Additive Fuzzy Multiple Goal Programming Model for the Unbalanced Transportation Problem with Hyperbolic Membership Function

Bodkhe S. G.[1], Omprakash S. Jadhav[2]

[1]Department of Statistics A. S. C. College, Badnapur, Jalna, Maharashtra, INDIA.

[2]Department of Statistics Dr. B. A. M. University, Aurangabad, Maharashtra, INDIA.

Corresponding Addresses:

[1]sgbodkhe@gmail.com, [2]drjadhav@gmail.com

Research Article

Abstract: In this paper, introduces a fuzzy goal programming approach to an unbalanced transportation problem with additive multiple fuzzy goals, when the goals are considered to be of equal importance. But in reality all goals may not be of equal importance. We have discussed assigning interval weights to goals, suggested by Narasimhan [11]. And those problems can be solved by non-linear membership function.

Keywords: unbalanced transportation problem, fuzzy nonlinear membership function, multiple goal programming.

1. Introduction

Goal programming (GP) is a suitable method to solve Multi criteria Decision Making (MCDM) problems with multiple conflicting objectives. The detailed analysis of GP has been given by Lee [9], Ignizio [5, 6, 7], and Romero [12]. Goal Programming is being used in multicriteria decision making problems where the alternatives cannot be compared on the basis of a single performance criterion. In goal formulation, the aspiration level is fixed and the decision maker has no control over the deviation from the aspiration level. To overcome these difficulties, we have investigated Fuzzy Goal Programming (FGP) of an unbalanced transportation problem with multiple fuzzy goals where all the goals are equality important. To reflect the equal importance of the goals, equal weights are assigned to them. In most of the MCDM problems in the real world, the articulation of the goals and objectives of the decision maker are fuzzy in nature. Bellman and Zadeh [1] dealt with the problems involving decision making under fuzzy environment. The application of the fuzzy set theory to GP has been made by Narasimhan [11] and Hannan [3, 4].Tiwari et al.[14] have investigated how the preemptive priority structure can be used in FGP problems. Tiwari et al. [15] have developed an additive model for fuzzy goal programming problems.

In many instances, the decision maker finds multiple conflicting objectives in a transportation problem. In such cases, the usual transportation method cannot be used. It is possible to apply the GP approach to such transportation problems. Lee and Moore [10] have shown the application of goal programming to multi-objective transportation problem. Kwak [8] considered a GP model for improved transportation problem. Weighted goal programming for unbalanced single objective transportation problem with budgetary constraint has been discussed by Singh and Kishore [13]. Bit et al [2] have presented a transportation problem model for the allocation of coal and its by products (soft coke, middling, and washed coal) from different sources and converting plants of coal mines to different consumption sites. The model has been formulated to meet the energy demand, and to minimize the transportation problem cost. A case study is presented and the problem is solved using goal-programming method in order to find a satisfactory solution.

In many decision-making situations, the decision maker faces an unbalanced transportation problem in which total supply is less than the total demand. Due to the scarcity of funds, the decision maker is not able to satisfy all the demand points fully with the existing availability. In such cases the solution is affected by the multiple objectives. This type of problem is faced by the government agencies like Food Corporation of India, (FCI) which supplies food grains from different warehouses (i.e., source points) to different distribution centers (i.e., demand points). Although the quantity of food grains supplied is not sufficient to meet the demand of each distribution center. It is not possible for the FCI to transport even the available capacity due to paucity of fund. It is very difficult to supply food grains even to all remote places where multiple objective values are very high due to lack of approachability to such places. But it is essential to supply a certain percentage of the demand to save the people from starvation.

2. Unbalanced Transportation Problem with Multiple Fuzzy Goals

In a typical transportation problem, a homogeneous product is to be transported from each of m sources to n destinations. The sources are

production facilities, warehouses, or supply point, characterized by available capacities a_i (i = 1,2,…, m). The destinations are consumption facilities, warehouses, or demand points, characterized by required levels of demand b_j (j = 1,2, …, n), A penalty C^p_{ij} is associated with transportation of a unit of the product from sources i to destination j corresponding to the p-th criterion. The penalty could represent transportation cost, delivery time, deterioration of total goods, quantity of goods delivered, under used capacity, etc. A variable X_{ij} represents the unknown quantity to be transported from origin O_i to destination D_j. In the real would, however, Multi-objective transportation problems are not balanced and objectives are not precisely specified (i.e., goals are fuzzy in nature). We have formulated the FGP model of an unbalanced multi-objective transportation problem in which the objectives and demands are specified imprecisely. The decision variable, supply constraints, fuzzy demand goals and fuzzy multiple goal are identified as follows:

- Decision variable:
  Decision variables for the model are defined as
  x_{ij}, i = 1,2,…,m; j = 1,2,…,n
  where $x_{ij} \geq 0$ for all i and j
- System or supply constraints:
  $\sum_{j=1}^{n} x_{ij} \leq a_i$, i = 1,2,…,m
  where $a_i > 0$ for all i
- Fuzzy demand goals:
  $\sum_{i=1}^{m} x_{ij} \gtrsim b_j$, j = 1,2,…,n
  where $b_j > 0$ for all j.

The symbol $\gtrsim$ stands for approximately or fuzzily nearly greater than or equal to.

- fuzzy budget goals:
  $Z_p = \sum_{i=1}^{m}\sum_{j=1}^{n} c^p_{ij} x_{ij} \lesssim B^p$, p=1,2,...,P

Where the superscript p denotes the p-th penalty criterion, B^p is the aspiration level of the pth objective, and the symbol $\lesssim$ stands for approximately or fuzzily nearly less than or equal to.

3. Mathematical Model

Mathematical model of an unbalanced transportation problem with multiple fuzzy goals is stated as follows:

Find x_{ij}, i = 1,2,…, m; j = 1,2,…,n so as to satisfy the following

constraints and fuzzy goals:

$$\sum_{j=1}^{n} x_{ij} \leq a_i, \quad i = 1,2,..., m$$

$$\sum_{i=1}^{m} x_{ij} \gtrsim b_j, \quad j = 1,2,\ldots,n$$

$$Z_p = \sum_{i=1}^{m}\sum_{j=1}^{n} c^p_{ij} x_{ij} \lesssim B^p, \quad p=1,2,...,P \tag{1}$$

$$x_{ij} \geq 0, \; a_i > 0, \; b_j > 0 \quad \text{for all i, and j}$$

$$\sum_{j=1}^{n} b_j \neq \sum_{i=1}^{m} a_i$$

In an unbalanced problem, the total market requirements either exceed the total plant capacity $\left(\text{i.e., } \sum_{j=1}^{n} b_j > \sum_{i=1}^{m} a_i\right)$ or less than the total plant capacity $\left(\text{i.e., } \sum_{j=1}^{n} b_j < \sum_{i=1}^{m} a_i\right)$. In our unbalanced transportation model, we have considered $\left(\text{i.e., } \sum_{j=1}^{n} b_j > \sum_{i=1}^{m} a_i\right)$.

4. Solution Procedure for the Proposed Model

We convert the model to a linear programming model by using non-linear membership functions and max-min operator. The solution of linear programming model gives an efficient solution for the fuzzy goal-programming model.

Define a hyperbolic membership function of the fuzzy demand goals are as follows:

$$\mu_{A_j}(x) = \begin{cases} 1 & \text{if } \sum_{i=1}^{m} x_{ij} \geq b_j \\ \frac{1}{2}\tanh\left(\left(\sum_{i=1}^{m} x_{ij} - \frac{b_j + b_j^*}{2}\right)\alpha_p\right) + \frac{1}{2}, & \text{if } b_j^* < \sum_{i=1}^{m} x_{ij} < b_j \\ 0 & \text{if } \sum_{i=1}^{m} x_{ij} \leq b_j^* \end{cases} \tag{2}$$

Where $\alpha_p = 6/(b_j - b_j^*)$

where b_j^* (j =1,2,…,n) is the lower tolerance limit of the j-th demand goal.

The membership functions of the multiple fuzzy goals are defined as

$$\mu_{A_{n+p}}(x) = \begin{cases} 1, & \text{if } \sum_{i=1}^{m}\sum_{j=1}^{n} c^p_{ij} x_{ij} \leq B^p \\ \frac{1}{2}\tanh\left(\left(\frac{B+B^*}{2} - \sum_{i=1}^{m}\sum_{j=1}^{n} c^p_{ij} x_{ij}\right)\alpha_p\right) + \frac{1}{2}, & \text{if } B^p < \sum_{i=1}^{m}\sum_{j=1}^{n} c^p_{ij} x_{ij} < B^p \\ 0, & \text{if } \sum_{i=1}^{m}\sum_{j=1}^{n} c^p_{ij} x_{ij} \geq B^p \end{cases} \tag{3}$$

Where Where $\alpha_p = 6/(B^* - B)$

Where B^* is the upper tolerance limit of the budget goal.

Use the max-min operator

As is well known problem (1) is equivalent to solving the following

Nonlinear Programming.

Max λ

$\lambda \le \mu_{A_j}(x), \quad j=1,2,...,n$

$\lambda \le \mu_{A_{n+p}}(x), \quad p=1,2,...,p$

Where

$$\lambda \le \text{Min}\left[\frac{1}{2}\tanh\left(\left(\left(\sum_{i=1}^{m} x_{ij} - \frac{b_j+b_j^*}{2}\right)\right)\alpha_p\right)+\frac{1}{2}, \quad \frac{1}{2}\tanh\left(\left(\frac{B+B^*}{2} - \sum_{i=1}^{m}\sum_{j=1}^{n} c^p_{ij}x_{ij}\right)\alpha_p\right)+\frac{1}{2}\right] \tag{4}$$

An equivalent nonlinear programming model of the unbalanced transportation problem with multiple fuzzy goals (1) can be stated as follow:

Max λ

Subject to $\sum_{j=1}^{m} x_{ij} \le a_i, \quad i=1,2,...,m$

$$\begin{aligned} &\lambda \le \mu_{A_j}(x), \quad j=1,2,...,n \\ &\lambda \le \mu_{A_{n+p}}(x), \quad p=1,2,...,n \\ &x_{ij} \ge 0 \text{ for all i, and j}, \lambda \ge 0 \end{aligned} \tag{5}$$

5. Interval weights with FGP

We formulate the initial fuzzy goal-programming model of unbalanced multi-objective Transportation problem stated in (6.1), and then transform it into interval weights FGP model (i.e., a linear programming problem)

The steps are as follows:

Step 1 Construct the membership function for the fuzzy demand goals and multiple Fuzzy goals is defined as (6.2) and (6.3)

Step 2 Use the max-min operator

Step 3 Formulate a linear programming problem equivalent to problem (6.1). By using the max-min operator, the overall achievement function is defined as 6.4) An equivalent linear programming model of the unbalanced transportation problem with multiple fuzzy goals (6.1) is defined as (6.5)

Step 4 The equivalent interval weights fuzzy goal programming model of the unbalanced Multi-objective transportation problem, stated in membership function of the demand goals and membership function of the multiple fuzzy goals. Let (W_j, w_j*) are the interval weights associated with the membership functions of demand goals μ_{Aj}, for j = 1,2,…, n and (W_{n+1}, W^*_{n+P}) is interval weight associated with the membership function of the multiple fuzzy goals.

Let us denote the weighted membership function of demand goals as μ_{wj}, J = 1,2,…, n is defined as follows:

$$\mu_{w_j} = \begin{cases} \dfrac{\mu_{A_j} - w_j}{w_j^* - w_j}, & w_j \le \mu_{A_j} \le w^* \\ 0, & \text{otherwise} \end{cases} \tag{6}$$

Or

$$\mu_{w_j}(x) = \begin{cases} \dfrac{(\sum_{i=1}^{m} x_{ij} - b_j)/(b_j - b^*_j) - w_j}{w^*_j - w_j}, & w_j \le (\sum_{i=1}^{m} x_{ij} - b_j)/(b_j - b^*_j) \le w^*_j \\ 0, & \text{otherwise} \end{cases}$$

The weighted membership functions of multiple fuzzy goals μ_{wn+p},

Can be defined as:

$$\mu_{w_{n+p}}(x) = \begin{cases} \dfrac{\mu_{A_{n+p}} - w_{n+p}}{w^*_{n+p} - w_{n+p}}, & w_{n+p} \le \mu_{A_{n+p}} \le w^*_{n+p} \\ 0, & \text{otherwise} \end{cases} \tag{7}$$

which is equivalent to

$$\mu_{w_{n+p}}(x) = \begin{cases} \dfrac{B^p_u - \sum_{i=1}^{m}\sum_{j=1}^{n} c^p_{ij}x_{ij}/(B^p_u - B^p) - W_{n+p}}{W^*_{n+p} - W_{n+p}}, & \text{if } W_{n+p} \le B^p_u - \sum_{i=1}^{m}\sum_{j=1}^{n} c^p_{ij}x_{ij}/(B^p_u - B^p) \le W^*_{n+p} \\ 0, & \text{if otherwise} \end{cases}$$

Thus the weighted structure with interval weights of the model (6.1) is formulated as follows:

Max λ

Subject to $\sum_{j=1}^{m} x_{ij} \le a_i, \quad i=1,2,...,m$

$$\begin{aligned} &\lambda \le w\mu_{A_j}(x), \quad j=1,2,...,n \\ &\lambda \le w\mu_{A_{n+p}}(x), \quad p=1,2,...,P \\ &x_{ij} \ge 0 \text{ for all i and j} \\ &\lambda \ge 0 \end{aligned} \tag{8}$$

where,

$$\lambda = \underset{j}{\text{Min}}\left[\frac{\sum_{i=1}^{m} x_{ij} - b_j/(b_j - b_j^*) - w_j}{w_j^* - w_j}, \frac{B^p_u - \sum_{i=1}^{m}\sum_{j=1}^{n} c^p_{ij}x_{ij}/(B^p_u - B^p) - W_{n+p}}{W^*_{n+p} - W_{n+p}}\right]$$

Which is linear programming problem can be solved by an appropriate linear programming algorithm.

6. Conclusion

In fuzzy goal programming formulation the aspiration levels of the fuzzy goals are fixed. The deviations from the goals in the fuzzy goal programming are under the control of the decision maker. Thus the present method is also suitable method for unbalanced transportation problem with fuzzy goals.

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