International Journal of Statistika and Mathematika, ISSN: 2277- 2790 E-ISSN: 2249-8605, Volume 4, Issue 1, 2012 pp 10-13

Applications of the Bi-Lateral Laplace-Mellin Integral Transform in the Range [0,0] to (∞,∞)

Sarita Poonia[1*], Rachana Mathur [2]

[2]Department of Mathematics, Govt. Dungar College, Bikaner, Rajasthan, INDIA.

[1] Research Scholar, Govt. Dungar College, Bikaner, Rajasthan, INDIA.

[*]Corresponding Address:

sari.abhi14@gmail.com

Research Article

Abstract: In this paper we discuss Bi-Lateral Laplace-Mellin Integral Transform technique for solving initial value problem. This transform is studied in the range of [0,0] to (∞,∞). We investigate the properties and theorems like inversion theorem, convolution theorem, Parseval's theorem and some properties by using Ramanujan's formula. To illustrate the advantages and use of this transformation Cauchy's differential equation have been solved. We have also studied graphical representation of Bi-Lateral Laplace-Mellin Integral Transform using Matlab.

Keywords: Laplace Transform, Finite Mellin Transform, Integral Transform, Double Bi Lateral Laplace Transform, Convolution and Parseval's theorem

1.Introduction

The Double Bi Lateral Laplace Transform is used to find the Bi-Lateral Laplace-Mellin Integral Transform in the range [0,0] to (∞,∞).We have derived the different properties like Linear property, Scaling Property, Power Property. Inversion Theorem, Convolution Theorem, Parseval Theorem, First and Second Shifting theorems are also obtained by using Ramanujan's formula. We study n^{th} order derivative, the solution of the Cauchy's Linear differential equation using the Bi-Lateral Laplace-Mellin Integral Transform in the range [0,0] to (∞,∞) and the solution is graphically represented by using Matlab.

2.Preliminary Result

The Double Bi-Lateral Laplace Integral Transform in $(-\infty,-\infty)$ to (∞,∞) is

$$L_L[f(x,t),s,p] = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} f(x,t)e^{-sx-pt}\,\text{dx dt} \quad (1)$$

Whenever this double integral exists and s > 0, p > 0

Substitute $y = e^{-t}$, $dy = -e^{-t}dt$, $dy = -y\,dt$ and $z = e^{-x}$,$dz = -e^{-x}dx$, $dz = -z\,dx$

If $t = -\infty$ then $y = \infty$ and $t = \infty$ then $y = 0$; if $x = -\infty$ then $z = \infty$ and $x = \infty$ then $z = 0$ then

$$L_L[f(z,y),s,p] = \int_0^{\infty}\int_0^{\infty} f(z,y)z^{s-1}\,y^{p-1}\,\text{dz dy}$$

Or $L_L[f(x,y),s,p] = \int_0^{\infty}\int_0^{\infty} f(x,y)\,x^{s-1}y^{p-1}\text{dx dy}$ (2)

Provided this double integral exists and s > 0 , p > 0

This is the Bi-Lateral Laplace-Mellin Integral Transform (BLLMIT) in the range [0,0] to (∞,∞).

It is denoted by $L_m\,[f(x,y),s,p]$, then

$$L_m[f(x,y),s,p] = \int_0^{\infty}\int_0^{\infty} f(x,y)\,x^{s-1}y^{p-1}\text{dx dy} \quad (3)$$

3. Result and Discussion

3.1. Properties

3.1.1: Linearity Property

The BLLMIT is a linear operation theorem for the functions f(x,y) and g(x,y) and α and β are constant ,then the BLLMIT in [0,0] to (∞,∞).is

$$L_m[f(x,y),s,p] = \int_0^{\infty}\int_0^{\infty} f(x,y)\,x^{s-1}y^{p-1}\text{dx dy}$$

then $L_m[\alpha f(x,y) + \beta\,g(x,y),s,p] = \alpha L_m[f(x,y),s,p] + \beta L_m[f(x,y),s,p]$ (4)

3.1.2:Scaling Property

The scaling property for BLLMIT in [0,0] to (∞,∞).is

$$L_m[f(x,y),s,p] = \int_0^{\infty}\int_0^{\infty} f(x,y)\,x^{s-1}y^{p-1}\text{dx dy}$$

Then

$$L_m[f(cx,dy),s,p] = \frac{1}{c^s d^p}L_m[f(r,q),s,p] \quad (5)$$

3.1.3:Power Property

The power property for BLLMIT in [0,0] to (∞,∞).is

$$L_m[f(x,y),s,p] = \int_0^{\infty}\int_0^{\infty} f(x,y)\,x^{s-1}y^{p-1}\text{dx dy}$$

Then

$$L_m[f(x^n,y^m),s,p] = \frac{1}{mn}L_m[f(r,q),\frac{s}{n},\frac{p}{m}] \quad (6)$$

3.2.Main Results

3.2.1:Inversion Theorem

Assume that $L_m[f(t,z),s,p]$ is a regular function in the strips $|Re(s)| < r$ ('r' be real number) of the s-plane and $0 < c < v_1$, $c_1 - i\infty \leq s \leq c_1 + i\infty$ where c_1 is constant and $|Re(p)| < q$ ('q' be real number) of the p-plane and $0 < c < v_2$, $c_2 - i\infty \leq p \leq c_2 + i\infty$ where c_2 is constant. Then the BLLMIT in [0,0] to (∞,∞).is

$$L_m[f(x,y),s,p] = \int_0^{\infty}\int_0^{\infty} f(x,y)\,x^{s-1}y^{p-1}\text{dx dy}$$

then

$$f(x,y) = \frac{1}{(2\pi i)^2}\int_{c1-i\infty}^{c1+i\infty}\int_{c2-i\infty}^{c2+i\infty}\frac{1}{sp}e^{sx}y^{-p}l_m[f(x,y),s,p]\,ds\,\text{dp}$$

Proof

If $L[f(t), s, o, \infty] = \int_0^\infty t^{s-1} f(t)dt$, then its inverse is

$$f(t) = \frac{1}{2\pi i}\int_{c-i\infty}^{c+i\infty} e^{st}\ L[f(t), s, o, \infty]\text{ds},$$

and the Mellin Transform is

$$M[f(z), p, 0, \infty] = \int_0^\infty f(z) z^{p-1}\, dz,$$

Then its inverse is

$$f(z) = \frac{1}{2\pi i}\int_{c-i\infty}^{c+i\infty} z^{-p}\, M[f(z), p, 0, \infty]dp.$$

The BLLMIT in [0,0] to (∞,∞) is

$$L_m[f(x,y), s, p] = \int_0^\infty\int_0^\infty f(x,y)\, x^{s-1} y^{p-1}\text{dx dy}$$

then

$$= \int_0^\infty\int_0^\infty x^{s-1}y^{p-1}\left[\frac{1}{(2\pi i)^2}\int_{c1-i\infty}^{c1+i\infty}\int_{c2-i\infty}^{c2+i\infty} e^{sx}\, y^{-p} l_m[f(x,y), s, p]\, ds\ \text{dp}\right]\text{dx dy}$$

$$=\frac{1}{(2\pi i)^2}\int_{c1-i\infty}^{c1+i\infty}\int_{c2-i\infty}^{c2+i\infty} e^{sx}\, y^{-p} l_m[f(x,y), s, p]\, ds\ \text{dp}\left[\int_0^\infty\int_0^\infty x^{s-1}y^{p-1}\text{dx dy}\right]$$

$$=\frac{1}{(2\pi i)^2}\int_{c1-i\infty}^{c1+i\infty}\int_{c2-i\infty}^{c2+i\infty} e^{sx}\, y^{-p} l_m[f(x,y), s, p]\, ds\ \text{dp}\left[\int_0^\infty x^{s-1}dx\ \int_0^\infty y^{p-1}dy\right]$$

$$=\frac{1}{(2\pi i)^2}\int_{c1-i\infty}^{c1+i\infty}\int_{c2-i\infty}^{c2+i\infty} e^{sx}\, y^{-p} l_m[f(x,y), s, p]\, ds\ \text{dp}\left[\left[\frac{x^s}{s}\right]_0^\infty\left[\frac{y^p}{p}\right]_0^\infty\right]$$

$$=\frac{1}{(2\pi i)^2}\int_{c1-i\infty}^{c1+i\infty}\int_{c2-i\infty}^{c2+i\infty} e^{sx}\, y^{-p} l_m[f(x,y), s, p]\, ds\ \text{dp}\left[\frac{1}{s}\right]\left[\frac{1}{p}\right]$$

$$=\frac{1}{(2\pi i)^2}\int_{c1-i\infty}^{c1+i\infty}\int_{c2-i\infty}^{c2+i\infty}\frac{1}{sp} e^{sx} y^{-p} l_m[f(x,y), s, p]\, ds\ \text{dp}$$

By assuming $\lim_{x\to\infty} x^s=1$ and $\lim_{y\to\infty} y^p=1$

This is the Inverse of Laplace–Mellin Integral Transform.

It is denoted by $f(x,y) = L_m^{-1}[f(x,y), s, p]$

$$f(t,z)=\frac{1}{(2\pi i)^2}\int_{c1-i\infty}^{c1+i\infty}\int_{c2-i\infty}^{c2+i\infty}\frac{1}{sp} e^{st} z^{-p} l_m[f(t,z), s, p]\, ds\ \text{dp}$$

Or

$$f(x,y) = \frac{1}{(2\pi i)^2}\int_{c1-i\infty}^{c1+i\infty}\int_{c2-i\infty}^{c2+i\infty}\frac{1}{sp} e^{sx} y^{-p} l_m[f(x,y), s, p]\, ds\ \text{dp} \quad (7)$$

3.2.2.Convolution Theorem

The BLLMIT in [0,0] to (∞,∞) is

$$L_m[f(x,y), s, p] = \int_0^\infty\int_0^\infty f(x,y)\, x^{s-1} y^{p-1}\text{dx dy}$$

Then $L_m[f(x,y)g(t-x,y), s, p] =$

$$\frac{1}{(2\pi i)^2}\int_{c1-i\infty}^{c1+i\infty}\int_{c2-i\infty}^{c2+i\infty}\frac{1}{sp} e^{sx} y^{-p} l_m[f(x,y), s, p]\ l_m[g(t-x,y), s, p]\, ds\ \text{dp}$$

Proof

The BLLMIT in [0,0] to (∞,∞) .is

$$L_m[f(x,y), s, p] = \int_0^\infty\int_0^\infty f(x,y)\, x^{s-1} y^{p-1}\text{dx dy}$$

then

$$L_m[f(x,y)g(t-x,y), s, p] = \int_0^\infty\int_0^\infty f(x,y)g(t-x,y)\, x^{s-1} y^{p-1}\text{dx dy}$$

$$= \int_0^\infty\int_0^\infty g(t-x,y)\, x^{s-1}y^{p-1}\left[\frac{1}{(2\pi i)^2}\int_{c1-i\infty}^{c1+i\infty}\int_{c2-i\infty}^{c2+i\infty}\frac{1}{sp} e^{sx} y^{-p} l_m[f(x,y), s, p]\, ds\ \text{dp}\right]\text{dx dy}$$

$$=\frac{1}{(2\pi i)^2}\int_{c1-i\infty}^{c1+i\infty}\int_{c2-i\infty}^{c2+i\infty}\frac{1}{sp} e^{sx} y^{-p} l_m[f(x,y), s, p]\, ds\ \text{dp}\left[\int_0^\infty\int_0^\infty g(t-x,y)\, x^{s-1}y^{p-1}dx\, dy\right]$$

$$=\frac{1}{(2\pi i)^2}\int_{c1-i\infty}^{c1+i\infty}\int_{c2-i\infty}^{c2+i\infty}\frac{1}{sp} e^{sx} y^{-p} l_m[f(x,y), s, p]\ l_m[g(t-x,y), s, p]\, ds\ \text{dp}$$

$$L_m[f(x,y)g(t-x,y), s, p] = \frac{1}{(2\pi i)^2}\int_{c1-i\infty}^{c1+i\infty}\int_{c2-i\infty}^{c2+i\infty}\frac{1}{sp} e^{sx} y^{-p} l_m[f(x,y), s, p]\ l_m[g(t-x,y), s, p]\, ds\ \text{dp} \quad (8)$$

3.2.3:Parsavals Theorem

The BLLMIT in [0,0] to (∞,∞) .is

$$L_m[f(x,y), s, p] = \int_0^\infty\int_0^\infty f(x,y)\, x^{s-1} y^{p-1}\text{dx dy}$$

Then

$$L_m[f(x,y)g(x,y), s, p] = \frac{1}{(2\pi i)^2}\int_{c1-i\infty}^{c1+i\infty}\int_{c2-i\infty}^{c2+i\infty}\frac{1}{sp} e^{sx} y^{-p} l_m[f(x,y), s, p]\ l_m[g(x,y), s, p]\, ds\ \text{dp}$$

Proof

The BLLMIT in [0,0] to (∞,∞) .is

$$L_m[f(x,y), s, p] = \int_0^\infty\int_0^\infty f(x,y)\, x^{s-1} y^{p-1}\text{dx dy}\quad \text{then}$$

$$L_m[f(x,y)g(x,y), s, p] = \int_0^\infty\int_0^\infty f(x,y)g(x,y)\, x^{s-1} y^{p-1}\text{dx dy}$$

$$= \int_0^\infty\int_0^\infty g(x,y)\, x^{s-1}y^{p-1}\left[\frac{1}{(2\pi i)^2}\int_{c1-i\infty}^{c1+i\infty}\int_{c2-i\infty}^{c2+i\infty}\frac{1}{sp} e^{sx} y^{-p} l_m[f(x,y), s, p]\, ds\ \text{dp}\right]\text{dx dy}$$

$$=\frac{1}{(2\pi i)^2}\int_{c1-i\infty}^{c1+i\infty}\int_{c2-i\infty}^{c2+i\infty}\frac{1}{sp} e^{sx} y^{-p} l_m[f(x,y), s, p]\, ds\ \text{dp}\left[\int_0^\infty\int_0^\infty g(x,y)\, x^{s-1}y^{p-1}dx\, dy\right]$$

$$= \frac{1}{(2\pi i)^2}\int_{c1-i\infty}^{c1+i\infty}\int_{c2-i\infty}^{c2+i\infty}\frac{1}{sp} e^{sx} y^{-p} l_m[f(x,y), s, p]\ l_m[g(x,y), s, p]\, ds\ \text{dp}$$

$$L_m[f(x,y)g(x,y), s, p] = \frac{1}{(2\pi i)^2}\int_{c1-i\infty}^{c1+i\infty}\int_{c2-i\infty}^{c2+i\infty}\frac{1}{sp} e^{sx} y^{-p} l_m[f(x,y), s, p]\ l_m[g(x,y), s, p]\, ds\ \text{dp} \quad (9)$$

3.2.4:Definitions

(a) Unit Step Function :-

If H(t) = U(t) = 1, when t > 0

= 0, when t < 0

Then H(t) = U(t) is known as the unit step function.

(b) Heviside Unit Step Function

If H(t-a) = U(t-a) = 1, when t > a

= 0, when t < a

Then H(t-a) = U(t-a) is known as the Heviside Unit Step Function.

3.2.5: First Shifting Theorem

The BLLMIT in [0,0] to (∞,∞) .is

$$L_m[f(x,y), s, p] = \int_0^\infty\int_0^\infty f(x,y)\, x^{s-1} y^{p-1}\text{dxdy}$$

then $L_m[x^n y^m f(x,y), s.p] = L_m[f(x,y), s+n, p+m]$

Proof

The BLLMIT in [0,0] to (∞,∞) .is

$$L_m[f(x,y), s, p] = \int_0^\infty\int_0^\infty f(x,y)\, x^{s-1} y^{p-1}\text{dxdy}\quad \text{then}$$

$L_m[x^n y^m f(x,y), s.p] =$
$\int_0^\infty \int_0^\infty f(x,y)\, x^n y^m x^{s-1} y^{p-1}$dxdy

$=\int_0^\infty \int_0^\infty f(x,y)\, x^{s+n-1} y^{p+m-1}$dxdy
$= L_m[f(x,y), s+n, p+m]$
$L_m[x^n y^m f(x,y), s.p] = L_m[f(x,y), s+n, p+m]$
(10)

3.2.6:Second Shifting Theorem
The BLLMIT in [0,0] to (∞,∞) .is
$L_m[f(x,y), s, p] = \int_0^\infty \int_0^\infty f(x,y)\, x^{s-1} y^{p-1}$dxdy then
$L_m[f(x-c,y)U(x-c), s, p] =$
$c^{s-1} L_m[f(u,y), s, -c, \infty; p, 0, \infty]$

Proof
The BLLMIT in [0,0] to (∞,∞) .is
$L_m[f(x,y), s, p] = \int_0^\infty \int_0^\infty f(x,y)\, x^{s-1} y^{p-1}$dxdy then
$L_m[f(x-c,y)U(x-c), s, p] = \int_0^\infty \int_0^\infty f(x-c,y) U(x-c)\, x^{s-1} y^{p-1}$dxdy
Substitute x-c=u, dx=du , if x=0 then u=-c and if x=∞ then u=∞
$L_m[f(x-c,y)U(x-c), s, p] = \int_{-c}^\infty \int_0^\infty f(u,y) U(u)\, (c+s)^{s-1} y^{p-1}$dudy

$= c^{s-1} \int_{-c}^\infty \int_0^\infty f(u,y) U(u)\, u^{s-1} y^{p-1}$dudy

$= c^{s-1} L_m[f(u,y), s, -c, \infty; p, 0, \infty]$
$L_m[f(x-c,y)U(x-c), s, p] =$
$c^{s-1} L_m[f(u,y), s, -c, \infty; p, 0, \infty]$ (11)

3.2.7:Theorem (Ramanujan's Formula)

If $\int_0^\infty x^{r-1} \left(\sum_{n=0}^{\infty} (-1)^n \emptyset(n)\, x^n \right) dx = \frac{\pi}{\sin \pi r} \emptyset(-r)$

Then

$$\int_0^\infty \int_0^\infty f(x,y) x^{s-1} y^{p-1} \left(\sum_{n=0}^{\infty} (-1)^n f(x,n) y^n \right) dxdy$$
$$= \int_0^\infty x^{s-1}\, dx \int_0^\infty y^{p-1} \left(\sum_{n=0}^{\infty} (-1)^n f(x,n) y^n\right) dy$$
$$= \int_0^\infty x^{s-1} \frac{\pi}{\sin \pi p} f(x,-p)\, dx$$
$$= \frac{\pi}{\sin \pi p} \int_0^\infty x^{s-1} f(x,-p) dx$$

$$\int_0^\infty \int_0^\infty f(x,y) x^{s-1} y^{p-1} \left(\sum_{n=0}^{\infty} (-1)^n f(x,n) y^n \right) dxdy\, \frac{\pi}{\sin \pi p} L[f(x,-p), s, 0, \infty]$$
(12)

Where $L[f(x,-p), s] = \int_0^\infty e^{-sx} f(x,-p) dx$
This is the Laplace transform of f (x,-p) w.r.t. parameter s > 0 , denoted by L[f(x,-p),s] .

3.3.Derivatives

Theorem: Suppose that $f(t,z)$ is continuous for all t ≥ 0 and z ≥ 0 satisfying (1.2) for some value γ,η and m has a derivative $f_t(t,z)$ which is piecewise continuous on every finite interval in the range of t ≥ 0 and z ≥ 0.Then by using the Bi-Lateral Laplace-Mellin Integral transform, the derivative of $f(t,z)$ exists when s > γ and p > η and $|f(t,z)| \le m e^{\gamma t + \eta z}$ for all t ≥ 0, z ≥ 0 for some constants.

3.3.1: BLLMIT of first order partial derivative of $y f(x,y)$ w.r.t. y
The BLLMIT in [0,0] to (∞,∞) .is
$L_m[f(x,y), s, p] = \int_0^\infty \int_0^\infty f(x,y)\, x^{s-1} y^{p-1}$dxdy then
$L_m[y f_y(x,y), s, p] = \int_0^\infty \int_0^\infty f_y(x,y) x^{s-1} y^p$dxdy
$= \int_0^\infty x^{s-1}\, dx \int_0^\infty y^p\, f_y(x,y) dy$
$= \int_0^\infty x^{s-1}\, dx[(f(x,y) y^p)_0^\infty - \int_0^\infty p y^p\, f(x,y) dy]$
$= \int_0^\infty x^{s-1} dx[f(x,\infty) - p \int_0^\infty y^{p-1} f(x,y) dy]$

By assuming $\lim_{y \to \infty} y^p = 1$

$= -p \int_0^\infty x^{s-1}\, y^{p-1} f(x,y) dxdy + \int_0^\infty x^{s-1} f(x,\infty) dx$
$L_m[y f_y(x,y), s, p] = -p L_m[f(x,y), s, p] + k$ (13)
Where $k = \int_0^\infty x^{s-1} f(x,\infty) dx$

3.3.2: BLLMIT of second order partial derivative of $y^2 f(x,y)$ w.r.t. y
$L_m[y^2 f_{yy}(x,y), s, p] = \int_0^\infty \int_0^\infty y^2\, f_{yy}(x,y) x^{s-1} y^{p-1} dxdy$
$= \int_0^\infty \int_0^\infty f_{yy}(x,y) x^{s-1} y^{p+1}\, dxdy$
$= \int_0^\infty x^{s-1} dx \int_0^\infty f_{yy}(x,y) y^{p+1} dy$
$= \int_0^\infty x^{s-1} dx[[f_y(x,y) y^{p+1}]_0^\infty - (p+1) \int_0^\infty f_y(x,y) y^p\, dy]$
$= -(p+1) \int_0^\infty \int_0^\infty x^{s-1}\, y^p f_y(x,y) dxdy + \int_0^\infty x^{s-1}\, f_y(0,\infty) dx$
$= -(p+1) \int_0^\infty \int_0^\infty x^{s-1}\, y^p f_y(x,y) dxdy + 0$(By Using DUIS)
$= -(p+1) \int_0^\infty \int_0^\infty x^{s-1}\, y^p f_y(x,y) dxdy$
$= -(p+1)[-p L_m[f(x,y), s, p] + k]$
$= p(p+1) L_m[f(x,y), s, p] - (p+1)k$
$L_m[y^2 f_{yy}(x,y), s, p] = p(p+1) L_m[f(x,y), s, p] - (p+1)k$
(14)
Where $k = \int_0^\infty x^{s-1} f(x,\infty) dx$

3.4: Applications:
The Cauchy's linear Differential Equation is
$y^2 f_{yy}(x,y) + y f_y(x,y) + f(x,y) = 0$,x is constant variable.
The **BLLMIT** of $y^2 f_{yy}(x,y) + y f_y(x,y) + f(x,y) = 0$
$L_m[\ y^2 f_{yy}(x,y) + y f_y(x,y) + f(x,y), s, p]$
$= \int_0^\infty \int_0^\infty [\ y^2 f_{yy}(x,y) + y f_y(x,y) + f(x,y)] x^{s-1}\, y^{p-1} dxdy$

$=\int_0^\infty \int_0^\infty y^2 f_{yy}(x,y)x^{s-1}y^{p-1}dxdy +$
$\int_0^\infty \int_0^\infty f_y(x,y)yx^{s-1}y^{p-1}dxdy + \int_0^\infty \int_0^\infty f(x,y)\, x^{s-1}y^{p-1}\text{dxdy}$

$$= p(p+1)L_m[f(x,y),s,p] - (p+1)k + (-p)L_m[f(x,y),s,p] + k + L_m[f(x,y),s,p]$$
$$= (p^2+1)L_m[f(x,y),s,p] - pk$$

If $y^2 f_{yy}(x,y) + y f_y(x,y) + f(x,y) = 0$ then

$$(p^2+1)L_m[f(x,y),s,p] - pk = 0$$
$$(p^2+1)L_m[f(x,y),s,p] = pk$$
$$L_m[f(x,y),s,p] = \frac{pk}{(p^2+1)} \qquad (15)$$

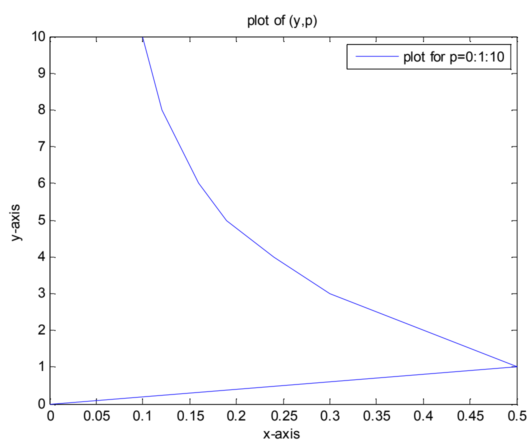
Where $k = \int_0^\infty x^{s-1} f(x,\infty)dx$

This is the required BLLMIT of Cauchy's Linear Differential Equation.

3.5. Graphical Representation of BLLMIT of Cauchy's Linear Differential Equation

$$L_m[f(x,y),s,p] = \frac{pk}{(p^2+1)}$$

where $k = \int_0^\infty x^{s-1} f(x,\infty)dx$



Plot of (y,p): We consider $y = L_m[f(x,y),s,p]$ i.e. BLLMIT in [0,0] to (∞,∞) on x-axis and p(parameter) on y-axis

4. Conclusion:

We have obtained interesting results of BLLMIT by using Ramanujan's formula .To illustrate the advantages and use of these transforms, some important differential equation has been solved. We have also studied graphical representation of the solution using matlab.

References

1. A .E. Gracc and M. Spann, A comparision between Fourier-Mellin descriptors and Moment based feactures for invariant object recognition using neural network, *Pattern Recog. Lett.*,12 (1991), 635-643.
2. A. H. Zemanian, Generalized Integal Transformation, Interscience Publication, New York (1968)
3. A. Z .Zemanian, The distributional Laplace and Mellin transformations, *J. SIAM*, 14(1) (1908)
4. Bruce Littlefield, Mastering Matlab, Prentice Hall ,Upper saddle River, New York.
5. C. Fox, Applications of Mellin's Transformation to the integral equations, (1933)
6. C.Fox Application of Mellin Transformation to Integral Equation, 3rd March, 1934, 495-502.
7. Derek Naylor, On a Mellin Type Integral Transforms, *Journal of Mathematics and Mechanics*, 12(2) (1963).
8. Ian N. Sneddon, The use of Integral Transforms, TMH edition 1974.
9. I. S. Reed, The Mellin Type Double Integral, Cambridge ,London.
10. Jean M. Tchuenche and Nyimvua S. Mhare, An applications of double Sumudu Transform, *Applied Mathematical Sciences*, 1 (2007), 31-39.
11. J. M. Mendez and J. R.Negrin , On the finite Hankel-Schwartz Transformation of Distributions, *Ganita*, 39(1) (1988).
12. Rugra Pratap, Getting Started With Matlab, Oxford University Press, NewYork, 2003.
13. Stephane Derrode and Faouzi Ghorbel, Robust and Efficient Fourier-Mellin Transform Appoximations for Gray-Level Image Reconsrruction and Complete Invarient Description, Computer Vision and Image Understandind 83 (2001), 57-78.
14. S. M. Khairnar, R.M. Pise and J. N. Salunke , Applications of the Mellin type integral transform in the range (1/a, ∞), IJMSA (Accepted).
15. S. M. Khairnar, R.M. Pise and J. N. Salunke , Study Of The Sumudu Mellin Integral Transform and Its Applications, *Int. J. Mat. Sci. and Engg. Appl.*, 4(IV) (2010), 307-325.
16. S. M. Khairnar, R.M. Pise and J. N. Salunke , Bilateral Laplace-Mellin Integral Transform and its Applications, International *Journal of Pure and Applied Sciences and Technology* (Accepted).