Comparative study of Multi-objective Unbalanced Transportation Problems with and without Budgetary Constraints

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Research Article

Abstract: A new method is proposed for finding an optimal solution for unbalanced transportation problems with budgetary constraints where demand and budget are imprecise. Further it is compared with the situation in which budgetary constraints are not known. The proposed method enables the decision makers to choose the optimal distribution according to their budget. Finally, with the help of numerical example, the proposed method is illustrated.

Keywords: *Transportation problem, Optimal solution, Budget, Fuzzy logic.*

Introduction

In logistics and supply-chain management for reducing cost and improving service, transportation models plays an important role. In real life problems, the supply and demand quantities are sometimes scarcely specified accurately because of changing economic conditions. In order to reduce information costs and also to construct a real model, the use of interval and fuzzy transportation problems are more appropriate. In an interval transportation problem, information about the range of variation of some (or all) of the parameters is available, which allows to specify a model with intervals. The transportation problem (TP) is a special class of linear programming problem, which deals with commodities from sources to destinations. In literature, a good amount of research has available to obtain an optimal solution for balanced transportation problems. But in real life situations, the decision maker faces an unbalanced transportation problem in which total supply is less than the total demand. Khanna et al. [6] introduced an algorithm for solving transportation flow under budgetary constraints. Tiwari et al. [7] investigated how the preemptive priority structure can be used in fuzzy goal programming problems. Weighted goal programming for unbalanced single objective transportation problem with budgetary constraint has been discussed by Kishore and Jayswal [1]. Kishore and Jayswal [2] introduced a method, called fuzzy approach, to solve unbalanced transportation problem with budgetary constraints. Peerayuth Charnsethikul and Saeree Sverasreni [4] discussed a method for solving the constrained bottleneck transportation problem under budgetary condition. Pandian and Natarajan [3] introduced the zero point method for finding an optimal solution to a classical transportation problem. Lin and Cheng [8] gave a genetic algorithm for solving a transportation network under a budget Constraint. Senapati and Tapan Kumar [5] investigated fuzzy multi-index transportation problem with budgetary restriction. In this paper, we have proposed interval-point method for finding an optimal solution for unbalanced transportation problems with budgetary constraints where demand and budget are imprecise. Further, it is compared with the situation in which budgetary constraints are not known. Here, our more emphasis on meeting the budget rather than on the fulfillment of the demand. For better understanding, the solution procedure is illustrated with a numerical example.

Research Methodology

Preliminaries:

We need the following definitions of the basic arithmetic operators and partial ordering on closed bounded intervals which can be found in [4].

Let D = $\{[a,b]$, $a \le b$ and a and b are in R$\}$ denote the set of all closed bounded intervals on the real line R.

Definition 1: Let $A=[a,b]$ and $B=[c,d]$ be in D. Then,

$$A \oplus B = [a+c, b+d];$$
$$A \otimes B = [p,q],$$

where $p = \min\{ac, ad, bc, bd\}$ and

$$q = \max\{ac, ad, bc, bd\}$$

Consider the following unbalanced transportation problem with budgetary constraints:

(A) Find the values of x_{ij}, i = 1,2,…m; j = =1,2,…n such that the following conditions are satisfied:

$$\sum_{j=1}^{n} x_{ij} \le a_i \quad i=1,2,....m \qquad (1)$$

$$\sum_{i=1}^{m} x_{ij} \in [l_j, u_j], \qquad j=1,2,....n \qquad (2)$$

$$\sum_{i=1}^{m}\sum_{j=1}^{n} c_{ij}x_{ij} \in [z_1, z_2] \qquad (3)$$

$$x_{ij} \ge 0, \quad i=1,2,\ldots m \text{ and } j=1,2,\ldots n \qquad (4)$$

Where c_{ij} is the cost of shipping one unit from supply point i to the demand point j;

a_i is the supply at supply point i; $[l_j, u_j]$ is the imprecise demand at demand point j;

x_{ij} is the number of units shipped from supply point i to demand point j and $[z_1,z_2]$ is the imprecise budget.

The unbalanced transportation problem related to the problem (p) is given below:

(IP) Minimize $[z_1, z_2] = \left[\sum_{i=1}^{m}\sum_{j=1}^{n} c_{ij}x_{ij}, \sum_{i=1}^{m}\sum_{j=1}^{n} c_{ij}x_{ij}\right]$

Subject to

$$\left[\sum_{j=1}^{n} x_{ij}, \sum_{j=1}^{n} x_{ij}\right] = [a_i, a_i], \quad i=1,2,....m$$

$$\left[\sum_{i=1}^{m} x_{ij}, \sum_{i=1}^{m} x_{ij}\right] = [l_j, u_j], \quad j=1,2,....n$$

$x_{ij} \ge 0$, i = 1,2,…m and j = 1,2,…n and integers.

Now, we need the following condition which finds a relation between optimal solutions of an interval transportation problem and a pair of induced transportation problems.

If the set { y^0_{ij} for all i and j} is an optimal solution of the upper bound transportation problem (UP) of (IP) where

(UP) Minimize $z_2 = \sum_{i=1}^{m}\sum_{j=1}^{n} c_{ij}x_{ij}$

subject to

$$\sum_{j=1}^{n} x_{ij} = a_i, \quad i=1,2,....m$$

$$\sum_{i=1}^{m} x_{ij} = u_j, \quad j=1,2,....n$$

$x_{ij} \ge 0$, i= 1,2,…m and j = 1,2,…n and are integers

And the set { x^0_{ij} for all i and j} is an optimal solution of the lower bound transportation problem (LP) of the problem (IP) where

(LP) Minimize $z_1 = \sum_{i=1}^{m}\sum_{j=1}^{n} c_{ij}x_{ij}$

Subject to

$$\sum_{j=1}^{n} x_{ij} = a_i \quad i=1,2,....m$$

$$\sum_{i=1}^{m} x_{ij} = l_j, \quad j=1,2,....n$$

$x_{ij} \ge 0$, i=1,2,…m and j=1,2,…n and are integers,

Then the set of intervals { $[x^0_{ij}, y^0_{ij}]$, for all i and j} is an optimal solution of the problem (IP) provided $x^0_{ij} \le y^0_{ij}$, for all i and j.

3. Interval-Point Method

We, now propose a new method namely, interval-point method for finding an optimal solution to the problem (P).

The interval-point method proceeds as follows:

Step 1: Construct an unbalanced interval transportation problem (IP) related to the given UTPBC.

Step 2: Construct the upper bound unbalanced transportation problem (UP) of the problem (IP) and solve the problem (UP) by the zero point method [3]. Let { y , for all i and j} be an optimal solution to the problem (UP).

Step 3: Construct the lower bound unbalanced transportation problem (LP) of the given the [problem (IP) and solve the problem (LP) with the upper bound constraints , $x_{ij} \le y^{\circ}_{ij}$ for all i and j by the zero point method [3]. Let { x^0_{ij} for all i and j} be the optimal solution to the problem (LP) with $x^0_{ij} \le y^0_{ij}$, for all i and j.

Step 4: The optimal solution to the problem (IP) is{ [$x^0_{ij} \le y^0_{ij}$],for all i and j } by the optimal objective values of the problem (IP) is $(Z_L - Z_U)$

Step 5: Let $Z \in [Z_1, Z_2]$ be the given budget cost for transportation cost of the given UTPBC. Now, we are write Z as in the form, $Z = Z_L + (Z_U - Z_L)\mu$ for some $\mu, 0 \le \mu \le 1$. This implies $\mu = \frac{Z - Z_L}{(Z_U - Z_L)}$

Here, we considered two cases,

Case 1: Values of Z is known i.e. fixed or given.

Case 2: Value of Z is not known.

Step 6: Compute the values of decision variables

$$x_{ij} = [x^0_{ij}, y^0_{ij}] = x^0_{ij} + (y^0_{ij} - x^0_{ij})\mu,$$

where μ is as in step 5(including 5.1 and 5.2).

Step 7: The optimal solution to the given UTPBC is $x_{ij} = [x^0_{ij}, y^0_{ij}] = x^0_{ij} + (y^0_{ij} - x^0_{ij})\frac{Z - Z_L}{(Z_U - Z_L)}$ for the given budget is Z.

Numerical Example

The interval-point method for solving unbalanced transportation problem with budgetary constraints is illustrated by the following example.

Example. There are four warehouses i.e. source from where the food grains are supplied to three different destinations i.e. demand stations. C_{ij}'s are the cost coefficients expressed in rupee per metric ton and a_i's , l_j's and u_j's are expressed in lakhs of metric ton. The transportation matrix is given in the following table:

	D_1	D_2	D_3	Supply
O_1	5	8	3	≤ 10
O_2	7	4	5	≤4
O_3	2	6	9	≤4
O_4	4	6	6	≤12
Demand	∈ [6,12]	∈ [7,14]	∈ [7,14]	

Determine an optimal distribution plan to transport the items from the source points to the destination points for the budget Rs.100.

Now, the interval transportation problem (IP) to the given problem is given below:

	D_1	D_2	D_3	Supply
O_1	[5,5]	[8,8]	[3,3]	[10,10]
O_2	[7,7]	[4,4]	[5,5]	[4,4]
O_3	[2,2]	[6,6]	[9,9]	[4,4]
O_4	[4,4]	[6,6]	[6,6]	[12,12]
Demand	[6,12]	[7,14]	[7,14]	

Now, the upper bound problem of (IP), (UP) of the problem (IP) is given below:

	D_1	D_2	D_3	Supply
O_1	5	8	3	10
O_2	7	4	5	4
O_3	2	6	9	4
O_4	4	6	6	12
Demand	12	14	14	

Now, using the zero point method, the optimal solution to the problem (UB) is:

$y^0_{13} = 10$, $y^0_{22} = 4$, $y^0_{31} = 4$, $y^0_{41} = 8$, and $y^0_{42} = 4$.

Now, the lower bound problem of interval transportation problem with the upper bounded (LB) constraints is:

	D_1	D_2	D_3	Supply
O_1	5	8	3	10
O_2	7	4	5	4
O_3	2	6	9	4
O_4	4	6	6	12
Demand	6	7	7	

With $x_{ij} \le y^0_{ij}$, i= 1, 2,…,m and j = 1,2,…n.

Now, using the zero point method, the optimal solution to the problem (LP) is

$x^0_{13} = 7$, $x^0_{22} = 4$, $x^0_{31} = 4$, $x^0_{41} = 2$, and $x^0_{42} = 3$.

Now, as in the step 3., we consider the optimal solution to the problem (IP) is $[x^\circ_{13}, y^\circ_{13}] = [7,10]$, $[x^\circ_{22}, y^\circ_{22}] = [4,4]$, $[x^\circ_{31}, y^\circ_{31}] = [4,4]$, $[x^\circ_{41}, y^\circ_{41}] = [2,8]$ and $[x^\circ_{42}, y^\circ_{42}] = [3,4]$ and also, the minimum interval transportation cost is [71,110].

Now, as in step 4., we have the total transportation cost as Z = 71 + 39 μ where Z is given budget. This implies that $\mu = \frac{Z-71}{39}$

Now, as in step 5. And the step 6., we have

$x_{13} = 7 + 3\left(\frac{Z-71}{39}\right)$; $x_{22}=4$; $x_{31}=4$;

$x_{41} = 2 + 6\left(\frac{Z-71}{39}\right)$; $x_{42} = 3 + \left(\frac{Z-71}{39}\right)$;

Case 1:

Given Z = 100. Then, the optimal solution to the given UTPBC for Z = 100 are:

$x_{13} = 9.23$, $x_{22} = 4$, $x_{31} = 4$, $x_{41} = 6.46$ and $x_{42} = 3.74$.

The total number of units transported = 27.43 tons.

Case 2:

We get, different values of Z for μ ($0 \le \mu \le 1$) is given as,

μ	Z	Decision variables	Transported Unit Cost
0	71	$x_{13} = 7$, $x_{22} = 4$, $x_{31} = 4$, $x_{41} = 2$ and $x_{42} = 3$	20
0.1	74.9	$x_{13} = 7.3$, $x_{22} = 4$, $x_{31} = 4$, $x_{41} = 2.6$ and $x_{42} = 3.1$	21
0.2	78.9	$x_{13} = 7.6$, $x_{22} = 4$, $x_{31} = 4$, $x_{41} = 3.2$ and $x_{42} = 3.2$	22
0.3	82.7	$x_{13} = 7.9$, $x_{22} = 4$, $x_{31} = 4$, $x_{41} = 3.8$ and $x_{42} = 3.3$	23
0.4	86.6	$x_{13} = 8.2$, $x_{22} = 4$, $x_{31} = 4$, $x_{41} = 4.4$ and $x_{42} = 3.4$	24
0.5	90.5	$x_{13} = 8.5$, $x_{22} = 4$, $x_{31} = 4$, $x_{41} = 5$ and $x_{42} = 3.5$	25
0.6	94.4	$x_{13} = 8.8$, $x_{22} = 4$, $x_{31} = 4$, $x_{41} = 5.6$ and $x_{42} = 3.6$	26
0.7	98.3	$x_{13} = 9.1$, $x_{22} = 4$, $x_{31} = 4$, $x_{41} = 6.2$ and $x_{42} = 3.7$	27
0.8	102.2	$x_{13} = 9.4$, $x_{22} = 4$, $x_{31} = 4$, $x_{41} = 6.8$ and $x_{42} = 3.8$	28
0.9	106.1	$x_{13} = 9.7$, $x_{22} = 4$, $x_{31} = 4$, $x_{41} = 7.4$ and $x_{42} = 3.9$	29
1	110	$x_{13} = 10$, $x_{22} = 4$, $x_{31} = 4$, $x_{41} = 8$ and $x_{42} = 4$	30

Conclusion

In this paper, we have considered the unbalanced transportation problem with and without budgetary constraints cases. The proposed without budgetary constraints method is useful for finding an optimal solution for decreasing or reducing the cost and finding a compromise solution. It is very easy to understand and easy to compute. Also, enables the decision makers to optimize the economical activities and make the correct managerial decisions depending on their financial position. The proposed method enables the decision makers to evaluate the economical activities and make self satisfied managerial decisions when they are handling a variety of unbalanced transportation problems with budgetary constraints.

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