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Fuzzy Average High Ranking Method for Solving Single Machine Scheduling Problem

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Research Article

Abstract: In this paper, we propose *n*-jobs to be processed on single machine scheduling problem involving fuzzy processing time and fuzzy due dates. The different due dates for each job be considered which meet the demand of customer with more satisfaction level. "*The main objective of this paper is the total penalty cost to be minimum in the schedule of the jobs on the single machine"*. This cost is composed of the total earliness and the total tardiness cost. Here, an algorithm is developed using Average High Ranking Method (AHRM) which minimizes the total penalty cost due to earliness of jobs in fuzzy environment was introduced and example was solved to show the effectiveness of the proposed approach.

Key Words: Fuzzy AHP Method, SMS Problem, Fuzzy Processing Time, Fuzzy Due Dates, etc.

1. Introduction

Job scheduling is a useful tool in decision-making problem. This technique is used to determine an optimal job scheduling problem. In most of the real world system, there are elements of uncertainty in process. Practical machine scheduling problems are numerous and varied. They arise in diverse areas such as flexible manufacturing systems, production planning, computer design, logistics, communication, etc. A scheduling problem is to find sequences of jobs (Tasks) on given machines (Resources) with the objective of minimising some function of the job completion times. In this paper, we propose *n*-jobs to be processed on Single Machine Scheduling Problem (SMSP) involving fuzzy processing time and fuzzy due dates. The different due dates for each job be considered which meet the demand of customer with more satisfaction level. The main objective of this paper is the total penalty cost to be minimum in the schedule of the jobs on the single machine. This cost is composed of the total earliness and the total tardiness cost. In Jadhav manufacturing scheduling, there are many different types of problems. These are V.S.Jadhav *et.al.* [1,7]:

a) Single Machine Scheduling Problems (b) Parallel Machine Scheduling Problems
c) Flow-shop Scheduling Problems (d) Job-shop Scheduling Problems
e) Manufacturing cells and assembly line scheduling problems.

Here, an algorithm is developed using Average High Ranking Method (AHRM) which minimizes the total penalty cost due to earliness of jobs in fuzzy environment. Finally, numerical example is given to illustrate proposed method. We consider generally the resources of two types: renewable or consumable. Renewable resources become available again after use (Reuse of Machine, File, Processor, Personal), whereas non-renewable resources disappear after use (Money, Raw materials.). The basic unit of the job shop process is the operation. There are three primary attributes of each operation which are given as part of the description of a particular job-shop problem (JSP) in Ramkumar R, A. *et al.* [10].They are:

- Identifying the operation with a particular job
- Identifying the operation with a particular machine
- Representing the processing time of the operation.

The short- term schedules show an optimal order (sequence) and time in which jobs are processed as well as show time tables for jobs, equipment, people, facilities and all other resources that are needed to support the production plan. The schedules should use resources efficiently to give '*low costs and high utilizations*'. Other purposes of scheduling are, '*minimizing customer wait time for a product, and meeting promised delivery dates, keeping stock levels low, giving preferred working pattern, minimizing waiting time of patients in a hospitals for different types of tests and so on*'. The general scheduling or sequencing problem may be described as: Let, there are *n*-jobs (Tasks) to be performed one at a time on each of *m*- machines (Processor). Slowinski, R; Hapke, M [6]. The study of earliness and tardiness penalties in scheduling models is a relatively recent area of inquiry. For many years scheduling research focused on single performance measures. Most of the literature deals with regular measures such as mean flow times, mean lateness, percentage of jobs tardy, mean tardiness etc. are having deterministic time. But the environment in real life is neither fixed nor probabilistic. So, here we are considering fuzzy environment i.e. the processing time of each job is having indeterministic environment. Here fuzzy processing time is considered in three situations *(a, b, c)* where, *a*- in Favorable (High) condition, b- Normal (Medium) condition and c- in Worse (Bad) condition. The mean tardiness criterion, in particular, has been a standard way of measuring conformance to due dates, although it ignores the consequences of jobs completed early. V.S.Jadhav *et al.* [12] Studied sequences with earliness and tardiness penalties in a JIT scheduling environment. Jobs that complete early must be held in finished goods inventory until their due date, while jobs that complete after their due date may cause a customer to shut down operations. Therefore, an optimal schedule is one in which all jobs finish on their assigned due dates. This can be translated to a scheduling objective in several ways. The most obvious objective is to minimize the deviation of job completion time around these due dates in non-deterministic time. The concept of penalizing both earliness and tardiness has spawned a new and rapidly developing line of research in the scheduling field. Because the use of both earliness and tardiness penalties in fuzzy environment give rise to a non-regular performance measure, it has led to new methodological issues in the design of solution procedures. This paper presents a special case of Early/Tardy (E/T) having distinct due dates (DDD) problem, when the earliness and tardiness are penalized at the rates fixed by demand maker for the jobs. The next sections introduce the concept of single machine and the processing time of the jobs in fuzzy environment Ghorbanali Mohammadi [11]. The average high ranking and the scheduling of some small systems are determined in these sections. An algorithm based on these arguments is developed and it is justified by a numerical example.

2. Concept of Single Machine

Now days, in competitive and flexible market, installing machine is very expensive, as the technology changes very frequently and the out dated machine can't satisfy the demands of the modern market. Secondly installing of more than one machines of the same type can speed up the work but needs more and more maintenance and supervision. Thirdly, installation of machines demands for more space to install, which also increases the idle cost of the project. So to reduce the expenditure, contractor wishes to process the work on single machine using an intelligent scheduling system and for the small systems single machine maximizes the profit of the whole project.

3. Assumptions and Notations

The single machine scheduling problem requires *n*-independent jobs $J_i (i = 1,2,3\ldots n)$ to be processed on a single machine with the following common assumptions [7]:

i. Only one job can be processed on a given machine at a time.
ii. All jobs are available for processing at time zero.
iii. The single machine can process at most one job at a time.
iv. The processing times are fixed and sequence independent.
v. The processing order of each job is given and fixed.

Notations:

S = Schedule for the n jobs.

a, b, c = Processing time of job i on the machine in fuzzy environment

A_i = A.H.R of the processing time a, b, c = [3b+ (c-a)] /3.

d_i = Due date for the job i.

C_i = Completion time of job

T_i = Max. $(0, c_i - d_i)$

E_i = Max. $(0, d_i - c_i)$

Sl_i = Slack time of job i

e_i = Penalty per unit time for the earliness of job i.

l_i = Penalty per unit time for the tardiness of job i.

4. Formulation of Single Machine Scheduling Problem

Let us consider the following single machine scheduling problem. There are a set of *n* jobs (Tasks) $\{J_1, J_2, \ldots, J_n\}$ to be scheduled non-preemptively for processing on a single machine. The machine is continuously available from $t = 0$, and can process only one job at a time. The jobs are also continuously available from $t = 0$, and require positive processing times $\{P_1, P_2, \ldots, P_n\}$ and Due Date (D.D) d_i are associated with each job J_i are positive integers. As completion time of job J_i passes between due date d_i and late date d^*_i, the customer satisfaction decreases until it vanishes in the delay case. The greater the delay, the lower the satisfaction.

Let C_i denotes completion time of job J_i and membership function $\mu_i(C_i)$ denote degree of satisfaction with respect to C_i, which is defined as follows:

$$\mu_i(C_i) = \begin{cases} 1, & C_i \le d_i, \\ 1 - (C_i - d_i) / (d^*_i - d_i), & d_i < C_i < d^*_i, \\ 0, & d^*_i \le C_i. \end{cases} \quad (1)$$

If completion time C_i is before due date d_i, there is maximum degree of satisfaction that is 1. When completion time is after due date d_i, level of satisfaction will decrease to zero. Generally, completion time C_i is within the interval [0, 1] .

The concepts of fuzzy processing time, fuzzy due date, fuzzy precedence relation etc. are introduced by various researchers. V.S.Jadhav & Leug *et al.* [1,4,] discussed about earliness & lateness of jobs in flow shop scheduling. [12,] studied the job sequencing problem when job processing time is represented with fuzzy numbers [5,8]. Thus the fuzzy due date is directly related to the earliness and tardiness penalty in conventional scheduling problems. In this paper, different due dates for each of the jobs is considered. Next jobs are scheduled in increasing order of their slack time.

This paper investigates a different approach to single machine under fuzzy environment with bi-objective criteria. On one side it minimizes the penalty cost of the tardy jobs and on the other side it minimizes the total flow time of all the jobs.

Consider the processing time in fuzzy environment (a, b, c), which is real time situation and is defuzzified [9,12] by Average High Ranking Method.

$$AHR = [\ 3b + (c - a)] / 3. \tag{2}$$

Where, a, b, c is fuzzy processing time.

5. Nature of Fuzzy Processing Time

The processing time of a job can vary in many ways, may be due to environmental factor or due to the different work places. We find that when a contractor takes the work from a department, he/she calculates total expenditure at the time of allotment. But due to many factors like non availability of labor, weather not favorable, or sometimes abnormal conditions, cost may vary. Hence due to these reasons work may be completed late and creates due date problem i.e. order can't be delivered on time, on the other hand if the work completes before the due time it arises the inventory problem. So to overcome these factors, the processing time of a job considered here is in three situations- favorable (High), Normal (Medium) and worse (Bad) conditions. In this chapter, a new concept of different processing time of each job is considered which helps the contractor to estimate the cost of the work at the time of allotment. Here different due dates for each of the job be considered which meets the demand maker with more satisfaction level. So using the Average High Ranking Method (AHRM) algorithm developed contractor can save the penalty cost and can satisfy the demand maker to great extent.

6. Proposed Methodology

6.1. Fuzzy Membership Function

An important special case in the family of E/T problems involves minimizing the sum of absolute deviations of job completion time form a DDD having processing time in fuzzy environment. In particular, the objective function can be written as:

$$f\ (s)\ = \Sigma\ \mid C_i - d_i \mid = \Sigma\ \left(E_i + T_i \right) \tag{3}$$

When we write the objective function in this form, it is clear that earliness and tardiness are penalized at the rate e_i and l_i for all the jobs. The processing time of the jobs considered in triangular fuzzy number (TFN) environment V.S.Jadhav *et al*., Abbasbandy *et al.* [1,2,3]:

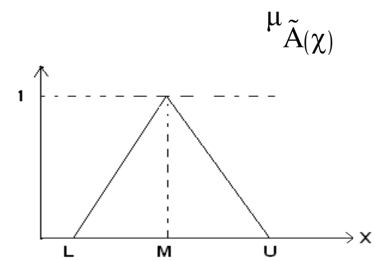

Fig 1: *The membership function of the triangular fuzzy number*

A TFN $\tilde{A}$ is specified by the triplet (a_1, a_2, a_3) and is defined by its continuous membership function $\mu_{\tilde{A}}(x): X \rightarrow [0,1]$.

6.2. Average High Ranking Method (AHRM) Algorithmic Approach

Step 1: Find Average High Ranking (AHR) of the fuzzy processing time (a, b, c) of all the jobs.

Step 2: Find the slack time of all the jobs $Sl_i = \mid A_i - d_i \mid$

Step 3: Arrange the jobs in increasing order of their slack time. If two jobs have the same slack times then consider the jobs of lowest processing time at the earlier position.

Step 4: Using the sequence obtained in step 3, find the total penalty of all the jobs using earliness (e_i) and lateness (l_i) penalty cost.

7. Numerical Example

Consider 7- jobs having fuzzy processing time, single machine and distinct due dates. Penalty cost (e_i) for earliness is also given in table 1.

Table 1

Job	P_i	AHR	D	Sl_i	e_i	l_i
1	3,4,6	5	6	1	2	3
2	5,7,9	25/3	8	1/3	2	3
3	11,13,15	43/3	9	16/3	2	3
4	7,9,11	31/3	12	5/3	2	3
5	6,8,10	28/3	10	2/3	2	3
6	8,9,11	10	17	7	2	3
7	12,14,16	46/3	12	10/3	2	3

7-Jobs having fuzzy processing time (a, b, c) are converted in to Average High Ranking (AHR) by using equation (2) i.e. AHR = [3b + (c - a)] /3 and as per mentioned in algorithm steps, the near optimal schedule is:

$$S = J_2 > J_5 > J_1 > J_4 > J_7 > J_3 > J_6$$

This is the final schedule for 7 – jobs having fuzzy processing time and fuzzy due date of customers.
Shown in table 2.

Table 2

Job	Processing Time	D	Sl_i	Cost
2	0-25	8	1/3	1/3*2
5	25/3 - 55/3	10	2/3	2/3*3
1	55/3 - 70/3	6	1	1*3
4	70/3 - 101/3	12	5/3	3/3*3
7	101/3 - 49	12	10/3	10/3*3
3	49 - 190/3	9	16/3	16/3*3
6	190/3 - 220/3	17	7	7*3

Total Penalty Cost is = 55.67 *

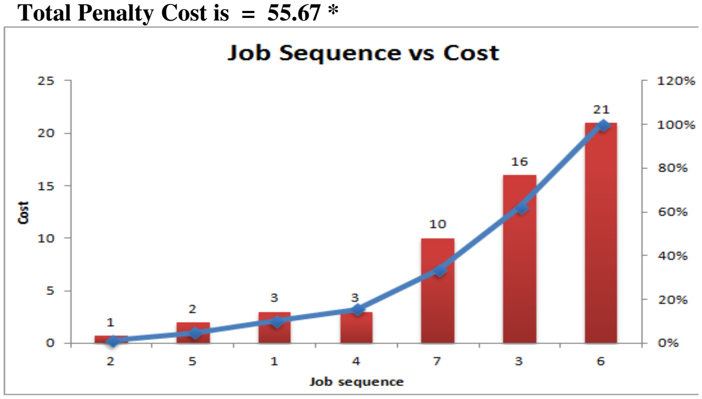

Figure 2: *Pareto Chart on Job Sequence Vs Cost*

8. Conclusion

We considered single machine scheduling problem (SMSP) with fuzzy processing time and fuzzy due dates. To minimize the total penalty cost for each job being late (the penalty is independent of the magnitude of the lateness). We have shown how to determine the optimal schedule according to the proposed fuzzy AHR algorithm approach. This method is very easy to understand and will help the decision maker in determining a best schedule for a given sets of jobs effectively to control penalty cost and provide a situation of job schedule.

References

1. V.S.Jadhav & V.H.Bajaj,(2012),"Flow-Shop Scheduling Problem Using Fuzzy Application" Int. Jr. of Agricult. Stat. Sci ; Vol.8, No.2, Dec-2012, pp. 405 - 413.
2. Abbasbandy S, Hajjari T; (2009), "A new approach for ranking of trapezoidal fuzzy numbers", Computer Math Appl, 57: 413- 419.
3. R. Tavakkoli-Moghaddam, G. Moslehi, et. al., (2005), "Optimal Scheduling for a Single Machine to Minimize the Sum of Maximum Earliness & Tardiness Considering Idle Insert", Applied Mathematics & Computation, Vol. 167, No. 2, pp. 1430 – 1450.
4. Leung, J. (Editor), (2004), "Handbook of Scheduling: Algorithms, Models, and Performance Analysis", (Chapman & Hall/ CRCRC Press: Boca Raton, Florida).
5. H.Ishii & M.dada, (1995),"Single Machine scheduling with fuzzy precedence relation", Europen Journal of Operation Research; 87, 284 - 288.
6. Slowinski, R; Hapke, M. (Editors),(2000),"Scheduling Under Fuzziness", (Physica Verlag: Heideberg).
7. V. S. Jadhav & V. H. Bajaj,(2011), "Application of Fuzzy TOPSIS Method for Solving Job-Shop Scheduling Problem", Int. Journal of Operation Research & Optimization, Vol.2, No.2, Dec-2011, pp. 333-342.
8. Chong T.C.E, (1995),"Optimal due date determination & sequence of n-jobs on a single machine", Journal Op. Res Soc;.35,(5) pp - 433.
9. C.S.Mc. Cahon & E.S.Lee; (1992)," Fuzzy Job Sequencing for a Job Shop", Europen Journal of Operation Research; 62, 294 -301.
10. Ramkumar R, A. Tamilarasi & T. Devi,(2012),"A Real Time Practical Approach for Multi-objective Job Shop Scheduling using Fuzzy Logic Approach", Journal of Computer Science, 8(4): PP. 606 – 612.
11. Ghorbanali Mohammadi, (2011), "Single Machine Common Flow Allowance Scheduling with a Fuzzy Rate-Modifying Activity", Indian Journal of Science & Tech., Vol.4, No.7, PP.726 -730.
12. V.S. Jadhav & V.H. Bajaj, (2012), "Selection of Job Shop Scheduling Problem Using Fuzzy Linguistic Variables", Int. Jr. of Statistika & Mathematika,Vol.3,Issue-3, 2012, pp.1-5.