open sets contained in S.

Definitions 2.1. Let (X, λ) be a generalized topological space and $A \subseteq X$, then A is said to be

(1) λ - semi open if $A \subseteq c_\lambda(i_\lambda(A))$

(2) λ - pre open if $A \subseteq i_\lambda(c_\lambda(A))$

(3) λ - α- open if $A \subseteq i_\lambda(c_\lambda(i_\lambda(A)))$

(4) λ - β open if $A \subseteq c_\lambda(i_\lambda(c_\lambda(A)))$

The complement of λ - semi open (resp λ - pre open , λ - α- open, λ - β open) is said to be λ - semi closed (resp λ - pre closed , λ - α- closed, λ - β closed). The class of all λ - semi open sets on X is denoted by $\sigma(\lambda_x)$(briefly σ_x or σ). The class of λ - pre open (λ - α- open and λ - β open) sets on X as [π(gx), α(gx), β(gx)] or briefly [[π, α, β].

Definition 2.3. [2] Let X be non-empty set and let λ_1 and λ_2 be generalized topologies on X. A triple (X, λ_1, λ_2) is said to be a bigeneralized topological space (briefly BGTS).

Let (X, λ_1, λ_2) be a bigeneralized topological space and A be a subset of X. The closure of A and the interior of A with respect to λ_m are denoted by $c_{\lambda m}(A)$ and $i_{\lambda m}(A)$, respectively, for m = 1,2.

Definition 2.4. [10]Let (X, λ) be a generalized topological space and $A \subseteq X$, then A is said to be λ -

J- open if $A \subseteq i_\lambda(c_\pi(A))$. The complement of λ - J- open is said to be λ *- J- closed*.

The class of all λ - J - open sets on X is denoted by λ – JO(X). The class of all λ - J - closed sets on X is denoted by λ – JC(X).The λ - J- closure of a subset S of X, denoted by $c_J(S)$ is the intersection of λ - J- closed sets including S. The λ - J- interior of a subset S of X, denoted by $i_J(S)$ is the union of λ - J- open sets contained in S.

The class of all λ - J - open sets is properly placed between λ - open sets and λ - pre – open sets.

3. $\lambda_{(m, n)}$ - J- closed sets

Definition 3.1. A Subset A of a bigeneralized topological space (X, λ_1, λ_2) is said to be $\lambda_{(m, n)}$ *– J - closed set* if $c_{\lambda n}(A) \subseteq U$, whenever $A \subseteq U$ and U is λ_m – J- open, where m, n = 1,2 and m≠n. The complement of $\lambda_{(m, n)}$ – J - closed set is said to be $\lambda_{(m, n)}$ – J-open set.

Remark 3.2. Every (m, n) - closed set is $\lambda_{(m, n)}$ – J - closed.

The converse is not true as can be seen from the following example.

Example 3.3. Let X = {a, b, c}. Consider two generalized topologies λ_1 = {ϕ,{a},{a, b}, X}, and λ_2 = {ϕ,{c},{b, c}},.Let A = {a, b}, then A is $\lambda_{(m, n)}$ – J- closed but not (m, n) - closed.

Proposition 3.4. Every $\lambda_{(m, n)}$ – closed set is $\lambda_{(m, n)}$ – J- closed.

Proof: Let U be a λ_m – J- open set such that $A \subseteq U$. Since A is $\lambda_{(m, n)}$ – closed set, $c_{\lambda n}(A) \subseteq U$. Therefore A is $\lambda_{(m, n)}$ – J- closed.

Proposition 3.5. Let (X, λ_1, λ_2) be a bigeneralized topological space and A be a subset of X. If A is λ_n – closed, then A is $\lambda_{(m, n)}$ – J- closed, where m, n = 1, 2 and m ≠ n.

Remark 3.6.(i) The union of two $\lambda_{(m, n)}$ – J - closed need not be $\lambda_{(m, n)}$ – J- closed.

(ii) The intersection of two $\lambda_{(m, n)}$ – J- closed sets need not be $\lambda_{(m, n)}$ – J – closed.

Proposition 3.7. Let (X, λ_1, λ_2) be a bigeneralized topological space. If A is $\lambda_{(m, n)}$ – J - closed and F is (m, n) – J- closed, then $A \cap F$ is $\lambda_{(m, n)}$ – J – closed, where m, n = 1, 2 and m ≠ n.

Proof: Let U be a λ_m – J- open set such that $A \cap F \subseteq U$. Then $A \subseteq U \cup (X - F)$ and so $c_{\lambda n}(A) \subseteq U \cup (X - F)$.Therefore $c_{\lambda n}(A) \cap F \subseteq U$ .Since F is (m, n) – J- closed, $c_{\lambda n}(A \cap F) \subseteq c_{\lambda n}(A) \cap c_{\lambda n}(F) \subseteq U$. Hence $A \cap F$ is $\lambda_{(m, n)}$ – J – closed.

Proposition 3.8. For each element x of a bigeneralized topological space (X, λ_1, λ_2), {x} is λ_m - J- closed or X – {x} is $\lambda_{(m, n)}$ - J - closed, where m, n = 1, 2 and m ≠ n.

Proof: Let $x \in X$ and the singleton {x} be not λ_m – J- closed. Then X – {x} is not λ_m –J- open, if $X \in \lambda_m$, then X is only λ_m – J- open set which contains X – {x}, hence X – {x} is $\lambda_{(m, n)}$ – J- closed and if $X \notin \lambda_m$, then X – {x} is $\lambda_{(m, n)}$ – J– closed.

Proposition 3.9. Let (X, λ_1, λ_2) be a bigeneralized topological space. Let $A \subseteq X$ be a $\lambda_{(m, n)}$ – J - closed subset of X, then $c_{\lambda n}(A) \setminus A$ does not contain any non-empty λ_m - J- closed set, where m, n = 1, 2 and m ≠ n.

Proof: Let A be a $\lambda_{(m, n)}$ – J - closed set and F ≠ ϕ is λ_m – J- closed such that $F \subseteq c_{\lambda n}(A) \setminus A$. Then $F \subseteq X \setminus A$ and hence $A \subseteq X \setminus F$. Since A is $\lambda_{(m, n)}$ – J- closed, $c_{\lambda n}(A) \subseteq X \setminus F$ and hence $F \subseteq X \setminus c_{\lambda n}(A)$ .So, $F \subseteq c_{\lambda n}(A) \cap (X \setminus c_{\lambda n}(A)) = \phi$.Therefore $c_{\lambda n}(A) \setminus A$ does not contain any non- empty λ_m - J- closed set.

Proposition 3.10. Let λ_1 and λ_2 be generalized topologies on X. If A is $\lambda_{(m, n)}$ - J- closed set, then $c_{\lambda m}(\{x\}) \cap A \neq \phi$ holds for each $x \in c_{\lambda n}(A)$, where m, n = 1, 2 and m ≠ n.

Proof: Let $x \in c_{\lambda n}(A)$. Suppose that $c_{\lambda m}(\{x\}) \cap A = \phi$. Then $A \subseteq X - c_{\lambda m}(\{x\})$. Since A is $\lambda_{(m, n)}$ – J– closed and $X - c_{\lambda m}(\{x\})$ is λ_m – J- open. Thus $c_{\lambda n}(A) \subseteq X - c_{\lambda m}(\{x\})$. Hence $c_{\lambda n}(A) \cap c_{\lambda m}(\{x\}) = \phi$. This is a contradiction.

Proposition 3.11. If A is a $\lambda_{(m, n)}$ – J - closed set of (X, λ_1, λ_2) such that $A \subseteq B \subseteq c_{\lambda n}(A)$ then B is $\lambda_{(m, n)}$ – J - closed set, where m, n = 1, 2 and m ≠ n.

Proof: Let A be a $\lambda_{(m, n)}$ – J- closed set and $A \subseteq B \subseteq c_{\lambda n}(A)$. Let $B \subseteq U$ and U is λ_m – J-open. Then $A \subseteq U$. Since A is $\lambda_{(m, n)}$ – J – closed, we have $c_{\lambda n}(A) \subseteq U$. Since $B \subseteq c_{\lambda n}(A)$, then $c_{\lambda n}(B) \subseteq c_{\lambda n}(A) \subseteq U$. Hence B is $\lambda_{(m, n)}$ – J – closed.

Remark.3.12. $\lambda_{(1,2)}$ – JC(X) is generally not equal to $\lambda_{(2,1)}$ – JC(X) as can be seen from the following example.

Example 3.13. Let X = {a, b, c}. Consider two generalized topologies λ_1 = {ϕ, {a}, {a, b}, X} and λ_2 = {ϕ, {c}, {b, c}}.Then $\lambda_{(1,2)}$ – JC(X) = {{a}, {b},{a, b}, {b, c},X} and $\lambda_{(2,1)}$ – JC(X) = {{b}, {c},{a, b}, {b, c},ϕ, X }. Thus $\lambda_{(1,2)}$ – JC(X) ≠ $\lambda_{(2,1)}$ – JC(X).

Proposition 3.14. Let λ_1 and λ_2 be generalized topologies on X. if $\lambda_1 \subseteq \lambda_2$, then $\lambda_{(2,1)}$ – JC(X) $\subseteq \lambda_{(1,2)}$ – JC(X)

Proposition 3.15. For a bigeneralized topological space (X, λ_1, λ_2), λ_m - JO(X) $\subseteq \lambda_n$ - JC(X) if and only if every subset of X is $\lambda_{(m, n)}$ - J - closed set, where m, n = 1, 2 and m $\neq$ n.

Proof: Suppose that λ_m - JO(X) $\subseteq \lambda_n$ - JC(X). Let A be a subset of X such that A $\subseteq$ U, where U $\in \lambda_m$ - JO(X). Then $c_{\lambda n}$(U) $\subseteq c_{\lambda n}$(A) = U and hence A is $\lambda_{(m, n)}$ – J – closed set.

Conversely, suppose that every subset of X is $\lambda_{(m, n)}$ – J – closed. Let U $\in \lambda_m$ - JO(X).Since U is $\lambda_{(m, n)}$ – J – closed, we have $c_{\lambda n}$ (U) $\subseteq$ U. Therefore, U $\in \lambda_n$ - JC(X) and hence λ_m - jO(X) $\subseteq \lambda_n$ - JC(X).

Proposition 3.16. Let (X, λ_1, λ_2) be a bigeneralized topological space. Let A $\subseteq$ X be a $\lambda_{(m, n)}$ – J - closed subset of (X, λ_1, λ_2). Then A is λ_m - J- closed if and only if $c_{\lambda n}$(A) \ A is λ_m - J- closed, where m, n = 1, 2 and m $\neq$ n.

Proof: Let A be a $\lambda_{(m, n)}$ – J- closed set. If A is λ_m – J- closed, then $c_{\lambda n}$(A) \ A = ϕ, but ϕ is λ_m – J- closed . Therefore $c_{\lambda n}$ (A) \ A is λ_m - J- closed.

Conversely, Suppose that $c_{\lambda n}$ (A) \ A is λ_m - J- closed, As A is $\lambda_{(m, n)}$ – J– closed, $c_{\lambda n}$ (A) \ A = ϕ, Consequently $c_{\lambda n}$(A) = A.

Proposition 3.17. Let (X, λ_1, λ_2) be a bigeneralized topological space. If A is $\lambda_{(m, n)}$ – J - closed and A $\subseteq$ B $\subseteq c_{\lambda n}$(A), then $c_{\lambda n}$ (B) \ B has no non empty λ_m - J- closed subset.

Proof: A $\subseteq$ B implies X – B $\subseteq$ X – A and B $\subseteq c_{\lambda n}$(B) implies $c_{\lambda n}$(($c_{\lambda n}$(A)) = $c_{\lambda n}$(A).Thus $c_{\lambda n}$(B) $\cap$ (X - B) $\subseteq c_{\lambda n}$(A) $\cap$ (X -A) which yields $c_{\lambda n}$(B) / B $\subseteq c_{\lambda n}$(A) / A. As A is $\lambda_{(m, n)}$ – J – closed, $c_{\lambda n}$ (A) \ A has no non empty λ_m– J- closed subset. Therefore $c_{\lambda n}$ (B) \ B has no non empty λ_m - J- closed subset.

Remark 3.18.(i) The intersection of two $\lambda_{(m, n)}$ – J- open sets need not be $\lambda_{(m, n)}$ – J- open

(ii) The union of two $\lambda_{(m, n)}$ – J- open sets need not be $\lambda_{(m, n)}$ – J – open.

Proposition 3.19. A subset of a bigeneralized topological space (X, λ_1, λ_2) is $\lambda_{(m, n)}$ - J - open iff every subset of F of X, F $\subseteq i_{\lambda n}$ (A) whenever F is λ_m – J- closed and F $\subseteq$ A, where m, n = 1, 2 and m $\neq$ n.

Proof: Let A be a $\lambda_{m, n}$ – J- open. Let F $\subseteq$ A and F is λ_m – J-closed. Then X – A $\subseteq$ X – F and X – F is λ_m – J- open, we have X – A is $\lambda_{(m, n)}$ – J– closed, then $c_{\lambda n}$(X - A) $\subseteq$ X - F. Thus X - $i_{\lambda n}$ (A) $\subseteq$ X – F and hence F $\subseteq i_{\lambda n}$ (A).

Conversely, let F $\subseteq i_{\lambda n}$ (A) where F is a λ_m – J- closed set such that F $\subseteq$ A. Let X - A $\subseteq$ U where U is a λ_m– J- open. Then X - U $\subseteq$ A and X - U is λ_m – J- closed. By the assumption, X - U $\subseteq i_{\lambda n}$ (A), then X - $i_{\lambda n}$ (A) $\subseteq$ U. Therefore, $c_{\lambda n}$(X - A) $\subseteq$ U. Hence X - A is $\lambda_{(m, n)}$ - J - closed and hence A is $\lambda_{(m, n)}$ -J - open.

Proposition 3.20. Let A and B be subsets of a bigeneralized topological space(X, λ_1, λ_2) such that $i_{\lambda n}$ (A) $\subseteq$ B $\subseteq$ A. If A is $\lambda_{(m, n)}$ - J - open then B is also $\lambda_{(m, n)}$ - J- open, where m, n = 1, 2 and m $\neq$ n.

Proof: Suppose that $i_{\lambda n}$ (A) $\subseteq$ B $\subseteq$ A. Let F be a λ_m – J- closed set such that F $\subseteq$ B. Since A is $\lambda_{(m, n)}$ - J - open, F $\subseteq i_{\lambda n}$. Since $i_{\lambda n} \subseteq$ B, we have $i_{\lambda n}$ ($i_{\lambda n}$(A)) $\subseteq i_{\lambda n}$(B). Consequently $i_{\lambda n}$(A) $\subseteq i_{\lambda n}$(B). Hence F $\subseteq i_{\lambda n}$(B). Therefore, B is $\lambda_{(m, n)}$ - J -open.

Proposition 3.21. If A be a subset of a bigeneralized topological space (X, λ_1, λ_2) is $\lambda_{(m ,n)}$ - J- closed , then $c_{\lambda n}$(A) - A is $\lambda_{m, n}$ – J – open, where m, n = 1, 2 and m $\neq$ n.

Proof: Suppose that A is $\lambda_{m, n}$ – J – closed. Let X – ($c_{\lambda n}$(A) - A) $\subseteq$ U and U is λ_m– J- open. Then X – U $\subseteq$ X – ($c_{\lambda n}$(A) - A) and X – U is λ_m– J- closed. Thus we have $c_{\lambda n}$(A) - A does not contain non – empty λ_m– J- closed by Proposition 3.10 . Consequently, X – U = ϕ, then U = X. Therefore, $c_{\lambda n}$(X- ($c_{\lambda n}$(A) – A)) $\subseteq$ U, so we obtain X – ($c_{\lambda n}$(A) – A) is $\lambda_{m, n}$ – J- closed. Hence $c_{\lambda n}$(A) - A is $\lambda_{m, n}$ - J - open.

4. $Jg_{(m, n)}$- continuous functions

Definition 4.1. Let (X,λ_X^1, λ_X^2) and (Y,λ_Y^1, λ_Y^2) be generalized topological spaces. A function f: (X,λ_X^1, λ_X^2) $\rightarrow$ (Y,λ_Y^1, λ_Y^2) is said to be (m, n) - J- generalized continuous (briefly $Jg_{(m, n)}$- continuous) if f^{-1}(F) is $\lambda_{(m, n)}$- J-closed in X for every λ_n- closed F of Y, where m, n = 1,2 and m$\neq$ n.

A function f: (X,λ_X^1, λ_X^2) $\rightarrow$ (Y,λ_Y^1, λ_Y^2) is said to be pairwise J-generalized continuous(briefly pairwise Jg-continuous) if f is $Jg_{(1,2)}$-continuous and $Jg_{(2,1)}$-continuous.

Proposition 4.2. For an injective function f: (X,λ_X^1, λ_X^2) $\rightarrow$ (Y,λ_Y^1, λ_Y^2), the following properties are equivalent:

(1) f is $Jg_{(m, n)}$- continuous,
(2) For each $x \in X$ and for every λ_n- open set V containing f(x), there exists a $\lambda_{(m, n)}$-J-open set U containing x such that $f(U) \subseteq V$:
(3) $f(c_{\lambda_X^n}(A)) \subseteq c_{\lambda_Y^n}(f(A))$ for every subset A of X;
(4) $c_{\lambda_X^n}(f^{-1}(B)) \subseteq f^{-1}(c_{\lambda_Y^n}(B)$ for every subset B of Y.

Proof: (1) ⇒(2): Let $x \in X$ and V be a λ_n-open subset of Y containing f(x). Then by (1), $f^{-1}(V)$ is $\lambda_{(m, n)}$-J-open of X containing x. If $U = f^{-1}(V)$, then $f(U) \subseteq V$.

(2)⇒(3): Let A be a subset of X and $f(x) \notin (c_{\lambda_Y^n}(f(A))$. Then, there exists a λ_n – open subset V of Y containing f(x) such that $V \cap f(A) = \phi$.Then by(2), there exist a $\lambda_{(m, n)}$-J-open set such that $f(x) \in f(U) \subseteq V$. Hence, $f(U) \cap f(A) = \phi$ implies $U \cap A = \phi$. Consequently, $x \notin c_{\lambda_X^n}(A)$ and $f(x) \notin f(c_{\lambda_X^n}(A))$.

(3) ⇒(4): Let B be a subset of Y. By (3) we obtain $f(c_{\lambda_X^n}(f^{-1}(B)) \subseteq c_{\lambda_Y^n}(f(f^{-1}(B))$. Thus $c_{\lambda_X^n}(f^{-1}(B)) \subseteq f^{-1}(c_{\lambda_Y^n}(B))$.

(4) ⇒(1): Let F be a λ_n – closed subset of Y. Let U be a λ_m – J- open subset of X such that $f^{-1}(F) \subseteq U$. Since $c_{\lambda_Y^n}(F) = F$ and by (4), $c_{\lambda_X^n}(f^{-1}(F)) \subseteq U$. Hence f is $Jg_{(m, n)}$ continuous.

Definition 4.3. Let $(X,\lambda_X^1, \lambda_X^2) \to (Y,\lambda_Y^1, \lambda_Y^2)$ be generalized topological spaces. A function f: $(X,\lambda_X^1, \lambda_X^2) \to (Y,\lambda_Y^1, \lambda_Y^2)$ is said to be Jg_m- continuous if $f^{-1}(F)$ is λ_m –J-closed in X for every λ_m – closed F of Y, for m = 1,2.

Definition 4.4. Let $(X,\lambda_X^1, \lambda_X^2) \to (Y,\lambda_Y^1, \lambda_Y^2)$ be generalized topological spaces. A function f: $(X,\lambda_X^1, \lambda_X^2) \to (Y,\lambda_Y^1, \lambda_Y^2)$ is said to be Jg_m- closed(resp. Jg_m-open) if f(F) is λ_m – J-closed(resp λ_m –J- open) of Y for every λ_m – closed (resp. λ_m – open) F of X, for m = 1,2.

Proposition 4.5. If f: $(X,\lambda_X^1, \lambda_X^2) \to (Y,\lambda_Y^1, \lambda_Y^2)$ is Jg_m - continuous and Jg_m - closed , then f(A) is $\lambda_{(m, n)}$-J-closed subset of Y for every $\lambda_{(m, n)}$- J-closed subset A of X, where m, n = 1,2 and m≠ n.

Proof: Let U be λ_m – J- open subset of Y such that $f(A) \subseteq U$. Then $A \subseteq f^{-1}(U)$ and $f^{-1}(U)$ is λ_m – J-open subset of X. Since A is $\lambda_{(m, n)}$- J-closed, $c_{\lambda_X^n}(f(A)) \subseteq f^{-1}(U)$ and hence $f(c_{\lambda_X^n}(A)) \subseteq U$. Therefore we have $c_{\lambda_Y^n}(f(A)) \subseteq c_{\lambda_Y^n}(f(c_{\lambda_X^n}(A))) = f(c_{\lambda_X^n}(A)) \subseteq U$. Therefore, f(A) is $\lambda_{(m, n)}$- J-closed subset of Y.

Lemma 4.6. If f: $(X,\lambda_X^1, \lambda_X^2) \to (Y,\lambda_Y^1, \lambda_Y^2)$ is Jg_m – closed, then for each subset S of Y and each λ_m – J-open subset U of X containing $f^{-1}(S)$, there exists a λ_m – J- open subset V of Y such that $f^{-1}(V) \subseteq U$.

Proposition 4.7. If f: $(X,\lambda_X^1, \lambda_X^2) \to (Y,\lambda_Y^1, \lambda_Y^2)$ is injective, Jg_m-closed and $Jg_{(m ,n)}$-continuous, then $f^{-1}(B)$ is $\lambda_{(m, n)}$- J-closed subset of X for every $\lambda_{(m, n)}$- J-closed subset of B of Y, where m, n = 1,2 and m≠ n.

Proof: Let B be a $\lambda_{(m, n)}$- J-closed subset of Y. Let U be a λ_m – J- open subset of X such that $f^{-1}(B) \subseteq U$. Since f is Jg_m-closed and by lemma 4.6, there exists a λ_m – J- open subset of Y such that $B \subseteq V$ and $f^{-1}(V) \subseteq U$. Since B is $\lambda_{(m, n)}$- J-closed set and $B \subseteq V$, then $c_{\lambda_Y^n}(B) \subseteq V$. Consequently, $f^{-1}(c_{\lambda_Y^n}(B)) \subseteq f^{-1}(V) \subseteq U$. By theorem 4.2, $c_{\lambda_X^n}(f^{-1}(B)) \subseteq f^{-1}(c_{\lambda_Y^n}(B) \subseteq U$ and hence $f^{-1}(B)$ is $\lambda_{(m, n)}$- J-closed subset of X.

Definition 4.8. A bigeneralized topological space $(X,\lambda_X^1, \lambda_X^2)$ is said to be $\lambda_{(m, n)}$ – $JT_{1/2}$ – space if, for every $\lambda_{(m, n)}$- J-closed set is λ_n – closed, where m, n = 1,2 and m≠ n.

Definition 4.9. A bigeneralized topological space $(X,\lambda_X^1, \lambda_X^2)$ is said to be pairwise λ – $JT_{1/2}$ – space if it is both $\lambda_{(1, 2)}$ –$JT_{1/2}$ – space and $\lambda_{(2, 1)}$ – $J_{1/2}$ – space.

Proposition 4.10. A bigeneralized topological space is a $\lambda_{(m, n)}$- $JT_{1/2}$-space if and only if {x} is λ_n - open or λ_m –J-closed for each $x \in X$, where m, n = 1,2 and m≠ n.

Proof: Suppose that {x} is not λ_m–J- closed. Then X – {x} is $\lambda_{(m, n)}$ -J-closed by proposition 3.9. Since X is $\lambda_{(1, 2)}$ -$T_{1/2}$ -space, X – {x} is λ_n -closed. Hence, {x} is λ_n - open.

Conversely, let F be a $\lambda_{(m, n)}$ - J-closed set. By assumption , {x} is λ_n - open or λ_m –J- closed for any $x \in c_{\lambda n}(F)$.Case (i) Suppose that {x}is λ_n - open. Since $\{x\} \cap F \neq \phi$, we have $x \in F$. Case (ii) Suppose that {x} is λ_m – J-closed. If $x \notin F$, then $\{x\} \subseteq c_{\lambda n}(F) – F$, which is a contradiction to proposition 3.9, Therefore, $x \in F$. Thus in both case, we conclude that F is λ_n - closed. Hence, $(X,\lambda_X^1, \lambda_X^2)$ is a $\lambda_{(m, n)}$ -$JT_{1/2}$ -space.

Definition 4.11. Let $(X,\lambda_X^1, \lambda_X^2)$ and $(Y,\lambda_Y^1, \lambda_Y^2)$ be generalized topological spaces. A function f: $(X,\lambda_X^1, \lambda_X^2) \to (Y,\lambda_Y^1, \lambda_Y^2)$ is said to be $Jg_{(m, n)}$ - irresolute if $f^{-1}(F)$ is

$\lambda_{(m, n)}$-J- closed in X for every $\lambda_{(m, n)}$ –J-closed F of Y, where m, n = 1,2 and m≠ n.

Proposition 4.12. Let f: $(X,\lambda_X^1, \lambda_X^2) \rightarrow (Y,\lambda_Y^1, \lambda_Y^2)$ and g: $(Y,\lambda_Y^1, \lambda_Y^2) \rightarrow (Z,\lambda_Z^1, \lambda_Z^2)$ be functions, the following properties hold:

(i) If f is $Jg_{(m, n)}$ - irresolute and $Jg_{(m, n)}$ - continuous, then g ∘ f is $Jg_{(m, n)}$ - continuous.

(ii) If f and g are $Jg_{(m, n)}$ - irresolute , then g ∘ f is $Jg_{(m, n)}$ - irresolute;

(iii) Let $(Y,\lambda_Y^1, \lambda_Y^2)$ be a $\lambda_{(m, n)}$ -$JT_{1/2}$ - space. If f and g are $Jg_{(m, n)}$ -continuous, then g ∘ f is $Jg_{(m, n)}$ -continuous.

Proof: (i) Let F be a λ_n -closed subset of Z. Since g is $Jg_{(m, n)}$ - continuous, then $g^{-1}(F)$ is $\lambda_{(m, n)}$ -J-closed subset of Y. Since f is $Jg_{(m, n)}$ - irresolute, then $(g \circ f)^{-1}(F) = f^{-1}(g^{-1}(F))$ is $\lambda_{(m, n)}$ -J-closed subset of X. Hence g ∘ f is $Jg_{(m, n)}$ -continuous.

(ii) Let F be a $\lambda_{(m, n)}$ -J- closed subset of Z. Since g is $Jg_{(m, n)}$ - irresolute, then $g^{-1}(F)$ is $\lambda_{(m, n)}$ -J-closed subset of Y. Since f is $Jg_{(m, n)}$ - irresolute, then $(g \circ f)^{-1}(F) = f^{-1}(g^{-1}(F))$ is $\lambda_{(m, n)}$ –J- closed subset of X. Hence, g ∘ f is $Jg_{(m, n)}$ - irresolute.

(iii) Let F be a λ_n -closed subset of Z. Since g is $Jg_{(m, n)}$ - continuous, then $g^{-1}(F)$ is $\lambda_{(m, n)}$ -J- closed subset of Y. Since $(Y,\lambda_Y^1, \lambda_Y^2)$ is a $\lambda_{(m, n)}$ -$JT_{1/2}$ -space, then $g^{-1}(F)$ is λ_n -closed subset of Y. Since f is $Jg_{(m, n)}$ -continuous, then $(g \circ f)^{-1}(F) = f^{-1}(g^{-1}(F))$ is $\lambda_{(m, n)}$ –J-closed subset of X. Hence then g ∘ f is $Jg_{(m, n)}$ -continuous.

Proposition 4.13. Let: $(X,\lambda_X^1, \lambda_X^2)$ be a $\lambda_{(m, n)}$ -$JT_{1/2}$ - space. If f: $(X,\lambda_X^1, \lambda_X^2) \rightarrow (Y,\lambda_Y^1, \lambda_Y^2)$ is surjective, g_n - closed and $Jg_{(m, n)}$ -irresolute, then $(Y,\lambda_Y^1, \lambda_Y^2)$ is a $\lambda_{(m, n)}$ -$JT_{1/2}$ -space, where m, n = 1,2 and m≠ n.

Proof: Let F be a $\lambda_{(m, n)}$ -J-closed subset of Y. Since f is $Jg_{(m, n)}$ - irresolute, we have $f^{-1}(F)$ is a $\lambda_{(m, n)}$ -J-closed subset of X. Since $(X,\lambda_X^1, \lambda_X^2)$ is a $\lambda_{(m, n)}$ -$JT_{1/2}$ -space, $f^{-1}(F)$ is a λ_n -closed subset of X. It follows by assumption that F is a λ_n -closed subset of Y. Hence $(Y,\lambda_Y^1, \lambda_Y^2)$ is a $\lambda_{(m, n)}$ -$JT_{1/2}$ space.

Proposition 4.14. Let: $(X,\lambda_X^1, \lambda_X^2)$ be a $\lambda_{(m, n)}$ - $JT_{1/2}$ space. If f: $(X,\lambda_X^1, \lambda_X^2) \rightarrow (Y,\lambda_Y^1, \lambda_Y^2)$ is bijective, g_n - open and $Jg_{(m, n)}$ – irresolute, then $(Y,\lambda_Y^1, \lambda_Y^2)$ is a $\lambda_{(m, n)}$ -$JT_{1/2}$space, where m, n = 1,2 and m≠ n.

5. Strongly-$\lambda_{(m, n)}$ - J - closed sets

Definition 5.1. A subset A of a bigeneralized topological space $(X, \lambda_1, \lambda_2)$ is said to be *strongly $\lambda_{(m, n)} – J^s$ - closed set* (briefly *$\lambda_{(m, n)} – J^s$ - closed*) if $c_{\lambda n}(A) \subseteq U$, whenever $A \subseteq U$ and U is $\lambda_{(m, n)}$ – J - open, where m, n = 1,2 and m≠n.

Definition 5.2. A subset A of a bigeneralized topological space $(X, \lambda_1, \lambda_2)$ is said to be pairwise strongly *$\lambda_{(m, n)}$ – P- J^s* closed (briefly *$\lambda_{(m, n)}$ – P- J^s* closed)if A is $\lambda_{(1,2)}$ -J^s – closed and $\lambda_{(2,1)}$ - J^s – closed..

Proposition 5.3. Every λ_n - closed set is $\lambda_{(m, n)}$ – J^s – closed.

Proof: Let U be a $\lambda_{(m, n)}$ – J – open set such that $A \subseteq U$ and A be λ_n - closed set. Then $c_{\lambda n}(A) = A$ and so $c_{\lambda n}(A) \subseteq U$. Thus A is $\lambda_{(m, n)}$ – J^s – closed.

Proposition 5.4. Let $(X, \lambda_1, \lambda_2)$ be a bigeneralized topological space. Let $A \subseteq X$ be a $\lambda_{(m, n)}$ – J^s - closed if and only if , then $c_{\lambda n}(A) \setminus A$ does not contain any non-empty $\lambda_{(m,n)}$ - J- closed set, where m, n = 1, 2 and m ≠ n.

Proof: Let F be a $\lambda_{(m, n)}$ – J - closed subset of $c_{\lambda n}(A) \setminus A$. Now, $F \subseteq c_{\lambda n}(A) \setminus A$ and $A \subseteq X \setminus F$ where A is $\lambda_{(m, n)}$ – J^s – closed and $X \setminus F$ is $\lambda_{(m, n)}$ – J – open. Thus $c_{\lambda n}(A) \subseteq X \setminus F$ and hence $F \subseteq X \setminus c_{\lambda n}(A)$ .So, $F \subseteq c_{\lambda n}(A) \cap (X \setminus c_{\lambda n}(A)) = \phi$.Therefore $F = \varphi$.

Conversely, assume $c_{\lambda n}(A) \setminus A$ contains no non- empty $\lambda_{(m,n)}$ - J- closed set. Let U be $\lambda_{(m,n)}$ - J- open such that $A \subseteq U$. Suppose that $c_{\lambda n}(A)$ is not contained in U, then $c_{\lambda n}(A) \cap U^c$ is a nonempty $\lambda_{(m, n)}$ - J- closed set of $c_{\lambda n}(A) \setminus A$, which is a contradiction. Therefore $c_{\lambda n}(A) \subseteq U$ and hence A is $\lambda_{(m, n)}$ – J^s – closed.

Proposition 5.5. Let $(X, \lambda_1, \lambda_2)$ be a bigeneralized topological space. Let $A \subseteq X$ be a $\lambda_{(m, n)}$ – J^s - closed and $A \subseteq B \subseteq c_{\lambda n}(A)$ then B is $\lambda_{(m, n)}$ – J^s - closed set, where m, n = 1, 2 and m ≠ n.

Proof: Let A be a $\lambda_{(m, n)}$ – J^s- closed set and $A \subseteq B \subseteq c_{\lambda n}(A)$. Let $B \subseteq U$ and U is $\lambda_{(m, n)}$ – J - open. Then $A \subseteq U$. Since A is $\lambda_{(m, n)}$ – J^s – closed, we have $c_{\lambda n}(A) \subseteq U$. Since $B \subseteq c_{\lambda n}(A)$, then $c_{\lambda n}(B) \subseteq c_{\lambda n}(A) \subseteq U$. Hence B is $\lambda_{(m, n)}$ – J^s – closed.

Definition 5.6. A subset A of a bigeneralized space X is called strongly $\lambda_{(m, n)}$ – J^s – open if A^c is $\lambda_{(m, n)}$ – J^s – closed.

Proposition 5.7. Let $(X, \lambda_1, \lambda_2)$ be a bigeneralized topological space. Let $A \subseteq X$ be a $\lambda_{(m, n)}$ – J^s – open if and only if $F \subseteq i_{\lambda n}(A)$ whenever $F \subseteq A$ and F is $\lambda_{(m, n)}$ – J – closed.

Proof: Let A be $\lambda_{(m,\ n)}$ – J^s- open and suppose $F \subseteq A$ where F is a $\lambda_{(m,\ n)}$ – J – closed. Then X \ A is $\lambda_{(m,\ n)}$ – J^s – closed and X \ A ⊆ X \ F, where X \ F is $\lambda_{(m,\ n)}$ – J – open set. This implies that $c_{\lambda n}$(X \ A) ⊆ X \ F. Now $c_{\lambda n}$(X \ A) = X \ $i_{\lambda n}$(A). Hence X \ $i_{\lambda n}$(A) ⊆ X \ F and F ⊆ $i_{\lambda n}$(A). Conversely. If F is an $\lambda_{(m,\ n)}$ – J – closed with F ⊆ $i_{\lambda n}$(A) whenever F ⊆ A. Then X \ A ⊆ X \F and X \ $i_{\lambda n}$(A) ⊆ X \ F. Thus $c_{\lambda n}$(X \ A) ⊆ X \ F. Hence X \ A is $\lambda_{(m,\ n)}$ – J^s- closed and A is $\lambda_{(m,\ n)}$ – J^s- open.

Proposition 5.8. Let (X, λ_1, λ_2) be a bigeneralized topological space. For each x ∈ X.{x} is $\lambda_{(m,\ n)}$ – J – closed or $\{x\}^c$ is $\lambda_{(m,\ n)}$ – J^s- closed.
Proof: If {x} is not $\lambda_{(m,\ n)}$ – J – closed, then the only $\lambda_{(m,\ n)}$ – J – open containing $\{x\}^c$ is X. Thus $c_{\lambda n}$ ($\{x\}^c$) ⊆ X and $\{x\}^c$ is $\lambda_{(m,\ n)}$ – J^s- closed.

6. On Strongly - $\lambda_{(m,\ n)}$ - J - continuous and irresolute functions

Definition 6.1. Let (X,λ_X^1, λ_X^2) and (Y,λ_Y^1, λ_Y^2) be generalized topological spaces. A function f: (X,λ_X^1, λ_X^2) →(Y,λ_Y^1, λ_Y^2) is said to be $\lambda_{(m,\ n)}$ -strongly J - continuous(briefly $\lambda_{(m,\ n)}$-J^s- continuous) if f^{-1}(F) is $\lambda_{(m,\ n)}$-J^s-closed in X for every λ_n- closed F of Y, where m, n = 1,2 and m ≠ n.

Definition 6.2. Let (X,λ_X^1, λ_X^2) and (Y,λ_Y^1, λ_Y^2) be generalized topological spaces. A function f: (X,λ_X^1, λ_X^2) →(Y,λ_Y^1, λ_Y^2) is said to be $\lambda_{(m,\ n)}$ -strongly J – irresolute (briefly $\lambda_{(m,\ n)}$-J^s- irresolute) if f^{-1}(F) is $\lambda_{(m,\ n)}$- J^s-closed in X for every $\lambda_{(m,\ n)}$- J^s-closed F of Y, where m, n = 1,2 and m ≠ n.

Proposition 6.3. Let f: (X,λ_X^1, λ_X^2) → (Y,λ_Y^1, λ_Y^2), then, (1) ⇒(2): (2)⇒(3): (3) ⇒(4)

1. f is $\lambda_{(m,\ n)}$-J^s- continuous,
2. For each x ∈ X and for every λ_n- open set V containing f(x), there exists a $\lambda_{(m,\ n)}$-J^s-open set U containing x such that f(U) ⊆ V:
3. f ($\lambda_{(m,\ n)}$-J^s c_λ(A)) ⊆ $\lambda_{n\text{-}}$ c_λ(f(A)) for every subset A of X;
4. $\lambda_{(m,\ n)}$-J^s c_λ (f^{-1}(B)) ⊆ f^{-1}($\lambda_{n\text{-}}$ c_λ (B)) for every subset B of Y.

Proof: (1) ⇒(2): Let x ∈ X and V be a λ_n-open subset of Y containing f(x). Then by (1), f^{-1}(V) is $\lambda_{(m,\ n)}$-J^s-open of X containing x. If U = f^{-1}(V), then f (U) ⊆ V.
(2)⇒(3): Let A be a subset of X and f(x) ∉($\lambda_{n\text{-}}$ c_λ (f(A)). Then, there exists a λ_n – open subset V of Y containing f(x) such that V ∩ f(A) = ϕ.Then by(2), there exist a $\lambda_{(m,\ n)}$-J^s-open set such that f(x) ∈ f(U) ⊆ V. Hence, f(U) ∩ f(A) = ϕ and U ∩ A = ϕ. Consequently, x ∉ $\lambda_{(m,\ n)}$-J^s c_λ(A) and f(x) ∉ f ($\lambda_{(m,\ n)}$-J^s c_λ(A)).
(3) ⇒(4): Let B be a subset of Y and A = f^{-1}(B). By (3) we obtain f($\lambda_{(m,\ n)}$ - J^s c_λ(f^{-1}(B))) ⊆ λ_n - c_λ (f(f^{-1}(B))) ⊆ λ_n - c_λ (B).Thus $\lambda_{(m,\ n)}$-J^s c_λ (f^{-1}(B)) ⊆ f^{-1}($\lambda_{n\text{-}}$ c_λ (B)).

Proposition 6.4. If f: (X,λ_X^1, λ_X^2) → (Y,λ_Y^1, λ_Y^2) and g: (Y,λ_Y^1, λ_Y^2) → (Z,λ_Z^1, λ_Z^2) be two functions, then the following will hold

(i) If g is λ_n – continuous and f is $\lambda_{(m,\ n)}$-J^s – continuous, then g°f is $\lambda_{(m,\ n)}$-J^s – continuous.
(ii) If g is $\lambda_{(m,\ n)}$ - J^s –irresolute and f is $\lambda_{(m,\ n)}$-J^s – irresolute, then g°f is $\lambda_{(m,\ n)}$-J^s –irresolute.
(iii) If g is $\lambda_{(m,\ n)}$ - J^s –continuous and f is $\lambda_{(m,\ n)}$-J^s–irresolute, then g°f is $\lambda_{(m,\ n)}$-J^s – continuous.

Proof: obvious

References:

[1] H.S.Al-Saadi and A.H.Zakari,On Some separation axioms and Strongly Generalized closed sets in Bitopological spaces, Int.Mat.Forum,3,(2008),1039-1054.
[2] C. Boonpok, Weakly open functions on bigeneralized topological spaces, Int. J. of Math. Analysis, 15 (5)(2010). 891 – 897.
[3] A. Csaszar, Generalized open sets, Acta Math. Hungar., 75(1-2)(1997), 65-87.
[4] A. Csaszar, Generalized topology, generalized continuity, Acta Math. Hungar., 96(2002), 351- 357.
[5] A. Csaszar, Generalized open sets in generalized topologies, Acta Math. Hungar., 106(2005), 53- 66.
[6] A. Csaszar, Modifications of generalized topologies via hereditary spaces, Acta Math. Hungar., 115(2007), 29- 36.
[7] T. Fukutake, On generalized closed sets in bitopological spaces, Bull. Fukoka Univ. Ed. Part III, 35(1985), 19-28.
[8] J.C.Kelly, Bitopological Spaces, Pro, London Math, Soc., 3(13) (1969), 71-79.
[9] N.Levine: Generalized closed sets in topology, Rend, Circ.Mat. Palermo ,19 (1970) ,89-96 .
[10] D.Sasikala and I.Arockiarani, λ_α- J- Closed sets in Generalized Topological Spaces, IJST,. 1 (2)., (2011), 200-210.
[11] W.K.Min, Some results on Generalized topological spaces, Acta Math. Hungar., 108(1-2) (2005), 171-181.
[12] W.K.Min, Weak continuity on Generalized topological spaces, Acta Math. Hungar., 124 (1-2)(2009), 73-81.
[13] W.K.Min, Almost continuity on generalized topological spaces, Acta Math. Hungar., 125 (2009), 121-125.