

# Optimization Fuzzy Multi-Objective Multi-Index Transportation Problem with Linear Membership Function

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## Research Article

**Abstract:** Fuzzy programming technique is used to solve multi-objective multi-index transportation problem in which objectives are conflicting in nature. The fuzzy programming method has been applied to find optimal compromise solution with the help of numerical example.

**Keywords:** multi-Objective transportation problem, multi-Index transportation problem fuzzy linear membership function,

### 1. Introduction

Fuzzy set theory was proposed by L. A. Zadeh (1965), and has been found extensive in various fields. Bellman and Zadeh (1970) were the first to consider the application of the fuzzy set theory in solving optimization problems in a fuzzy environment, these investigators constraints that both the objective function and the constraints that exist in the model could be represented by corresponding fuzzy set and should be treated in the same manner. The earliest application of it to transportation problems include Prade (1980), O'he'igeartaigh (1982), Chanas et al. (1984), Vedegay (1984) and Delgado et al. (1987). But these researcher emphases on investigating theory and algorithm. Furthermore, these above investigations are illustrated with simple instance slacking in actual cases of submission. On the other hand, these models are only of single objective and are classical two index transportation problems. In actual transportation problem, the multi-objective functions are generally considered, which includes average delivery time of the commodities, minimum cost, etc. The first attempt to fuzzify a linear programming with multiple objective functions was made by Zeleny (1973). Zimmermann (1978) applied the fuzzy set theory to the linear multicriteria decision making problem. It used the linear fuzzy membership function and presented the application of fuzzy linear vector maximum problem. He showed that solutions obtained by fuzzy linear programming always provide efficient solutions and also an optimal compromised solution. Aneja and Nair (1988) Showed that the problem model and Klingman and Phillips (1988) developed a model / solution procedure for adjusting to obtain an equitably infeasible solution for an infeasible solution for an infeasible transportation problem. Klingman and Phillips (1988)

showed that the problem can be modeled and solved as a preemptive, multicriteria, and capacitated transportation problem. Multi-index transportation problem are the extension of conventional transportation problems, and are appropriate for solving transportation problems with multiple supply points, multiple demand points as well as problems using diverse modes of transportation demand or delivering different kinds of merchandises. Thus, the forwarded problem would be more complicated than conventional transportation problems. Junginger (1993) who proposed a set of logic problems, to solve multi-index transportation problems, has also conducted a detailed investigation regarding the characteristics of multi-index transportation problem model. Rautman et al. (1993) used multi-index transportation problem model to solve the shipping scheduling suggested that the employment of such transportation efficiency but also optimize the integral system. These references are only a single objective model and its constraints are not fuzzy numbers.

In this study, a model is developed, and it combines fuzzy multi-objective programming and multi-index transportation problem to solve problem. This model cannot only satisfy more of the actual requirements of the integral system but is also more flexible than conventional transportation problems. Furthermore, it can offer more information to the decision maker (DM) for reference, and then it can raise the quality for decision-making. The fuzzy multi-objective multi-index transportation problem model is presented.

### 2. Mathematical Model

#### Multi-Objective Multi-index Transportation Problem

Let  $a_{ijl}$  be multi-dimensional array  $1 \leq i \leq m, 1 \leq j \leq n, 1 \leq l \leq k$  and let

$A=(a_{ij}), B=(b_{jl}), C=(c_{il})$  be multi-matrices then multi-index transportation problem is defined as follows

$$\text{Minimize } \sum_i \sum_j \sum_l a_{ijl} X_{ijl} \quad (1)$$

Subject to

$$\begin{aligned} \sum_j X_{ij} &= a_{ij} \quad \forall(i,j) & (2) \\ \sum_j X_{ij} &= c_{ij} \quad \forall(i,l) \\ \sum_i X_{ij} &= b_{ij} \quad \forall(j,l) \\ X_{ij} &\geq 0 \quad \forall(i,j,l) \end{aligned}$$

It is immediate that

$$\sum_i a_{ij} = \sum_j b_{ij}; \quad \sum_j a_{ij} = \sum_l c_{il}; \quad \sum_j b_{ij} = \sum_l c_{il} \quad (3)$$

are three necessary conditions however they are noted to be non sufficient.

Multi-objective double transportation problem as follows

$$\text{Minimize } Z_p = \sum_{i=1}^m \sum_{j=1}^n k_{ij}^{(1)} x_{ij} + \sum_{i=1}^m \sum_{j=1}^n k_{ij}^{(2)} x_{ij} \quad (4)$$

Subject to

$$\sum_{j=1}^n x_{ij}^{(1)} = a_{i1} \quad \forall i \quad (5)$$

$$\sum_{j=1}^n x_{ij}^{(2)} = a_{i2} \quad \forall i \quad (6)$$

$$\sum_{i=1}^m x_{ij}^{(1)} = b_{j1} \quad \forall j \quad (7)$$

$$\sum_{i=1}^m x_{ij}^{(2)} = b_{j2} \quad \forall j \quad (8)$$

$$x_{ij}^{(1)} + x_{ij}^{(2)} = c_{ij} \quad \forall i,j \quad (9)$$

$$x_{ij}^{(1)}, x_{ij}^{(2)} \geq 0 \quad \forall i,j \quad (10)$$

It may be easily seen that for existence of solution following set of conditions are necessary.

$$\sum_{j=1}^n c_{ij} = a_{i1} + a_{i2} \quad \forall i \quad (11)$$

$$\sum_{i=1}^m c_{ij} = b_{j1} + b_{j2} \quad \forall j \quad (12)$$

$$\sum_{i=1}^m a_{i1} = \sum_{j=1}^n b_{j1} \quad (13)$$

$$\sum_{i=1}^m a_{i2} = \sum_{j=1}^n b_{j2} \quad (14)$$

$$\sum c_{ij} \leq \text{Min}(a_{i1} + b_{j1}) + \text{Min}(a_{i2} + b_{j2}) \quad \forall(i,j) \quad (15)$$

It may be easily seen that DTP is composed of two transportation tables and one C matrix as given below.

$$C_1 = \begin{bmatrix} k_{11}^{(1)} & k_{12}^{(1)} \dots & k_{1n}^{(1)} \\ k_{21}^{(1)} & k_{22}^{(1)} \dots & k_{2n}^{(1)} \\ \dots & \dots & \dots \\ k_{m1}^{(1)} & k_{m2}^{(1)} \dots & k_{mn}^{(1)} \\ b_{11} & b_{12} \dots & b_{1n} \end{bmatrix} a_{i1} \quad (16)$$

$$C_2 = \begin{bmatrix} k_{11}^{(2)} & k_{12}^{(2)} \dots & k_{1n}^{(2)} \\ k_{21}^{(2)} & k_{22}^{(2)} \dots & k_{2n}^{(2)} \\ \dots & \dots & \dots \\ k_{m1}^{(2)} & k_{m2}^{(2)} \dots & k_{mn}^{(2)} \\ b_{11} & b_{12} \dots & b_{1n} \end{bmatrix} a_{i2} \quad \text{and } C = (c_{ij})_{m \times n} \quad (17)$$

$$T_1 = \begin{bmatrix} k_{11}^{(1)} & k_{12}^{(1)} \dots & k_{1n}^{(1)} \\ k_{21}^{(1)} & k_{22}^{(1)} \dots & k_{2n}^{(1)} \\ \dots & \dots & \dots \\ k_{m1}^{(1)} & k_{m2}^{(1)} \dots & k_{mn}^{(1)} \\ b_{11} & b_{12} \dots & b_{1n} \end{bmatrix} a_{i1} \quad (18)$$

$$T_2 = \begin{bmatrix} k_{11}^{(2)} & k_{12}^{(2)} \dots & k_{1n}^{(2)} \\ k_{21}^{(2)} & k_{22}^{(2)} \dots & k_{2n}^{(2)} \\ \dots & \dots & \dots \\ k_{m1}^{(2)} & k_{m2}^{(2)} \dots & k_{mn}^{(2)} \\ b_{11} & b_{12} \dots & b_{1n} \end{bmatrix} a_{i2} \quad \text{and } C = (c_{ij})_{m \times n} \quad (19)$$

### 3. Fuzzy Algorithm to solve multi-objective multi-index transportation problem

#### Step 1:

Solve the multi-objective multi-index transportation problem as a single objective transportation problem P times by taking one of the objectives at a time

#### Step 2 :

From the results of step 1, determine the corresponding values for every objective at each solution derived. According to each solution and value for every objective, we can find pay-off matrix as follows

$$\begin{matrix} & Z_1(X) & Z_2(X) & \dots & Z_p(X) \\ X^{(1)} & \begin{bmatrix} Z_{11} & Z_{12} & \dots & Z_{1p} \\ Z_{21} & Z_{22} & \dots & Z_{2p} \\ \dots & \dots & \dots & \dots \\ Z_{p1} & Z_{p2} & \dots & Z_{pp} \end{bmatrix} \\ X^{(2)} & \\ \dots & \\ X^{(p)} & \end{matrix}$$

Where,  $X^{(1)}, X^{(2)}, \dots, X^{(p)}$  are the isolated optimal solutions of the P different transportation problems for P different objective functions  $Z_j = Z_j(X^i)$  ( $i=1,2,\dots,p$  &  $j=1,2,\dots,p$ ) be the i-th row and j-th column element of the pay-off matrix.

#### Step 3:

From step 2, we find for each objective the worst ( $U_p$ ) and the best ( $L_p$ )

values corresponding to the set of solutions, where,  $U_p = \max(Z_{1p}, Z_{2p}, \dots, Z_{pp})$  and  $L_p = Z_{pp}$   $p=1,2,\dots,P$

An initial fuzzy model of the problem (4)-(10) can be stated as

Find  $X_{ij}$   $i=1,2,\dots,m$   $j=1,2,\dots,n$ ,  
so as to satisfy  $Z_p \leq L_p$   $p=1,2,\dots,P$   
subject to (4)-(10)

#### Step 4: Case (i)

Define membership function for the p-th objective function as follows:

$$\mu_p(X) = \begin{cases} 1 & \text{if } Z_p(X) \leq L_p \\ \frac{U_p - Z_p(X)}{U_p - L_p} & \text{if } L_p < Z_p < U_p \\ 0 & \text{if } Z_p \geq U_p \end{cases} \quad (20)$$

**Step 5:**

Find an equivalent crisp model by using a linear membership function for the initial fuzzy model

$$\begin{aligned} & \text{Maximize } \lambda \\ & \lambda \leq \frac{U_p - Z_p(X)}{U_p - L_p} \quad (21) \\ & \text{subject to (5)-(10)} \end{aligned}$$

**Step 6:** Solve the crisp model by an appropriate mathematical programming algorithm.

$$\begin{aligned} & \text{Maximize } \lambda \\ & \text{Subject to} \\ & C_{ij}^p X_{ij} + \lambda(U_p - L_p) \leq U_p \quad p=1,2,\dots,P \\ & \text{subject to (5)-(10)} \end{aligned} \quad (22)$$

**4. Numerical Examples**

Example 1

$$\begin{matrix} C_1 & C_2 & C \\ \begin{bmatrix} 4 & 3 & 5 \\ 8 & 6 & 2 \\ 7 & 4 & 1 \\ 9 & 10 & 12 \\ 14 & 12 & 10 \end{bmatrix} & \begin{bmatrix} 8 & 6 & 3 \\ 5 & 4 & 1 \\ 9 & 2 & 6 \\ 4 & 9 & 3 \\ 5 & 8 & 11 \end{bmatrix} & \begin{bmatrix} 5 & 7 & 3 \\ 8 & 4 & 9 \\ 4 & 1 & 6 \\ 2 & 8 & 3 \end{bmatrix} \end{matrix} \quad (27)$$

Example 2

$$\begin{matrix} T_1 & T_2 & C \\ \begin{bmatrix} 5 & 6 & 7 \\ 4 & 5 & 2 \\ 1 & 3 & 4 \\ 4 & 2 & 3 \\ 14 & 12 & 10 \end{bmatrix} & \begin{bmatrix} 10 & 9 & 9 \\ 7 & 9 & 2 \\ 8 & 7 & 9 \\ 8 & 4 & 5 \\ 5 & 8 & 11 \end{bmatrix} & \begin{bmatrix} 5 & 7 & 3 \\ 8 & 4 & 9 \\ 4 & 1 & 6 \\ 2 & 8 & 3 \end{bmatrix} \end{matrix} \quad (28)$$

Example 1 is simplified as

$$\begin{aligned} \text{Minimize } Z_1 = & 4X_{11}^{(1)} + 3X_{12}^{(1)} + 5X_{13}^{(1)} + 8X_{21}^{(1)} + 6X_{22}^{(1)} + 2X_{23}^{(1)} + 7X_{31}^{(1)} + 4X_{32}^{(1)} + \\ & X_{33}^{(1)} + 9X_{41}^{(1)} + 10X_{42}^{(1)} + 12X_{43}^{(1)} + 8X_{11}^{(2)} + 6X_{12}^{(2)} + 3X_{13}^{(2)} + 5X_{21}^{(2)} + \\ & 4X_{22}^{(2)} + X_{23}^{(2)} + 9X_{31}^{(2)} + 2X_{32}^{(2)} + 6X_{33}^{(2)} + 4X_{41}^{(2)} + 9X_{42}^{(2)} + 3X_{43}^{(2)} \end{aligned} \quad (29)$$

$$\begin{aligned} & \text{Subject to} \\ & X_{11}^{(1)} + X_{12}^{(1)} + X_{13}^{(1)} = 9 \\ & X_{21}^{(1)} + X_{22}^{(1)} + X_{23}^{(1)} = 14 \\ & X_{31}^{(1)} + X_{32}^{(1)} + X_{33}^{(1)} = 6 \\ & X_{41}^{(1)} + X_{42}^{(1)} + X_{43}^{(1)} = 7 \\ & X_{11}^{(2)} + X_{12}^{(2)} + X_{13}^{(2)} = 6 \\ & X_{21}^{(2)} + X_{22}^{(2)} + X_{23}^{(2)} = 7 \\ & X_{31}^{(2)} + X_{32}^{(2)} + X_{33}^{(2)} = 5 \\ & X_{41}^{(2)} + X_{42}^{(2)} + X_{43}^{(2)} = 6 \\ & X_{11}^{(1)} + X_{21}^{(1)} + X_{31}^{(1)} + X_{41}^{(1)} = 14 \\ & X_{12}^{(1)} + X_{22}^{(1)} + X_{32}^{(1)} + X_{42}^{(1)} = 12 \\ & X_{13}^{(1)} + X_{23}^{(1)} + X_{33}^{(1)} + X_{43}^{(1)} = 10 \\ & X_{11}^{(2)} + X_{21}^{(2)} + X_{31}^{(2)} + X_{41}^{(2)} = 5 \\ & X_{12}^{(2)} + X_{22}^{(2)} + X_{32}^{(2)} + X_{42}^{(2)} = 8 \\ & X_{13}^{(2)} + X_{23}^{(2)} + X_{33}^{(2)} + X_{43}^{(2)} = 11 \\ & X_{11}^{(1)} + X_{11}^{(2)} = 5 \\ & X_{12}^{(1)} + X_{12}^{(2)} = 7 \\ & X_{13}^{(1)} + X_{13}^{(2)} = 3 \\ & X_{21}^{(1)} + X_{21}^{(2)} = 8 \\ & X_{22}^{(1)} + X_{22}^{(2)} = 4 \\ & X_{23}^{(1)} + X_{23}^{(2)} = 9 \\ & X_{31}^{(1)} + X_{31}^{(2)} = 4 \\ & X_{32}^{(1)} + X_{32}^{(2)} = 1 \\ & X_{33}^{(1)} + X_{33}^{(2)} = 6 \\ & X_{41}^{(1)} + X_{41}^{(2)} = 2 \\ & X_{42}^{(1)} + X_{42}^{(2)} = 8 \\ & X_{43}^{(1)} + X_{43}^{(2)} = 3 \end{aligned} \quad (30)$$

Example 2 is simplified as

$$\begin{aligned} \text{Minimize } Z_2 = & 5X_{11}^{(1)} + 6X_{12}^{(1)} + 7X_{13}^{(1)} + 4X_{21}^{(1)} + 5X_{22}^{(1)} + 2X_{23}^{(1)} + 1X_{31}^{(1)} + 3X_{32}^{(1)} + \\ & 4X_{33}^{(1)} + 4X_{41}^{(1)} + 2X_{42}^{(1)} + 3X_{43}^{(1)} + 10X_{11}^{(2)} + 9X_{12}^{(2)} + 9X_{13}^{(2)} + 7X_{21}^{(2)} + \\ & 9X_{22}^{(2)} + 2X_{23}^{(2)} + 8X_{31}^{(2)} + 7X_{32}^{(2)} + 9X_{33}^{(2)} + 8X_{41}^{(2)} + 4X_{42}^{(2)} + 5X_{43}^{(2)} \end{aligned} \quad (31)$$

Subject to

$$\begin{aligned} & X_{11}^{(1)} + X_{12}^{(1)} + X_{13}^{(1)} = 9 \\ & X_{21}^{(1)} + X_{22}^{(1)} + X_{23}^{(1)} = 14 \\ & X_{31}^{(1)} + X_{32}^{(1)} + X_{33}^{(1)} = 6 \\ & X_{41}^{(1)} + X_{42}^{(1)} + X_{43}^{(1)} = 7 \\ & X_{11}^{(2)} + X_{12}^{(2)} + X_{13}^{(2)} = 6 \\ & X_{21}^{(2)} + X_{22}^{(2)} + X_{23}^{(2)} = 7 \\ & X_{31}^{(2)} + X_{32}^{(2)} + X_{33}^{(2)} = 5 \\ & X_{41}^{(2)} + X_{42}^{(2)} + X_{43}^{(2)} = 6 \\ & X_{11}^{(1)} + X_{21}^{(1)} + X_{31}^{(1)} + X_{41}^{(1)} = 14 \\ & X_{12}^{(1)} + X_{22}^{(1)} + X_{32}^{(1)} + X_{42}^{(1)} = 12 \\ & X_{13}^{(1)} + X_{23}^{(1)} + X_{33}^{(1)} + X_{43}^{(1)} = 10 \\ & X_{11}^{(2)} + X_{21}^{(2)} + X_{31}^{(2)} + X_{41}^{(2)} = 5 \\ & X_{12}^{(2)} + X_{22}^{(2)} + X_{32}^{(2)} + X_{42}^{(2)} = 8 \\ & X_{13}^{(2)} + X_{23}^{(2)} + X_{33}^{(2)} + X_{43}^{(2)} = 11 \\ & X_{11}^{(1)} + X_{11}^{(2)} = 5 \\ & X_{12}^{(1)} + X_{12}^{(2)} = 7 \\ & X_{13}^{(1)} + X_{13}^{(2)} = 3 \\ & X_{21}^{(1)} + X_{21}^{(2)} = 8 \\ & X_{22}^{(1)} + X_{22}^{(2)} = 4 \\ & X_{23}^{(1)} + X_{23}^{(2)} = 9 \\ & X_{31}^{(1)} + X_{31}^{(2)} = 4 \\ & X_{32}^{(1)} + X_{32}^{(2)} = 1 \\ & X_{33}^{(1)} + X_{33}^{(2)} = 6 \\ & X_{41}^{(1)} + X_{41}^{(2)} = 2 \\ & X_{42}^{(1)} + X_{42}^{(2)} = 8 \\ & X_{43}^{(1)} + X_{43}^{(2)} = 3 \end{aligned} \quad (32)$$

For objective  $Z_1$ , we find the optimal solution as

$$X^{(1)} = \begin{cases} X_{11}^{(1)} = 5; X_{12}^{(1)} = 4; X_{21}^{(1)} = 8, X_{22}^{(1)} = 2, \\ X_{23}^{(1)} = 4; X_{33}^{(1)} = 6; X_{41}^{(1)} = 1; X_{42}^{(1)} = 6; \\ X_{12}^{(2)} = 3; X_{13}^{(2)} = 3; X_{22}^{(2)} = 2; X_{23}^{(2)} = 5, \\ X_{31}^{(2)} = 4; X_{32}^{(2)} = 1; X_{41}^{(2)} = 1; X_{42}^{(2)} = 2; \\ X_{43}^{(2)} = 3 \end{cases}$$

$$Z_1 = 300$$

For objective  $Z_2$ , we find the optimal solution as

$$X^{(2)} = \begin{cases} X_{11}^{(1)} = 4; X_{12}^{(1)} = 5; X_{21}^{(1)} = 8, X_{22}^{(1)} = 4, \\ X_{23}^{(1)} = 2; X_{31}^{(1)} = 1; X_{33}^{(1)} = 5; X_{41}^{(1)} = 1; \\ X_{42}^{(1)} = 3; X_{43}^{(1)} = 3; X_{11}^{(2)} = 1; X_{12}^{(2)} = 2; \\ X_{13}^{(2)} = 3; X_{23}^{(2)} = 7; X_{31}^{(2)} = 3; X_{32}^{(2)} = 1; \\ X_{33}^{(2)} = 1; X_{41}^{(2)} = 1; X_{42}^{(2)} = 5; \end{cases}$$

$$Z_2 = 283$$

Now for  $X^{(1)}$  we can find out  $Z_2$ ,  $Z_2(X^{(1)}) = 291$

Now for  $X^{(2)}$  we can find out  $Z_1$ ,  $Z_1(X^{(2)}) = 330$

Pay-off matrix is

$$\begin{matrix} & Z_1 & Z_2 \\ X^{(1)} & [300 & 291] \\ X^{(2)} & [330 & 283] \end{matrix}$$

From this matrix  
 $U_1=330, U_2=291, L_1=300, L_2=283$

Find  $\{X_{ij}, i=1,2,3, j=1,2,3\}$ , so as to satisfy  $Z_1 \leq 300$  and  $Z_2 \leq 283$ ,

Define membership function for the objective functions  $Z_1(X)$  and  $Z_2(X)$  respectively

$$\mu_1(X) = \begin{cases} 1, & \text{if } Z_1(X) \leq 300 \\ \frac{330-Z_1(X)}{330-300}, & \text{if } 300 < Z_1(X) < 330 \\ 0, & \text{if } Z_1(X) \geq 330 \end{cases};$$

$$\mu_2(X) = \begin{cases} 1, & \text{if } Z_2(X) \leq 283 \\ \frac{291-Z_2(X)}{291-283}, & \text{if } 283 < Z_2(X) < 291 \\ 0, & \text{if } Z_2(X) \geq 291 \end{cases}$$

Find an equivalent crisp model

Maximize  $\lambda$ ,  $\lambda + Z_1(X) \leq 330$  and  $5\lambda + Z_2(X) \leq 291$

Solve the crisp model by using an appropriate mathematical algorithm.

$$4X_{11}^{(1)} + 3X_{12}^{(1)} + 5X_{13}^{(1)} + 8X_{21}^{(1)} + 6X_{22}^{(1)} + 2X_{23}^{(1)} + 7X_{31}^{(1)} + 4X_{32}^{(1)} + X_{33}^{(1)} + 9X_{41}^{(1)} + 10X_{42}^{(1)} + 12X_{43}^{(1)} + 8X_{11}^{(2)} + 6X_{12}^{(2)} + 3X_{13}^{(2)} + 5X_{21}^{(2)} + 4X_{22}^{(2)} + X_{23}^{(2)} + 9X_{31}^{(2)} + 2X_{32}^{(2)} + 6X_{33}^{(2)} + 4X_{41}^{(2)} + 9X_{42}^{(2)} + 3X_{43}^{(2)} + 30\lambda \leq 330$$

$$5X_{11}^{(1)} + 6X_{12}^{(1)} + 7X_{13}^{(1)} + 4X_{21}^{(1)} + 5X_{22}^{(1)} + 2X_{23}^{(1)} + 1X_{31}^{(1)} + 3X_{32}^{(1)} + 4X_{33}^{(1)} + 4X_{41}^{(1)} + 2X_{42}^{(1)} + 3X_{43}^{(1)} + 10X_{11}^{(2)} + 9X_{12}^{(2)} + 9X_{13}^{(2)} + 7X_{21}^{(2)} + 9X_{22}^{(2)} + 2X_{23}^{(2)} + 8X_{31}^{(2)} + 7X_{32}^{(2)} + 9X_{33}^{(2)} + 8X_{41}^{(2)} + 4X_{42}^{(2)} + 5X_{43}^{(2)} + 8\lambda \leq 291$$

Subject to (30)

The optimal compromise solution of the problem is represented as

$$\lambda = 0.6521$$

$$X^{(*)} = \left\{ \begin{array}{l} X_{11}^{(1)}=5; X_{12}^{(1)}=2.2608; X_{13}^{(1)}=1.7391; X_{21}^{(1)}=8; \\ X_{22}^{(1)}=3.7391; X_{23}^{(1)}=2.2608; X_{33}^{(1)}=6; X_{41}^{(1)}=1; \\ X_{42}^{(1)}=6; X_{43}^{(1)}=4.7391; X_{13}^{(2)}=1.2608; X_{23}^{(2)}=6.7391; \\ X_{31}^{(2)}=4; X_{32}^{(2)}=1; X_{41}^{(2)}=1; X_{42}^{(2)}=2; X_{43}^{(2)}=3 \\ Z_1^*=309.3902 \quad \text{and} \quad Z_2^*=283.4329 \end{array} \right.$$

### 5. Conclusions

In this paper multi-objective multi-index transportation problem is defined and problem is solved by using fuzzy programming technique (Linear membership function).

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