Squeeze Film Lubrication between Parallel Stepped Plates with Couplestress Fluids

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Research Article

Abstract: In this paper, a theoretical analysis of the effects of couple stresses on the squeeze film lubrication between parallel stepped plates is presented. The modified Reynolds type equation is derived on the basis of Stokes microcontinuum theory of couplestress fluids. The closed form solution is obtained. According to the results obtained, the influence of couple stresses enhances the squeeze film pressure, load carrying capacity and decreases the response time as compared to the classical Newtonian-lubricant case. The load carrying capacity decreases as the step height increases.

Keywords: Squeeze film, Parallel stepped plates, couple stresses.

1 Introduction

The squeeze film lubrication phenomenon is widely observed in several applications such as gears, bearings, machine tools, rolling elements and automotive engines. The squeeze film action is also seen during approach of faces of disc clutches under lubricated condition. The squeeze film phenomenon arises when the two lubricating surfaces move towards each other in the normal direction and generates a positive pressure and hence supports a load. This phenomenon arises from the fact that a viscous lubricant present between the two surfaces cannot be instantaneously squeezed out when the two surfaces moving towards each other and this action provides a cushioning effect in bearings. The squeeze film lubrication between two infinitely long parallel plates is studied by Cameron (1981). The flow of an incompressible fluid between two parallel plates due to normal motion of the plates is investigated by Bujurke. *et al* (1995). The unsteady flow between two parallel discs with arbitrary varying gap width was studied by Ishizawa (1996). The squeeze film with Newtonian lubricants has been studied by several investigators (Jackson, 1963; Burbidge and Colin servais, 2004; Gupta, 1977). The Rayleigh step-bearings with non-Newtonian fluids has been studied by many researchers (Hughes, 1963; Bujurke *et al*., 1987; Maiti, 1973; Elkouh and Yang, 1991).

The use of different liquids as lubricants under different circumstances has gained its importance with a development of modern machines. In most of these lubricating oils the additives of high molecular weight polymers are present as a kind of viscosity index improvers. The presence of these additives in the lubricant prevents the viscosity variation of the lubricants with a change in temperature. The lubricants with additives causes the non-Newtonian behavior of the lubricating oils since the classical continuum mechanics of fluids neglects the size of fluid particles in the flow of fluids and hence several microcontinuum theories has been proposed to take into account of the intrinsic motion of material constituents(Ariman *et al.,* 1973,1974). The Stokes (1966) microcontinuum theory of couplestress fluid accounts for the polar effects such as the couple stresses, body couples and asymmetric tensor. During last few decades Stokes microcontinuum theory has been extensively used to study the effect of couple stresses on the performance of various bearing systems viz: the slider bearings (Bujurke *et al.,* 1990; Naduvinamani *et al.*,2003), Journal bearings (Mak and Conway, 1978; Guha, 2004), Squeeze film bearings (Lin, 1998; Naduvinamani *et al.*, 2001) and thrust bearings (Ramesh and Dubey, 1975). These studies predicted the advantages of couple stress fluid lubricants over the Newtonian lubricants such as the increased load carrying capacity, decreased coefficient of friction in the slider bearings and delayed time of approach in squeeze film bearings.

So far no attempt has been made to study the squeeze film characteristics with couplestress lubricants between flat stepped plates. Hence, in this chapter an attempt has been made to analyze the effect of couple stresses on these bearings.

2 Basic Equations

The constitutive equations for force and couple stresses proposed by Stokes are

$$T_{(ij)} = \left(-P + \lambda D_{kk}\right)\delta_{ij} + 2\mu D_{ij} \qquad (2.1)$$

$$T_{[ij]} = -2\eta W_{ij,kk} - \frac{\rho}{2}\varepsilon_{ijs} G_s \qquad (2.2)$$

and $$M_{ij} = 4\eta W_{j,i} + 4\eta' W_{i,j}, \qquad (2.3)$$

where

$$D_{ij} = \frac{1}{2}\left(V_{i,j} + V_{j,i}\right),$$

$$W_{ij} = \frac{1}{2}\left(V_{i,j} - V_{j,i}\right),$$

and $$w_i = \frac{1}{2}\varepsilon_{ijk} V_{k,j},$$

The dimensions of λ and μ are those of Newtonian viscosity and the dimensions of η and η' are those of momentum and the ratio $\left(\eta/\mu\right)$ has a dimension of length square. For the incompressible fluids when the body forces and body moments are absent the equations of motion derived by Stokes are

$$\rho\frac{D\vec{V}}{Dt} = -\nabla p + \mu\nabla^2\vec{V} - \eta\nabla^4\vec{V} \qquad (2.4)$$

$$\nabla\vec{V} = 0 \qquad (2.5)$$

3 Mathematical formulation of the problem

The squeeze film between parallel stepped plates approaching each other with a normal velocity V is shown in figure 1. The lubricant in the film region is considered to be an incompressible Stokes couple stress fluid. When body forces and body couples are absent, under the usual assumptions of hydrodynamic lubrication applicable to thin films the equations of motion (2.4) and (2.5) for the couple stress fluids in Cartesian coordinates take the form

$$\mu\frac{\partial^2 u}{\partial y^2} - \eta\frac{\partial^4 u}{\partial y^4} = \frac{\partial p}{\partial x}, \qquad (3.1)$$

$$\frac{\partial p}{\partial y} = 0, \qquad (3.2)$$

$$\frac{\partial u}{\partial x} + \frac{\partial u}{\partial y} = 0. \qquad (3.3)$$

The relevant boundary conditions are

(i) At the upper surface $y = h$

$$u = 0 \qquad \frac{\partial^2 u}{\partial y^2} = 0 \qquad (3.4a)$$

$$v = -V \qquad (3.4b)$$

(ii) At the bearing surface $y = 0$

$$u = 0 \qquad \frac{\partial^2 u}{\partial y^2} = 0 \qquad (3.5a)$$

$$v = 0. \qquad (3.5b)$$

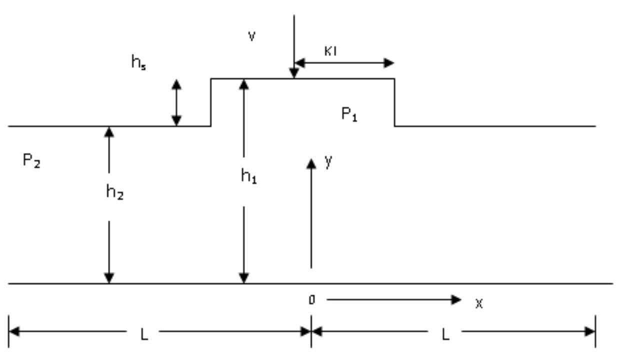

Fig. 1 Squeeze film between parallel stepped plates

4 Solution of the problem

Solution of equation (3.1) subject to the boundary conditions (3.4a) and (3.5a) is

$$u = \frac{1}{2\mu}\frac{dp}{dx}\left\{y^2 - yh + 2l^2\left[1 - \frac{\cosh\left(\frac{2y-h}{2l}\right)}{\cosh\left(\frac{h}{2l}\right)}\right]\right\} \qquad (4.1)$$

Where $l = \sqrt{\eta/\mu}$ is the couplestress parameter.

The volume flux of the lubricant is given by

$$Q = b\int_0^h u\, dy \qquad (4.2)$$

where b is the width of the bearing.

On using equation (4.1) in (4.2) gives

$$Q = -\frac{b}{12\mu}\frac{dp}{dx}\, s(h,l) \qquad (4.3)$$

Where

$$s(h,l) = h^3 - 12\, l^2 h + 24\, l^3 \tanh\left(\frac{h}{2l}\right)$$

Integration of the continuity equation (4.3) over the film thickness and the use of boundary conditions (3.4a) and (3.5a) gives

$$\frac{\partial Q}{\partial x} = bV. \qquad (4.4)$$

Integration of equation (4.4) w.r.t x and using the condition $Q = 0$ at x = 0 gives

$$Q = b\,V\,x. \qquad (4.5)$$

The modified Reynolds type equation for determining the pressure is obtained from equations (4.3) and (4.5) in the form

$$\frac{dp_i}{dx} = -\frac{12\,\mu\, V\, x}{g_i(h_i, l)} \qquad (4.6)$$

where

$h_i = h_1 \; for \quad 0 \le x \le KL$;

$\quad = h_2 \; for \quad KL \le x \le L.$

$$s_i(h_i, l) = h_i^3 - 12 l^2 h_i + 24 l^3 \tanh\left(\frac{h_i}{2l}\right) .$$

The relevant boundary conditions for the pressure are

$p_1 = p_2$ at $x = KL$, (4.7a)

$p_2 = 0$ at $x = L$ (4.7b)

Solution of equation (4.6) subject to the boundary conditions (4.7a) and (4.7b) is

$$p_1 = 6\,\mu V\left(\frac{K^2L^2 - x^2}{g_1(h_1, l)} + \frac{L^2(1-K^2)}{g_2(h_2, l)}\right) \qquad (4.8) \text{ and}$$

$$p_2 = \frac{6\,\mu\, V}{g_2(h_2, l)}(L^2 - x^2) \qquad (4.9)$$

The load carrying capacity, W is obtained in the form

$$W = 2\,b\int_0^{KL} p_1\, dx + 2\,b\int_{KL}^{L} p_2\, dx \qquad (4.10)$$

Which in nondimensional form

$$\bar{W} = \frac{W h_2^3}{8\,\mu b V L^3} = \left[\frac{K^3}{g_1(\bar{H}, \bar{l})} + \frac{1-K^3}{g_2(1, \bar{l})}\right] \qquad (4.11)$$

Where $\bar{H} = \frac{h_1}{h_2}$ and $\bar{l} = \frac{2l}{h_2}$

$$g_1(\bar{H}, \bar{l}) = \bar{H}^3 - 3\bar{l}^2\bar{H} + 3\bar{l}^3\tanh\left(\frac{\bar{H}}{\bar{l}}\right)$$

$$g_2(1, \bar{l}) = 1 - 3\,\bar{l}^2 + 3\,\bar{l}^3\tanh\left(\frac{1}{\bar{l}}\right)$$

Writing $V = -\frac{dh_2}{dt}$ in equation (4.11), the squeezing time for reducing the initial film thickness h_0 of h_2 to a final thickness h_f of h_2 is given by

$$t = -\frac{8\,\mu b L^3}{W}\int_{h_0}^{h_f}\left(\frac{K^3}{g_1(h_1, l)} + \frac{1-K^3}{g_2(h_2, l)}\right)dh_2 \qquad (4.12)$$

Which in nondimensional form

$$\bar{t} = \frac{W h_0^2\, t}{8\mu b L^3} = \int_{h_f^*}^{1}\{K^3\left[f_1(\bar{h}_s, \bar{h}_2, \tilde{l})\right] + f_2(\bar{h}_2, \bar{l})\}\; d\bar{h}_2 \qquad (4.13)$$

Where

$$g_1(\bar{h}_s, \bar{h}_2, \bar{l}) = \left[\bar{h}_2^3 + \bar{h}_s^3 + 3\bar{h}_2^2\bar{h}_s + 3\bar{h}_2\bar{h}_s^2 - 3\bar{l}^2\bar{h}_2\left(1 + \frac{\bar{h}_s}{\bar{h}_2}\right) + 3\bar{l}^3\tanh\left(\frac{\bar{h}_2\left(1 + \frac{\bar{h}_s}{\bar{h}_2}\right)}{\bar{l}}\right)\right]^{-1},$$

$$g_2(\bar{h}_2, \bar{l}) = \frac{1-K^3}{\bar{h}_2^3 - 3\bar{l}^2\bar{h}_2 + 3\bar{l}^3\tanh\left(\frac{\bar{h}_2}{\bar{l}}\right)},$$

$$\bar{h}_f = \frac{h_f}{h_0}, \quad \bar{h}_2 = \frac{h_2}{h_0}, \quad \bar{h}_s = \frac{h_s}{h_0}, \quad \bar{l} = \frac{2l}{h_0}.$$

5 Results and Discussion

This paper predicts the influence of couple stresses on the squeeze film characteristics of step bearings on the basis of Stokes couplestress fluid theory. The effect of couple stresses can be observed with the aid of nondimensional couplestress parameter $\bar{l} = \left(\frac{2l}{h_2}\right)$, where $l = \sqrt{\eta/\mu}$ which has the dimension of length and this length can be identified as the chain length of the polar additives in a non-polar lubricant. Hence the parameter, $\bar{l}$ provides the mechanism of the interaction of the lubricant with the bearing geometry.

5.1 Load carrying capacity

The variation of nondimensional load carrying capacity, $\bar{W}$ as a function of $\bar{H}$ for different values of couple stress parameter $\bar{l}$ with $K = 0.7$ is as shown in the Fig. 2. The dashed curve in the graph corresponds to the Newtonian case. Compared with the Newtonian lubricant case, the effect of couple stresses increase the load carrying capacity and this increase in $\bar{W}$ is more accentuated for larger values of $\bar{l}$. An increase of nearly 40% in $\bar{W}$ observed in the present study when $\bar{l} = 0.3$ and $K = 0.7$. Figure 3 depicts the variation of nondimensional load carrying capacity, $\bar{W}$ as a function of $\bar{H}$ for different values of K for both Newtonian lubricants $(\bar{l} = 0)$ and couplestress lubricants $(\bar{l} = 0.2)$. It is observed that, $\bar{W}$ increases for decreasing value of K and this increase in $\bar{W}$ is more pronounced for larger values of $\bar{H}$. The relative percentage increase in $\bar{W}$,

$$R_{\bar{W}}\left(= \left(\frac{\left(\bar{W}_{couplestress} - \bar{W}_{Newtonian}\right)}{\bar{W}_{Newtonian}}\right) X\,100\right)$$

for different values of K is given in the table 1

5.2 Time-height relationship

The most important characteristics of the squeeze film bearings is the squeeze film time i.e. the time required for reducing the initial film thickness h_2 of h_0 to a final value h_f. Figure 4 shows the variation of the nondimensional time of approach $\bar{t}$ as a function of $\bar{h}_f$ for different values of $\bar{l}$ with $K = 0.5, \bar{h}_s = 0.15$. It is observed that, the presence of couple stresses provides an increase in the response time as compared to the Newtonian lubricant case. The relative increase in $\bar{t}$, $R_{\bar{t}}$ $\left(= \left(\frac{\left(\bar{t}_{couplestress} - \bar{t}_{Newtonian} \right)}{\bar{t}_{Newtonian}} \right) X\, 100 \right)$ is given in the Table 1 for various values of $\bar{l}$ and K. It is found that, an increase of nearly 20% in $\bar{t}$ for $\bar{l} = 0.3$ and $K = 0.7$. The variation of $\bar{t}$ with $\bar{h}_f$ for different values of *K* for both Newtonian $(\bar{l} = 0)$ and couplestress lubricants $(\bar{l} = 0.2)$ is depicted in the Fig. 5. It is observed that, $\bar{t}$ increases for decreasing values of *K*. The variation of $\bar{t}$ with $\bar{h}_s$ for different values of the step height $\bar{h}_s$ with $K = 0.6$ is shown in the Fig. 6. It is observed that, $\bar{t}$ decreases for increasing values of nondimensional step height $\bar{h}_s$.

Table 1: Values of $R_{\bar{W}}$ and $R_{\bar{t}}$ for different values of $\bar{l}$ and *K* with $\bar{H} = 1.6$ $\bar{h}_f = 0.6$.

K	$\bar{l}$	$R_{\bar{W}}$	$R_{\bar{t}}$
0.5	0.1	2.719	5.008
	0.3	22.865	42.393
	0.5	62.24	116.205
0.7	0.1	2.58	4.753
	0.3	21.757	40.213
	0.5	59.209	110.176
0.8	0.1	2.435	4.513
	0.3	20.488	38.167
	0.5	55.727	104.519

6 Conclusions

The squeeze film lubrication between parallel stepped bearings with couplestress fluid as lubricant is studied on the basis of Stokes microcontinuum theory for couplestress fluids. On the basis of the numerical computations of the results presented the following conclusions are drawn

1. The effect of couple stresses enhances the load carrying capacity significantly.
2. The relative increase in the load carrying capacity $R_{\bar{W}}$ is found to be a function of *K*.
3. The relative squeeze film time $R_{\bar{t}}$ is found to be a function of $\bar{l}$ and *K* and $R_{\bar{t}}$ increases for increasing values of $\bar{l}$ and decreases for increasing values of *K*.

In view of the above the squeeze film characteristics between parallel stepped plates can be improved by the use of lubricants with microstructure additives.

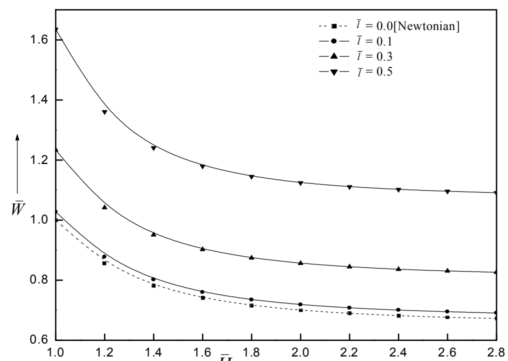

Fig.2 Variation of nondimensional load carrying capacity $\bar{W}$ with $\bar{H}$ with different values of $\bar{l}$ with K=0.7.

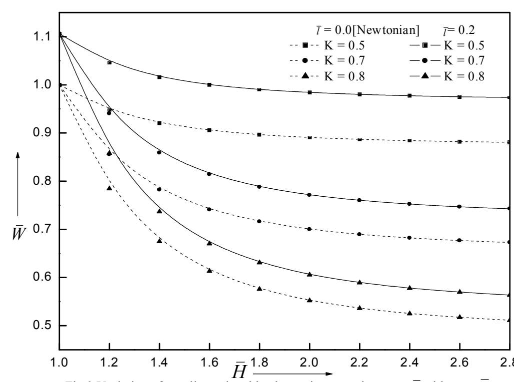

Fig.3 Variation of nondimensional load carrying capacity $\bar{W}$ with $\bar{H}$ for different values of K.

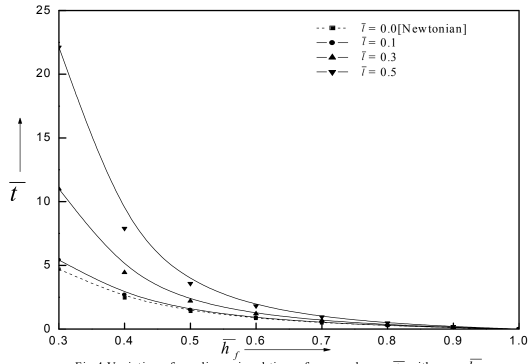

Fig.4 Variation of nondimensional time of approach $\overline{t}$ with $\overline{h}_f$ different values of l with K = 0.5, $\overline{h}_s$ = 0.15.

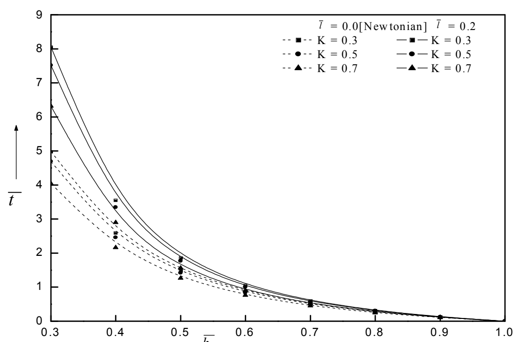

Fig.5 Variation of nondimensional time of approach $\overline{t}$ with $\overline{h}_f$ for different values of K with $\overline{h}_s$ = 0.15.

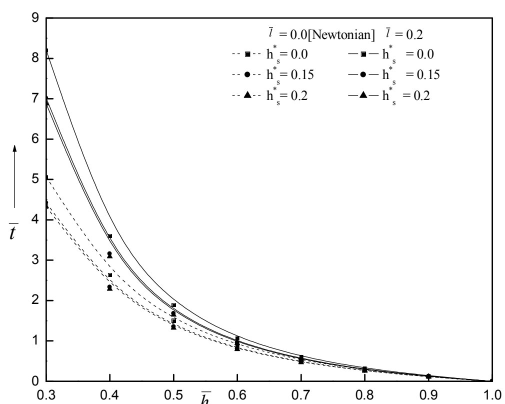

Fig.6 Variation of nondimensional time of approach $\overline{t}$ with $\overline{h}_f$ for different values of $\overline{h}_s$ with K=0.6.

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