FLC Modeling of Classical EEG Signals Model by the Technique Tsukamoto Fuzzy Rule Base

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Research Article

Abstract: In this paper we consider a system (plant) of "Hodgkin-Hoxley classical mathematical model of EEG signals" as an input-output map $y = f(x)$. We assume that the internal structure of this system is unknown, but qualitative knowledge about the behavior is available in the form of "If - Then" rules. We construct a mathematical description of the system, based on available information, so that it will represents faithfully the true system of "Tsukamoto Fuzzy Control Model". The construction process consists of translating linguistic rules into mathe-matical expression using fuzzy sets and fuzzy logic with the technique of Tsukamoto fuzzy inference rules so that desired output (o/p) result is achieved. In essence Tsukamoto Fuzzy Controlled Model is constructed by fusing multiple local models that associated with fuzzy subspaces of the given inputs (I/Ps) space. These I/Ps are nothing but I/Ps of the classical EEG signal model. Furthermore the set of fuzzy I/Ps subspaces form a fuzzy decomposition of the I/Ps space. Finally the result of fusing multiple local models (in terms of Fuzzy "If - Then" rules) by the technique of Tsukamoto Fuzzy Rule Base method gives a final output result which is equivalent to the final output result of classical EEG signal model. The obtained fuzzy controlled system is shown to be within the class of designs capable of approximating the true input - output relation to the required degree of accuracy.

Key words: Mathematical model of EEG signals, inputs - output linguistic variables, Tsukamoto fuzzy inference rules, weighted average formula.

2000 Mathematics Subject Classification: Primary: 94C42, Secondary: 68T27, 68T37.

1 Introduction

The conventional PID controller EEG signal model: A conventional (classical) proportional-integral-derivative (PID) controller of Hodgkin-Huxley mathematical model of EEG signal is based on a rigorous mathematical model of some linear process. This model uses a set of equations that describes the stable equilibrium state of the control surface through coefficients originated to the PID aspect of the system. Conventional controller reads a sensor value, applies mathematical model and produces desired output by the mathematical algorithm. It is to be noted that the conventional mathematical EEG signal model is deceptively complex. It run up against computationally complex problems that they simply could not address without consuming prohibitive amount of computer power - if they address them at all. Hence need of Tsukamoto Fuzzy Controlled Model is essential. Since fuzzy system is universal approximator and is well suited to modeling highly complex system, it is able to approximate the behavior of system displaying a variety of poorly understood and /or linear nonlinear properties. Fuzzy rule based system usually execute faster than conventional rule based system and requires fewer rules with the ability to explain their reasoning, it provides an ideal way of addressing these difficult problems.

Fuzzy control model: Fuzzy logic controller (FLC) serves the same function as the conventional PID controller. PID manages a complex control surface by reading sensor information, executing a mathematical model and making changes to the device actuators. However the fuzzy logic controller manages this complex control surface through heuristic rather than a mathematical model. Further a fuzzy system is able to approximate to any level of precision to any continuous linear/non-linear function. A fuzzy controller is fuzzy system model. It employs fuzzy sets to represent the semantic properties of each control rule and solution variable and processes its input-output using the set of production 'If-Then' rules that associates an input value, through a collection of fuzzy sets, into a desired output representation.

In the "Tsukamoto fuzzy reasoning method" the consequent part of each fuzzy "If-Then" rule is represented by a fuzzy set with a monotonic membership function as shown in **Figure 1**. Tsukamoto fuzzy model like a classical EEG signal model is based on the I/Ps process and output (O/P) flow concepts. This has two important benefits over classical EEG signal model: i) The model can generally be modified with fewer induced error; ii) The model can be located orfixed in a minimum amount of time.

Because of the practical merits, Tsukamoto fuzzy model have been recognized over classical EEG signal model. It has been applied very effectively (and efficiently) to provide O/P result which is as good as the O/P result of classical EEG signal model.

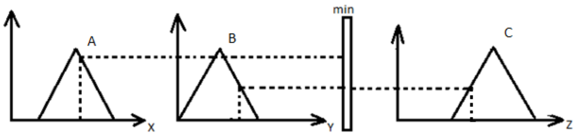

Figure 1: Tsukamoto fuzzy reasoning method representing monotonic consequent part and 'α_i' is the minimum matching degree between $A_i(x_0)$ and $B_i(y_0)$.

2 Classical Mathematical Model of EEG Signals

This EEG signal model is based on the Hodgkin - Huxley **Nobel prize** winning model for the squid axon published in1952[5].

2.1) Mechanism: A nerve axon may be stimulated and the activated sodium (Na^+) and potassium (k^+) channels produced in the vicinity of the cell membrane may lead to the electrical excitation of the nerve axon. Prominently, the electrical excitation arises: (a) from the effect of membrane potential on the movement of ions, and (b) from interaction of the potential with the opening and closing of voltage activated membrane channels. The membrane potential increases when the membrane polarized with a net negative charges lining in the inner surface and equal but apposite net positive charge on the outer surface. This potential (E) may be related to the amount of electrical charge (Q), using the relation,

$$E = \frac{Q}{c_m}, \tag{1}$$

where E, electrical potential (or membrane potential or electrical force) is measured in the unit of volts; Q, electrical charge is measured in terms of coulombs/cm^2; and Cm, is the measure of capacity of membrane in units of farad/cm^2.

In practice, in order to model the action potential (APs) the amount of charge Q^+ on the inner surfaces (and Q^- on the outer surface) of the cell membrane has to be mathematically related to the stimulating current (I_{steam}) flowing into the cell through the stimulating electrodes. The Hodgkin-Huxley model is shown in ***Figure 2***.

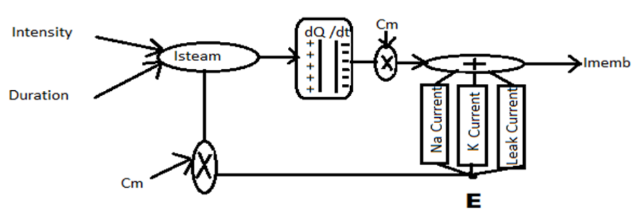

Figure 2: Hodgkin-Huxley excitation model.

In this ***Figure 2*** membrane current (I_{memb}) is the result of positive charges flowing out of cell. This current consist of three currents namely, sodium (Na), potassium (K) and leak currents (the leak current is due to fact that the inner and outer Na and K ions are not exactly equal).Hodgkin and Huxley estimated the activation and inactivation functions for the Na and K currents and derived a mathematical model to describe an action potential AP similar to that of a giant squid. The model is neuron model that usages voltage gated channels. This model describes the change in membrane potential (E) with respect to time. The overall membrane current is the sum of capacity current and ionic current as follows,

$$I_{memb} = c_m \frac{dE}{dt} + I_i \tag{2}$$

Where $\boldsymbol{I_i}$, is the ionic current as indicated in **Figure 3.** It consists of the sum of three individual components as follows,

$$I_i = I_{Na} + I_k + I_{leak} \tag{3}$$

where I_{Na}, can be related to the maximal conductance $\bar{g}_{Na}$; activation variable a_{Na};
inactivation variable h_{Na} and driving force $(E - E_{N_a})$ through

$$I_{N_a} = \bar{g}_{Na} h_{Na} (E - E_{N_a}) a^3 Na \tag{4}$$

Similarly I_k and I_{leak} can be described.

The change in the variables a_{Na}, a_k and h_{Na} vary from 0 to 1 (time in ms) according to the following equations:

$$\frac{d}{dt}(a_{Na}) = \lambda_t\, [\alpha_{Na}(E)(1-a_{Na}) - \beta_{Na}(E) a_{Na}] \tag{5}$$

where, $\alpha(E)$ and $\beta(E)$ are forward and backward rate functions respectively and λ_t is a temperature dependent factor. Similarly, $\frac{d}{dt}(h_{Na})$ and $\frac{d}{dt}(a_k)$ can be described. The forward and backward parameters were empirically estimated by Hodgkin and Huxley as follows:

$$\alpha_{Na}(E) = \frac{3.5+0.1E}{1-e^{-(3.5+0.1E)}}\ \beta_{Na}(E) = 4e^{\frac{-(E+60)}{80}}, \text{ etc.} \tag{6}$$

As stated in the simulator for neural network and action potential (SNNPA) $literature$[5]. The parameters $\alpha(E)$ and $\beta(E)$ have been converted from the original Hodgkin-Huxley version to a version agreeing with physiological practice taking depolarization of the membrane as positive. Resting potential has been shifted to -60mV (from original 0mV). A simulated action potential is illustrated in Figure 3. For this model, the parameters are set to be, $c_m = 1.1\mu F/cm^2$, $\bar{g}_{Na} = 100 ms/cm^2$,
$\bar{g}_k = 35\, ms/cm^2$, $\bar{g}_l = 0.35\, ms/cm^2$, d$E_{Na}$=60mV. Using the values of c_m, $\bar{g}_k$, g_l etc in the above related equations (1)-(6), one gets

$$\boldsymbol{I_{memb} = 80\mu A/cm^2} \text{ (see Figure 3)} \tag{7}$$

2.2) Brief algorithm of EEG signal modeling: The information transmitted by nerve in the central nerves system (CNS) is called an action potential (AP). APs are caused by an exchange of ions across the neuron membrane and are a temporary change in the membrane potential that transmitted along the axon. As soon as the stimulus strength goes above the threshold, an action potential appears and travels down the nerve. The membrane potential **depolarizes** (becomes more positive) producing spike. After the peak of the spike (having sodium (+) channels close and the potassium (+) open), the membrane potential **repolarizes** (becomes more negative). The potential becomes more negative than the resting potential is called **hyper polarization** and return to the normal called **resting potential a**s shown in **Figure 3**. ***It is important to note that the action potential of the most nerves system last up to 5 to 10ms.***

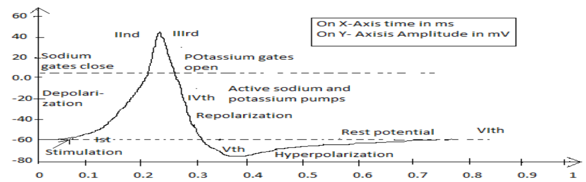

Figure 3: A single AP in response to a transient stimulation based on Hodgkin –Huxley model. The initiated time is t=0.4ms and the injected current i.e.$I_{memb} = 80\mu A/cm^2$ for duration of 0.1ms

This model is complex due to imprecise linguistic I/P-variables and coupling of different parameters. The technique of Tsukamoto-fuzzy controllers on EEG signal modeling is more convenient under these conditions.

3. Tsukamoto fuzzy controller on EEG signal Modeling

The system of the classical EEG signal model consist of two fuzzy I/ Ps intensity (I) and duration (τ) as the stimulator for dendrites of the nerve cell and one fuzzy o/p namely membrane current (I_{memb}) to be computed. A general scheme for controlling a desired value by the technique of Tsukamoto - FLC over the classical EEG signal model is shown in **Figure 4**.

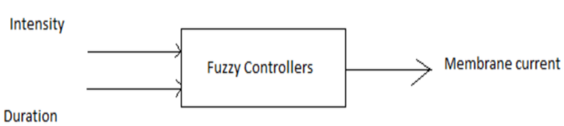

Figure 4: A general scheme of Tsukamoto - FLC for controlling desired value.

The general inference process based on the TSukamoto - FLC proceeds in four steps:

a) Construction of fuzzy sets and fuzzifications;
b) Formation of fuzzy inference rules;
c) Measurement of the adaptability and infer the conclusion and
d) Aggregate the individual conclusion to obtain the overall conclusion.

Step-(a) Construction of fuzzy sets and fuzzifications: After identifying the relevant *I/Ps* and *O/P* - variables of the classical controller, our first step in designing the FLC should be to characterize the range of values for the *I/Ps* and *O/P* - variables. Since the duration of the action potential of a nerve system in the classical controllers is in the range of 5 to 10 ms, so that we have chosen the range of values for the both *I/P* - variables, 'intensity' and 'duration' in the time interval of 0 to 10 ms in FLC. And since final injected current in EEG signal model is $\boldsymbol{I_{memb} = 80\mu A/cm^2}$**,** accordingly we have chosen range of values for *O/P*- variable 'membrane current' as 0 to 100 µA $/cm^2$ in FLC.

Further we have to select meaningful linguistic states for each of the three variable and express them by appropriate fuzzy sets. Accordingly we choose as: Negative Large(NL); Negative Medium (NM); Negative Slow(NS); Almost zero(AZ); Positive Slow(PS); Positive Medium(PM)and Positive Large(PL). We elaborate these seven linguistic verbal adjectives to their corresponding numerical descriptions as : "about and below 0.13 "; "about 0.26 "; "about 0.39 "; "about 0.52 ";"about 0.65 "; "about 0.78"; "about and above 0.91 " respectively.

Representing these seven linguistic states of I/P and O/P linguistic variables by triangular shape fuzzy numbers as in Figure 5 and Figure 6 respectively.

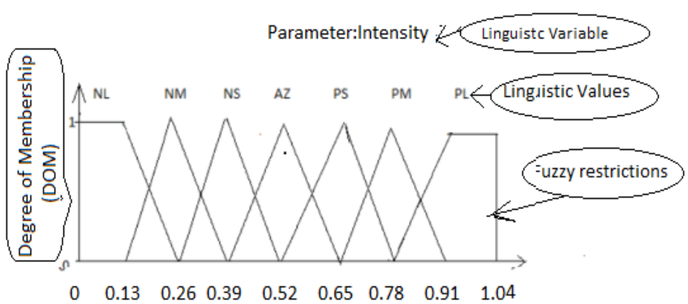

Figure 5: Fuzzy sets and decomposition for I/P variable intensity/ duration over the range [0, 1]-is the time in ms.

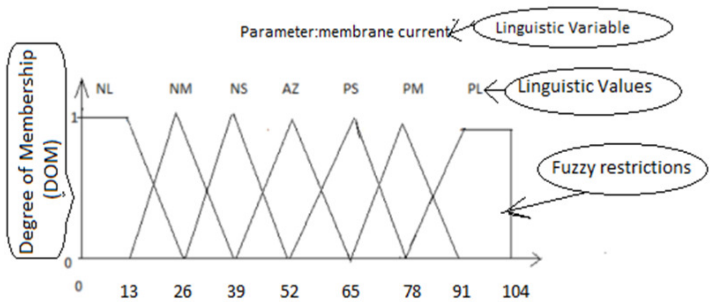

Figure 6: Fuzzy sets and decomposition for O/P variable 'membrane current' (I_{memb}) over the range [0,100] is the injected current in$\mu A/cm^2$

Fuzzification of I/P-variables: The main purpose of the fuzzification is to interpret measurement of I/P -variables (each expressed by the fuzzy approximation of the respective real number) and to express the associated measurement uncertainties. Let us consider an illustration. A fuzzification process (function) applied to the I/P - variable 'intensity' (I), is represented by f_I.Then the fuzzification function has the form$f_I: [0,1] \rightarrow R$, where R denote the set of all fuzzy numbers. Then $f_I(x_0 = 0.40)$ is a fuzzy number chosen by $f_I$as a fuzzy approximation of the measurement (sensor reading) intensity (I) at $x_0 = 0.40$.

The computation of fuzzy membership values from Figure 5, for which $f_I(x_0 = 0.40) \neq 0$, is as below and shown in Figure 7.

NS(0.40sec)= $\frac{0.40-0.52}{0.39-0.52} = \frac{0.12}{0.13} = 0.92$; AZ$(0.40sec) = \frac{0.40-0.39}{0.52-0.39} = \frac{0.01}{0.13} = 0.08$.

Remaining all fuzzy membership values (from Figure 5) are zero such as,

N L (0.40) = N M (0.40) = P S (0.40) = P M (0.40) = PL (0.40) = 0

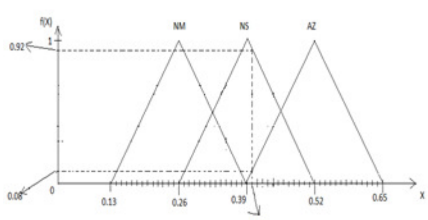

Figure 7

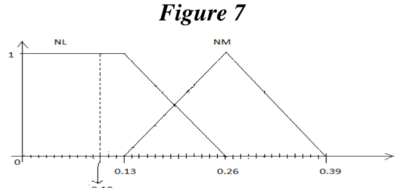

Figure 8

Figures (7 and 8): Fuzzification of I/P variables intensity for $x_0 = 0.40$ and duration $y_0 = 0.10$.

The computation of fuzzy membership values from Figure 5, for which $f_\tau(y_0 = 0.10) \neq 0$, is carried out as below and is as shown in Figure 8.

$NL(0.10) = 1$.

Remaining all memberships values from Fig(5) are zero such as *NS*(0.10) = *AZ*(0.10) = *PL*(0.10) = *PM*(0.10) = *PS*(0.10) = *NM*(0.10) = 0. This shows that only one rule fires, namely *NL*(0.10) = 1.

Step-(b) Formation of fuzzy inference rules: The knowledge pertaining to the given control problem is formulated in terms of a set of fuzzy inference rules. To elicit fuzzy inference rules, for the *I/P*-variables intensity (*I*), duration (τ) and *O/P*-variable membrane current (I_{memb}) in our problem, the inference rules have the canonical form of the following type,

If I = A_i and τ = B_i then $I_{memb} = C_i$, I = 1, 2… n, (8)

where A_i,B_i and C_i are fuzzy numbers chosen from the set of fuzzy numbers(on the domains X,Y&Z- axes respectively)that represent the linguistic states NL, NM, NS, AZ, PM, PS and PL and $\mu_{C_i}(z)$ is a monotonic function.

Since each I/P- variable has, seven linguistic states, the total number of possible non- conflicting fuzzy inference rules are $7^2 = 49$.In practice, instead of these 49 rules, a small subset of all possible fuzzy inference rules is often sufficient to obtain acceptable performance of the fuzzy controllers.

An appropriate subset of fuzzy rules derived intuitively by common sense reasoning is as follows:

Rule (1): If I is AZ and τ is NL thenI_{memb} is PL
Rule (2): If I is NS and τ is NL then I_{memb} is PM
Rule (3): If I is NM and τ is NL then $I_{memb}$is NS
Rule (4): If I is NM and τ is AZ then$I_{memb}$is AZ
Rule (5): If I is NS and τ is PS then$I_{memb}$is PL
Rule (6): If I is PS and τ is NS then I_{memb} is PS
Rule (7): If I is PL and τ is AZ then$I_{memb}$is PL
Rule (8): If I is AZ and τ is NS then $I_{memb}$is PS
Rule (9): If I is AZ and τ is NM then$I_{memb}$is PM

Step-(c) Measurement of the adaptability and infer the conclusion: Measurements of I/P-variables of fuzzy controller must be properly combined with relevant fuzzy information rules to make inference regarding the O/P-variables. This is the purpose of the inference engine. This process of finding inferred crisp O/P by inference is called **rule strength computation** or **adaptability** the rule or **firing strength**. We note that by the Tsukamoto fuzzy rules in the form given by (8), the consequence part of each rule is represented by fuzzy set C_i with monotonic membership function $\mu_{C_i}(w)$and that α_i is the matching degree of the ith rule. For the singleton input values (sensor readings) of the linguistic variables intensity($I = x_0$) and duration(τ= y_0) the matching degree α_i is obtained by

$$\alpha_i = \mu_{A_i}(x_0) \wedge \mu_{B_i}(y_0), i = 1,2 \dots n \tag{9}$$

Where "∧" denote the minimum operation.

The overall inferred O/P result is taken as the weighted average of each rule's output is given by

$$w_i = \mu_{C_i}^{-1}(\alpha_i), i = 1,2 \dots n \tag{10}$$

The final result is derived from the weighted average formula which is expressed as,

$$w_0 = \frac{\sum_{i=1}^{n} \alpha_i w_i}{\sum_{i=1}^{n} \alpha_i} \tag{11}$$

Where 'n' is a finite positive integer. Since each rule infers a crisp result, the Tsukamoto fuzzy model aggregates each rule's O/P by the weighted average method. Therefore, this method avoids the time consuming process of defuzzication.

Following the above mathematical algorithmic steps of the Tsukamoto fuzzy control model for the computation of final o/p result we proceed as:

Utilizing fuzzy membership values from Figure 7 and Figure 8 and appropriate subset fuzzy rules that fired only (1 and 2), we write these rules for the values(sensor readings) of the I/P variables intensity (at $x_0 = 0.40$) and duration (at $y_0 = 0.10$) as below.

Rule (1): If x [= I=0.40] is A_1[AZ=0.08] and y [=τ =0.10] is B_1[=NL=1] then z [I_{memb}] is C_1[=PL].
Rule (2): If x [= I=0.40] is A_2[NS=0.92] and y [=τ =0.10] is B_2[=NL=1] then z [I_{memb}] is C_2[=PM].

The computation for measure of adaptability of each rule is as follows:

Adaptability rule-1:α_1= μ_{A_1}(x_0= 0.40) ∧ μ_{B_1}(y_0= 0.10) = min (0.08, 1) = 0.08
Adaptability rule-2:α_2= μ_{A_2}(x_0= 0.40) ∧ μ_{B_1}(y_0= 0.10) = min (0.92, 1) = 0.92
Where " ∧" represents minimum -operation.

We can check very easily adaptability of remaining six rules are zero:
min(0,0) = min(0, 0) = min (0.920, 0) = min (0, 0)=min(0,0) = min (0.0799,0) =0.

The calculations in the conclusion rules 1 and 2 correspond with cutting the fuzzy sets in the consequence part by height of the adaptability of the premise part are shown in Figure 9.

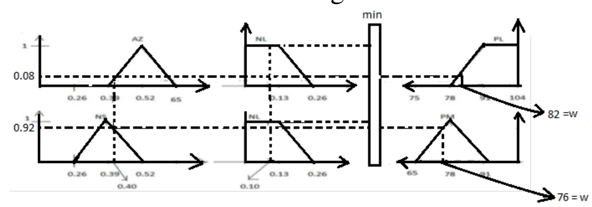

Figure 9: Graphical representation of Tsukamoto method

Step-(d) Aggregate the individual conclusion to obtain the overall conclusion: -Final O/P result is derived from the weighted average formula as follows when there are two 'If-Then' rules are in action,

$$w_0 = \frac{\alpha_1 w_1 + \alpha_2 w_2}{\alpha_1 + \alpha_2}. \tag{12}$$

Now using the valuesα_1,α_2,w_1 *and* $w_2$in the above equation we get,

$$w_0 = \frac{0.08*82+0.92*76}{0.08+0.92} = 76.48 \tag{13}$$

Thus for the singleton I/Ps (x_0, y_0) = (0.40, 0.10) of the linguistic variables intensity (I) and duration (τ) respectively by Tsukamoto fuzzy control we get desired O/P result i.e. membrane current (I_{memb}) is,
$\boldsymbol{I_{memb} = 76.48}\ \mu A/cm^2$.

4. Conclusion

Tsukamoto FLC modeling provides a better, more consistent and more mathematical sound method of handling uncertainty raised due to the I/P linguistic variables of the classical EEG signal model and provide O/P result equivalent to the O/P result of classical EEG signal model (see relations 7 and 13). Further we note that FLC engineer is looking for a good enough solution and not necessarily optimal one. The overall study signify that the Tsukamoto– fuzzy control model have better performance in comparison to the classical mathematical model of EEG signals. Hence the model of the Tsukamoto -fuzzy control obtained from the classical mathematical model of EEG signal is catering the actual dynamics of the system.

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