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A Size-Biased Probability Model on Rural Out-Migration

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Research Article

Abstract: This paper deals with the study of probability model for rural out-migration through the size-biased probability model based on certain assumptions. Expressions for the parameters involved in this model have been estimated by the suitable estimation techniques. Various set of data from RDPG for Uttar Pradesh has been used to discuss the applicability of the proposed model.

Key Word: Sized-Biased Poisson distribution, Risk of Migration, Households, Probability model, Estimation Technique, Migrants.

1. Introduction

The migration of human being is not an immediate action but it is generated by simple or single impulse that may vary from one person to another. When an individual or a family changes residence, the move is made for certain reasons and the choice of destination is made on some logic and linings. Due to these reasons, sometimes, it may not be even clear perceptive but may be the result of economic, social, political, ethic or other factors. These causes may occur in different combinations also. The micro level research on both residential mobility and migration has played a decisive role in the development of theory of migration (Bilsborrow *et al.* 1987; Dejong *et al.* 1981; Singh *et al.* 1982; Singh and Yadava, 1981) and investigation of factors affecting movement process. In this respect Singh *et al.* (1985), Ojha and Pandey (1991) Pandey *et.al.* (2012) suggested various probability models for the out-migrant. In this present paper a size-Biased probability model is develop for the number of male migrants.

2. Model

If $\omega(x,\beta) = X$, $x^{\omega} = X$ is called size-biased of x and its distribution is called the size-biased distribution with probability function as:

$$f^*(x,\theta) = \frac{x.f(x,\theta)}{\mu}$$

Where $\mu = E(X)$

f^* is the size-biased from of f.

Let X denote the random number of rural out-migration from a household. Probability model for describing the variation in the number of single male migrants has been obtained on the basis of the following assumption.

1. The risk of migration occurs at the time of survey due to their establishment or disestablishment in the village/ at the origin. Let α be the probability that a household is exposed to the risk of migration at the time of survey and $(1-\alpha)$ be the probability that a household is not exposed to the risk of migration.
2. x male migrating from a household is follows a size-biased Poisson distribution with parameter λ, i.e.

$$f^*(x,\lambda) = \frac{e^{-\lambda}\lambda^{x-1}}{(x-1)!} \quad \text{, where } x = 1,2,3, \ldots$$

From assumption (1) and (2), the model given by the probability function P_x as follows.

$$P(X=x) = 1-\alpha \qquad ,x = 0$$

$$P(X=x) = \frac{\alpha e^{-\lambda}\lambda^{x-1}}{(x-1)!}, x = 1,2,3, \ldots \qquad (2.1)$$

3. Estimation

Method of Moment: The proposed probability model (2.1) consist two parameters i.e. α and λ. The parameter are estimated by equating zeroth cell theoretical frequencies to the observed frequencies and observed mean with their corresponding theoretical values which gives the following relations.

$$1-\alpha = f_0/f \qquad (3.1)$$

$$Mean = \bar{X} = E(X) = \alpha(1+\lambda) \qquad (3.2)$$

Where $f_0 =$ Number of observations in the zeroth cell

$f =$ Total Number of observation

$\bar{X} =$ Observed mean

Maximum Likelihood Method: Consider a sample consisting of f observations of the random variable X with probability function (2.1) in which f_0 designates the number of zeroth observation; f_1 the number of one observation and f the total number of observations. Now for the model likelihood function takes the following form.

$$L = (1-\alpha)^{f_0}(\alpha e^{-\lambda})^{f_1}\{\alpha(1-e^{-\lambda})\}^{f-f_0-f_1} \quad (3.3)$$

Taking logarithmic on both sides we get

$$\log L = f_0 \log(1-\alpha) + f_1 \log(\alpha e^{-\lambda}) + (f - f_0 - f_1)\log\{\alpha(1-e^{-\lambda})\} \quad (3.4)$$

Differentiating partiality w.r.to α and λ and equating it to be zero.

$$\frac{\partial \log l}{\partial \alpha} = \frac{-f_0}{(1-\alpha)} + \frac{(f-f_0)}{\alpha} = 0 \quad (3.5)$$

$$\frac{\partial \log l}{\partial \lambda} = -f_0 + \frac{(f-f_0-f_1)e^{-\lambda}}{(1-e^{-\lambda})} = 0 \quad (3.6)$$

The required Maximum Likelihood Estimates α and λ can be obtained by simultaneously solving (3.5) and (3.6). To facilitate their solutions, the above equations are reduced to

$$\alpha = \frac{(f-f_0)}{f}$$

$$e^{-\lambda} = \frac{f_1}{(f-f_0)}$$

The asymptotic variances of (α, λ) is obtained by inverting the information matrix whose elements are negatives of expected values of the second order derivatives of logarithmic of the likelihood function. The second derivatives of L follows from (3.5) and (3.6) as:

$$\frac{\partial^2 \log l}{\partial \alpha^2} = \frac{-f_0}{(1-\alpha)^2} - \frac{(f-f_0)}{\alpha^2} \quad (3.7)$$

$$\frac{\partial^2 \log l}{\partial \lambda^2} = -f_0 + \frac{(f-f_0-f_1)e^{-\lambda}}{(1-e^{-\lambda})} \quad (3.8)$$

$$\frac{\partial^2 \log l}{\partial \alpha \partial \lambda} = \frac{\partial^2 \log l}{\partial \lambda \partial \alpha} = 0 \quad (3.9)$$

Now using the fact

$$E(f_0) = f(1-\alpha)$$

$$E(f_1) = f\alpha e^{-\lambda}$$

$$E(f - f_0 - f_1) = f\alpha(1-e^{-\lambda})$$

Where E denote for the expectation.

The elements of information matrix follows from (3.7), (3.8) and (3.9) as:

$$\phi_{11} = E\left(\frac{\partial^2 \log l}{\partial \alpha^2}\right)/f = \left[\frac{1}{1-\alpha} + \frac{1}{\alpha}\right]$$

$$\phi_{22} = E\left(\frac{\partial^2 \log l}{\partial \lambda^2}\right)/f = \left[\frac{\alpha e^{-\lambda}}{(1-e^{-\lambda})}\right]$$

$$\phi_{12} = E\left(\frac{\partial^2 \log l}{\partial \alpha \partial \lambda}\right)/f = \phi_{21} = E\left(\frac{\partial^2 \log l}{\partial \lambda \partial \alpha}\right)/f = 0$$

On inverting the information matrix, the asymptoticvariances as;

$$V(\hat{\alpha}) = \left[\frac{\phi_{22}}{\phi_{11}\phi_{22} - \phi_{12}^2}\right]/f \quad (3.10)$$

$$V(\hat{\lambda}) = \left[\frac{\phi_{11}}{\phi_{11}\phi_{22} - \phi_{21}^2}\right]/f \quad (3.11)$$

Application

The proposed size-biased probability model (2.1) has been applied to the migration data taken from the two surveys i.e. "Demographic survey of Varanasi (Rural) 1969" and "Rural development and population Growth 1978" both were conducted by the Centre of Population Studies Varanasi (India). From table I and II, it is observed that the value of χ^2 is found to be insignificant for set of migration data at 5% level of significance. This shows that the proposed model describe satisfactorily well to the rural out-migration. Thus the present model may be taken as useful tool in calculating the various probabilities of migrants connected with the process of migration from a household and also for predictions in a specified population. From the estimated value of the proposed model, it is conclude that the risk of migration α is higher in the family which has higher household size group i.e. (7-9) and the average number of migration is also higher in these families.

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Table 1

Number of Migrants	**Household Size Group**					
	(4-6)			**(7-9)**		
	Observed	**Expected**		Observed	**Expected**	
		M.M.	M.L.E.		M.M.	M.L.E.
0	651	651.00	651.00	362	362.00	362.00
1	120	119.15	120.00	114	110.50	114.00
2	28	29.01	28.37	39	45.64	43.53
3	3			12	9.42	8.31
4	1			2		
5	0			0		
6	0	3.84	3.63	0		
7	0			0	1.44	1.16
8 and over	0			0		
Total	803	803.00	803.00	529	529.00	529.00
$\hat{\alpha}$		0.1893	0.1893		0.3157	0.3157
$\hat{\lambda}$		0.2435	0. 2364		0.4130	0.3818
$Var(\hat{\alpha})$			0.00019			0. 00041
$Var(\hat{\beta})$			0.00175			0.00278
χ^2		0.0484	0.0425		1.9948	2.7182
$d.f.$		1	1		2	2

Observed and Expected Number of Households According to the Number of Migrants for Different Household Size Group (1969 Survey).

Table 2

Number of Migrants	**Household Size Group**					
	(4-6)			**(7-9)**		
	Observed	**Expected**		Observed	**Expected**	
		M.M.	M.L.E.		M.M.	M.L.E.
0	388	388.02	388.02	188	188.00	188.00
1	61	61.88	60.98	61	59.72	61.00
2	12	10.20	10.95	17	18.94	18.94
3	0			3		
4	0			1		
5	0			0		
6	0	0.8896	1.05	0	3.34	2.96
7	0			0		
8 and over	0			0		
Total	461	461.00	461.00	270	270.00	270.00
$\hat{\alpha}$		0.1583	0.1583		0.3037	0.3037
$\hat{\lambda}$		0.1649	0.1796		0.3171	0.2958
$Var(\hat{\alpha})$			0.00029			0. 00078
$Var(\hat{\beta})$			0.00269			0.00420
χ^2		1.2181	1.1507		0.3531	0.4253
$d.f.$		1	1		1	1

Observed and Expected Number of Households According to the Number of Migrants for Different Household Size Group (1978 Survey).