International Journal of Statistika and Mathematika, ISSN: 2277- 2790 E-ISSN: 2249-8605, Volume 4, Issue 1, 2012 pp 01-05

Efficiency of Nearest Neighbour Balanced Block Designs for Correlated Observations (ARMA Models)

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Research Article

Abstract: Neighbour Balanced Designs, permitting the estimation of direct and neighbour effects, are used when the treatment applied to one experimental plot may affect the response on neighbouring plots as well as the response on the plot to which it is applied. The allocation of treatments in these designs is such that every treatment occurs equally often with every other treatment as neighbours. Neighbour Balanced Block Designs for Observations Correlated within a block have been investigated. The performance of a series of Complete Neighbour Balanced Design for Moving Average with First order (MA(1)) and Nearest Neighbour (NN) error correlation structure is studied when generalized least squares estimation is used by Senthil Kumar & Santharam (2012). In this paper, we have investigated that, the efficiency of Neighbour Balanced Block Designs for AutoRegressive Moving Average with First Order (ARMA(1,1)) and NN correlation structures. The performance of Nearest neighbour balanced block designs is satisfactory for ARMA(1,1) models. However, the efficiency of left and right neighbour effects is more as compared to direct effects of treatments under NN correlation structure.

Key Words: Neighbour balanced design; Correlated observations; Generalized least squares; AutoRegressive Moving Average; Nearest neighbour; Efficiency.

1. Introduction

Experiments in agriculture, horticulture and forestry often show neighbour effects; that is, the response on a given plot is affected by the treatments on neighbouring plots as well as by the treatment applied to that plot. When treatments are varieties, neighbour effects may be caused by differences in height, root vigour, or germination date, especially on small plots, such as are used in plant-breeding experiments. Treatments such as fertilizer, irrigation, or pesticide may spread to adjacent plots causing neighbour effects. Such experiments exhibit neighbour effects, because the effect of having no treatment as a neighbour is different from the neighbour effects of any treatment. In case of block design setup, if each block is a single line of plots and blocks are well separated, extra parameters are needed for the effect of left and right neighbours. An alternative is to have border plots on both ends of every block. Each border plot receives an experimental treatment, but it is not used for measuring the response variable. These border plots do not add too much to the cost of one-dimensional experiments. The estimates of treatment differences may therefore deviate because of *interference* from neighbouring units.

Neighbour balanced block designs, where in the allocation of treatments is such that every treatment occurs equally often with every other treatment as neighbours, are used for modeling and controlling interference effects between neighbouring plots. Azais *et al*. (1993) obtained a series of efficient neighbour designs with border plots that are balanced in $v-1$ blocks of size v and v blocks of size $v-1$, where v is the number of treatments. Druilhet (1999) studied optimality of circular neighbour balanced block designs obtained by Azais *et al*. (1993) Bailey (2003) has given some designs for studying one-sided neighbour effects. These neighbour balanced block designs have been developed under the assumption that the observations within a block are uncorrelated. In situations where the correlation structure among the observations within a block is known, may be from the data of past similar experiments, it may be advantageous to use this information in designing an experiment and analyzing the data so as to make more precise inference about treatment effects (Gill and Shukla, 1985). Kunert *et al*. (2003) considered two related models for interference and have shown that optimal designs for one model can be obtained from optimal designs for the other model. Martin and Eccelston (2004) have given variance balanced designs under interference and dependent observations. Tomar and Seema Jaggi (2007) observed that efficiency is quite high, in case of complete block designs for both AR(1) and NN correlation structures. Ruban, Santharam and Ramesh (2012) observed that the MV-optimality Nearest Neighbour Balanced Block Designs (NNBD) using AR(1), MA(1), ARMA(1,1), AR(2) and MA(2) models. Senthil kumar & Santharam

(2012) observed that the efficiency is high, in case of Complete Block Designs for both MA(1) and NN correlation structures. However, the efficiency of left and right neighbour effects is more as compared to direct effects of treatments under both the structures.

In this article, neighbour balanced block designs for observations correlated within a block have been investigated for the estimation of direct as well as left and right neighbour effects of treatments. The performance of these designs for AutoRegressive Moving Average with first order and nearest neighbour error correlation structure is studied when generalized least squares estimation is used. The efficiency of the designs as compared to a neighbour balanced complete block design of Azais *et al*.(1993) under correlated error structure is computed and tabulated for some cases.

2. Model and Definition

Let Δ be a class of binary neighbour balanced block designs with $n = bk$ units that form b blocks each containing k units. Y_{ij} be the response from the i^{th} plot in the j^{th} block $(i = 1,2,\ldots,k; j = 1,2,\ldots,b)$. The layout includes border plots at both ends of every block, i.e. at 0^{th} and $(k+1)^{th}$ position and observations for these units are not modeled. It is further assumed that the design is **circular**, that is the treatment on border plots is same as the treatment on the inner plot at the other end of the block. The following fixed effects additive model is considered for analyzing a neighbour balanced block design under correlated observations:

$$Y_{ij} = \mu + \tau_{(i,j)} + l_{(i-1,j)} + \gamma_{(i+1,j)} + \beta_j + e_{ij} \qquad (2.1)$$

where μ is the general mean, $\tau_{(i,j)}$ is the direct effect of the treatment in the i^{th} plot of j^{th} block, β_j is the effect of the j^{th} block. $l_{(i-1,j)}$ is the left neighbour effect due to the treatment in the $(i-1)^{th}$ plot of j^{th} block. $\gamma_{(i+1,j)}$ is the right neighbour effect due to the treatment in the $(i+1)^{th}$ plot in j^{th} block. e_{ij} are error terms distributed with mean zero and a variance-covariance structure $\Omega = I_b \otimes \Lambda$ (I_b is an identity matrix of order b and $\otimes$ denotes the kronecker product). Assuming no correlation among the observations between the blocks and correlation structure between plots within a block to be the same in each block, Λ is the correlation matrix of k observations within a block.

The error-in variable model (Besag, 1977) is closely related to the smooth trend plus error model of Wilkinson *et al*. (1983). This is a general model which gives a better fit in situations where the error structure is non stationary (Besag, 1977; Wilkinson *et al*. 1983; Patterson, 1983). Gill and Shukla, (1985) studied universal optimality of NNBD using AR(1) and MA(1) models. In the present paper, we have introduced an ARMA(1,1) model along with AR(1) and MA(1) and explored the performance of NNBD for $\rho = -0.4(-0.4)0.4$.

If the errors within a block follow an ARMA model (ARMA(1,1)) then $\Omega = I_b \otimes \Lambda$. Where I_b is an identity matrix of order b and

$$\Lambda = \begin{bmatrix} r_0 & r_1 & r_2 & \cdots & r_{k-1} \\ r_1 & r_0 & r_1 & \cdots & r_{k-2} \\ r_2 & r_1 & r_0 & \cdots & r_{k-3} \\ & & \vdots & & \\ r_{k-1} & r_{k-2} & r_{k-3} & \cdots & r_0 \end{bmatrix},$$

where $r_0 = \frac{1+2\rho_1\rho_2+\rho_2^2}{1-\rho_1^2}$, $r_1 = \frac{\rho_1(1+\rho_2^2)+\rho_2(1+\rho_1^2)}{1-\rho_1^2}$, $r_k = \rho_1^k(k-1)$ for $k \ge 2$ (Santharam and Ponnuswamy, 1997). The Nearest Neighbour (NN) correlation structure, Λ is a matrix with diagonal entries as 1 and off-diagonal entries as ρ.

Model (2.1) can be rewritten in the matrix notation as follows:

$$Y = \mu 1 + \Delta'\tau + \Delta_1' l + \Delta_2'\gamma + D'\beta + e \qquad (2.2)$$

Where Y is $n \times 1$ vector of observations, 1 is $n \times 1$ vector of ones, Δ' is an $n \times v$ incidence matrix of observations versus direct treatments, τ is $v \times 1$ vector of direct treatment effects, Δ_1' is a $n \times v$ matrix of observations versus left neighbour treatment, Δ_2' is a $n \times v$ matrix of observations versus right neighbour treatment, l is $v \times 1$ vector of left neighbour effects, γ is $v \times 1$ vector of right neighbour effects, D' is an $n \times b$ incidence matrix of observations versus blocks, β is $b \times 1$ vector of block effects and e is $n \times 1$ vector of errors.

The joint information matrix for estimating the direct and neighbour (left and right) effects under correlated observations estimated by generalized least squares is obtained as follows:

$$C = \begin{bmatrix} \Delta(I_b \otimes \wedge^*)\Delta' & \Delta(I_b \otimes \wedge^*)\Delta_1' & \Delta(I_b \otimes \wedge^*)\Delta_2' \\ \Delta_1(I_b \otimes \wedge^*)\Delta' & \Delta_1(I_b \otimes \wedge^*)\Delta_1' & \Delta_1(I_b \otimes \wedge^*)\Delta_2' \\ \Delta_2(I_b \otimes \wedge^*)\Delta' & \Delta_2(I_b \otimes \wedge^*)\Delta_1' & \Delta_2(I_b \otimes \wedge^*)\Delta_2' \end{bmatrix} \qquad (2.3)$$

with .

$$\wedge^* = \wedge^{-1} - \left(1_k^{'} \wedge^{-1} 1_k\right)^{-1} \wedge^{-1} 1_k 1_k^{'} \wedge^{-1}$$

The above $3v \times 3v$ information matrix (C) for estimating the direct effects and neighbour effects of treatments in a block design setting is symmetric, non-negative definite with row and column sums equal to zero.

The information matrix for estimating the direct effects of treatments from (2.3) is as follows:

$$C_\tau = C_{11} - C_{12} C_{22}^{-1} C_{21} \qquad (2.4)$$

where $C_{11} = \Delta(I_b \otimes \wedge^*)\Delta^{'}$

$C_{12} = \left[\Delta(I_b \otimes \wedge^*)\Delta_1^{'} \quad \Delta(I_b \otimes \wedge^*)\Delta_2^{'}\right]$ and

$$C_{22} = \begin{bmatrix} \Delta_1(I_b \otimes \wedge^*)\Delta_1^{'} & \Delta_1(I_b \otimes \wedge^*)\Delta_2^{'} \\ \Delta_2(I_b \otimes \wedge^*)\Delta_1^{'} & \Delta_2(I_b \otimes \wedge^*)\Delta_2^{'} \end{bmatrix}$$

Similarly, the information matrix for estimating the left neighbour effect of treatments (C_l) and right neighbour effect of treatments (C_γ) can be obtained. We have given some definitions associated with the neighbour balanced block designs under correlated observations.

Definition 2.1. A block design is *neighbour balanced* if every treatment has every treatment appearing as a neighbour (left and right) constant number of times (say, λ).

Definition 2.2. A neighbour balanced block design is called *pair-wise uniform* on the plots if each treatment $s(=1,2,\ldots,v)$ occurs equally often in each plot position $i(=1,2,\ldots,k)$ and each pair of treatments s and $s^{'}$, $s \neq s^{'}(=1,2,\ldots,v)$ occurs equally often (α times) within the same block in each unordered pair of plot positions i and $i^{'}$, $i \neq i^{'}(=1,2,\ldots,k)$.

Definition 2.3. A neighbour balanced block design with correlated observations permitting the estimation of direct and neighbour (left and right) effects, is called *variance balanced* if the variance of any estimated elementary contrast among the direct effects is constant, say V_1, the variance of any estimated elementary contrast among the left neighbour effect is constant, say V_2, and the variance of any estimated elementary contrast among the right neighbour effects is constant, say V_3. The constant V_1, V_2 and V_3 may not be equal. A block design is *totally balanced* if $V_1 = V_2 = V_3$.

3. Design

Tomer *et al.* (2005) has constructed neighbour balanced block design with parameters v (prime or prime power), $b = v(v-1)$, $r = (v-1)(v-m)$, $k = (v-m)$, $m = 1,2,\ldots,v-4$ and $\lambda = (v-m)$ using Mutually Orthogonal Latin Squares (MOLS) of order v. This series of design bas been investigated under the correlated error structure. It is seen that the design turns out to be pair-wise uniform with $\alpha = 1$ and also variance balanced for estimating direct (V_1) and neighbour effects $(V_2 = V_3)$.

Example 3.1.

Let $v = 5$ and $m = 0$. The following is a neighbour balanced pair-wise uniform complete block design with parameters $v = 5$, $b = 20$, $r = 20$, $k = 5$, $\lambda = 5$ and $\alpha = 1$:

2	3	4	5	1	2	3
3	4	5	1	2	3	4
4	5	1	2	3	4	5
5	1	2	3	4	5	1
1	2	3	4	5	1	2
3	4	5	1	2	3	4
4	5	1	2	3	4	5
5	1	2	3	4	5	1
1	2	3	4	5	1	2
2	3	4	5	1	2	3
4	5	1	2	3	4	5
5	1	2	3	4	5	1
1	2	3	4	5	1	2
2	3	4	5	1	2	3
3	4	5	1	2	3	4
5	1	2	3	4	5	1
1	2	3	4	5	1	2
2	3	4	5	1	2	3
3	4	5	1	2	3	4
4	5	1	2	3	4	5
1	2	3	4	5	1	2
2	3	4	5	1	2	3
3	4	5	1	2	3	4
4	5	1	2	3	4	5
5	1	2	3	4	5	1

The information matrices for estimating the direct and neighbour effects (left and right) of treatments for ARMA(1,1) structure with $\rho = 0.2$ is obtained as given below:

$C_\tau = 16.35390\left[I - \frac{J}{5}\right]$ and

$C_l = C_\gamma = 19.69317\left[I - \frac{J}{5}\right]$

Similarly for NN structure,

$C_\tau = 18.71826\left[I - \frac{J}{5}\right]$ and

$C_l = C_\gamma = 19.57275\left[I - \frac{J}{5}\right]$

These matrices have been worked out using **R** package.

Example 3.2.

Let $v = 6$ and $m = 0$. The following is a neighbour balanced pair-wise uniform complete block design with parameters $v = 6$, $b = 30$, $r = 30$, $k = 6$, $\lambda = 6$ and $\alpha = 1$:

4	5	6	1	2	3	4	5
5	6	1	2	3	4	5	6
6	1	2	3	4	5	6	1
1	2	3	4	5	6	1	2
2	3	4	5	6	1	2	3
3	4	5	6	1	2	3	4
5	6	1	2	3	4	5	6
6	1	2	3	4	5	6	1
1	2	3	4	5	6	1	2
2	3	4	5	6	1	2	3
3	4	5	6	1	2	3	4
4	5	6	1	2	3	4	5
6	1	2	3	4	5	6	1
1	2	3	4	5	6	1	2
2	3	4	5	6	1	2	3
3	4	5	6	1	2	3	4
4	5	6	1	2	3	4	5
5	6	1	2	3	4	5	6
1	2	3	4	5	6	1	2
2	3	4	5	6	1	2	3
3	4	5	6	1	2	3	4
4	5	6	1	2	3	4	5
5	6	1	2	3	4	5	6
6	1	2	3	4	5	6	1
2	3	4	5	6	1	2	3
3	4	5	6	1	2	3	4
4	5	6	1	2	3	4	5
5	6	1	2	3	4	5	6
6	1	2	3	4	5	6	1
1	2	3	4	5	6	1	2
3	4	5	6	1	2	3	4
4	5	6	1	2	3	4	5
5	6	1	2	3	4	5	6
6	1	2	3	4	5	6	1
1	2	3	4	5	6	1	2
2	3	4	5	6	1	2	3

The information matrices for estimating the direct and neighbour effects (left and right) of treatments for ARMA(1,1) structure with $\rho = -0.2$ was obtained as given below:

$C_\tau = 19.91324\left[I - \frac{J}{5}\right]$ and

$C_l = C_\gamma = 21.34191\left[I - \frac{J}{5}\right]$

Similarly for NN structure,

$C_\tau = 19.22633\left[I - \frac{J}{5}\right]$ and $C_l = C_\gamma = 20.24410\left[I - \frac{J}{5}\right]$

4. Comparison of Efficiency

In this section, a quantitative measure of efficiency of the designs in Section 3 has been made in comparison to the universally optimal neighbour balanced design for v treatments in $(v-1)$ complete blocks of Azais *et al.* (1993) considering observations to be correlated within the blocks. We compare the average variance of an elementary treatment contrast $\hat{\tau}_s - \hat{\tau}_{s'}$ in both cases. The average variance of an elementary treatment contrast (Kempthorne, 1956) for direct effects of the neighbour balanced design of Azais *et al.* (1993) estimated by generalized least squares methods, is given by

$$V_A = \frac{2\sigma^2}{v-1}\sum_{s=1}^{v-1}\theta_s^{-1}$$

Where θ_s's are the $(v-1)$ non-zero eigen values of C_τ for Azais *et al.* (1993), σ^2 is the variance of an observation. The efficiency factor (E_τ) for direct effects of the neighbour balanced pair-wise uniform block design is thus given as:

$$E_\tau = \frac{(v-1)\sum_{s=1}^{v-1}\theta_s^{-1}}{(v-m)\sum_{s=1}^{v-1}\delta_s^{-1}}$$

δ_s's are the $(v-1)$ non-zero eigen values of C_τ of the design given in section 3. Similarly the efficiency (E_l) and (E_γ) for neighbour effects (left and right) of treatments is obtained. The ranges of correlation coefficient (ρ) for different correlation structures investigated are $|\rho| \le 0.40$ for ARMA(1,1) and NN correlation structures. For these ranges, the matrix of correlation coefficients among observations within a block is positive definite. For $\rho = 0$, the efficiency is that of totally balanced designs obtained by Tomer *et al.* (2005).

5. Conclusion

From Table 1, we conclude that, the parameters of neighbour balanced pair-wise uniform complete block design as given in Section 3 for $v = 5$ and 6 $(m = 0)$ along with the efficiency for direct, left and right neighbour effects. The efficiencies have been reported under the ARMA(1,1) and NN correlation structures with ρ in the interval -0.4 to 0.4. The performance of Nearest neighbour balanced block designs is satisfactory for ARMA(1,1) models. When the correlation between the adjacent plots increases, the efficiency also increases, which clearly reveals that Nearest neighbour block designs is more efficient than ARMA(1,1) models. However, the efficiency of left and right neighbour effects is more as compared to direct effects of treatments under NN correlation structure. When block sizes are large and neighbouring plots are highly correlated, generalized least squares for estimation of direct and neighbour effects (left and right) can be used.

6. References

1. AZAIS, J.M., BAILEY, R. A., and MONOD, H. (1993) A catalogue of efficient neighbour designs with border plots, *Biometrics*, 49, 1252-1261.
2. BAILEY, R. A. (2003) Designs for one-sided neighbour effects, *J. Ind. Soc. Agric. Statistics*, 56(3), 302-314.
3. BESAG, J. 1977: Errors-in-variables estimation for Gaussian lattice schemes, *J. Roy. Statis. Soc.* B 39, 73-78.
4. CHENG, C. S. and WU, C. F. (1981) Nearly balanced incomplete block designs, *Biometrika*, 68, 493-500.
5. DRUILHET, P. (1999) Optimality of neighbour balanced designs, *J. Statist. Plan & Inference*, 81, 141-152.
6. GILL, P. S. and SHUKLA, G. K. (1985) Efficiency of nearest neighbour balanced block designs for correlated observations, *Biometrika*, 72, 539-544.
7. KEMPTHORNE, O. (1956) The efficiency factor of an incomplete block design, *Ann. Math. Statist.*, 27, 846-849.
8. KUNERT, J., MARTIN, R. J., and POOLADSAZ, S. (2003) Optimal designs under two related models for interference, *Metrika*, 57, 137-143.
9. KUNERT, J., 1987: Neighbour balanced block designs for correlated errors. *Biometrika* 74, 4, 717-724.
10. MARTIN, J. P. and CHAKRAVARTI, I. M., 1998: Block designs for first and second order neighbour correlations. *The Ann. of Statist*. 16, 1206-1224.
11. MARTIN, R. J. and ECCLESTON, J. A. (2004) Variance-balanced designs under interference for dependent observations, *J. Statist Plan & Inference*, 119, 207-223.
12. RUBAN RAJA. B, SANTHARAM. C and RAMESH KUMAR, (2012) MV-Optimality of Nearest Neighbour Balanced Block Designs using First order and Second order correlated models, *International journal of Statistika and Mathematika*, (ISSN: 2277-2790 E-ISSN: 2249-8605)
13. C.SANTHARAM & K.N.PONNUSWAMY, (1997) On the Efficiency of Nearest Neighbour Balanced Block Designs with Correlated Error Structure, *Biometrics*, J. 39, 85-98
14. SENTHIL KUMAR & SANTHARAM (2012), Efficiency of Neighbour Balanced Block Designs for Correlated Observations, *International Journal of Statistika and Mathematika*, ISSN: 2277-2790 E-ISSN: 2249-8605, Volume 3, Issue 3, 2012 pp 115-120.
15. TOMAR, J.S., JAGGI, S., and VARGHESE, C. (2005) On totally balanced block designs for competition effects, *Jour. Applied Statistics*, 32 (1), 87-97.
16. TOMAR, J.S., JAGGI, S. (2007) Efficient neighbour balanced block designs for correlated observations, *METRON – International Journal of Statistics*, Vol. LXV, n. 2, pp. 229-238.
17. WILKINSON,G. N., ECKERT, S. R., HANCOCK, T. W., and MAYO, O., 1983: Nearest Neighbour (NN) analysis of field experiments (with discussion). *J. Roy. Statist. Soc.* B 45,151-2

APPENDIX: 1

Table 1: Efficiency of Neighbour Balanced Pair-Wise Uniform Complete Block Designs.

Parameters					Correlation Structure							
					ARMA(1,1)				NN			
v	b	m	r	$k=\lambda$	$\rho=(\rho_1,\rho_2)$	E_τ	E_l	E_γ	ρ	E_τ	E_l	E_γ
5	20	0	20	5	(-0.4,-0.4)	0.32275	0.41998	0.34200	-0.4	0.56287	0.53710	0.56352
					(-0.3,-0.3)	0.44297	0.25651	0.27193	-0.3	0.62530	0.60242	0.58364
					(-0.2,-0.2)	0.58318	0.58497	0.55448	-0.2	0.66174	0.65598	0.66174
					(-0.1,-0.1)	0.69314	0.70919	0.71026	-0.1	0.71680	0.71779	0.72109
					(0,0)	0.80000	0.80000	0.80000	0	0.80000	0.80000	0.80000
					(0.1,0.1)	0.83073	0.82819	0.82177	0.1	0.89193	0.94829	0.90732
					(0.2,0.2)	0.78885	0.73788	0.75456	0.2	0.99359	0.99120	1.02672
					(0.3,0.3)	0.63023	0.34204	0.18718	0.3	1.13873	1.17027	1.15689
					(0.4,0.4)	0.58709	0.06418	0.30422	0.4	1.37226	1.36867	1.33303
6	30	0	30	6	(-0.4,-0.4)	0.36038	0.42080	0.62225	-0.4	0.58753	0.58692	0.58983
					(-0.3,-0.3)	0.49118	0.36038	0.45027	-0.3	0.63785	0.64068	0.62985
					(-0.2,-0.2)	0.62365	0.60793	0.61215	-0.2	0.68384	0.69532	0.65391
					(-0.1,-0.1)	0.83020	0.74801	0.74261	-0.1	0.75234	0.76089	0.77723
					(0,0)	0.80000	0.80000	0.80000	0	0.80000	0.80000	0.80000
					(0.1,0.1)	0.86001	0.84611	0.84876	0.1	0.93761	0.91002	0.93524
					(0.2,0.2)	0.80037	0.75366	0.75237	0.2	1.05008	1.04844	1.04667
					(0.3,0.3)	0.65451	0.70426	0.60153	0.3	1.20284	1.20351	1.20192
					(0.4,0.4)	0.56920	0.70866	0.43601	0.4	1.40943	1.40854	1.40744