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Controlling of Chaos in Some Nonlinear Maps

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Research Article

Abstract: In this paper we have considered two non linear maps which follow a period –doubling route to chaos beyond accumulation point [3, 4]. In general , the key idea underlying in most controlling-chaos schemes is to take advantage of the unstable steady states and unstable periodic orbits of the system (infinite in number) that are embedded in the chaotic attractor characterizing the dynamics in phase space [8]. There are many methods available for controlling chaos in different models. In this paper, the periodic proportional pulses technique which is proposed by N.P.Chau [2] is applied to stabilise unstable periodic orbits embedded in the chaotic attractors of one and two dimensional non linear discrete maps and OGY method is also applied to the two dimensional map[9,11] .

1. Introduction[1,2,5,7,9,10,11]

In the last decade, controlling chaos has become on focus in the non linear problems ranging from physics and chemistry to biology and economics. Recently, there has been increasing interest in the research area of controlling chaotic dynamical systems. There are many practical reasons for controlling chaos. First of all, chaotic system response with little meaningful information content is unlikely to be useful. Secondly, chaos can lead systems to harmful situations, therefore chaos should be reduced as much as possible [1]. Chaos is observed as undesirable part in engineering control practice. So, controlling of chaos is an essential part of study of chaos [5]. The idea of "controlling chaos" was first suggested in a famous paper by Ott, Grebogi and Yorke in1990 and known as OGY method [11]. After that many techniques for controlling chaos have been proposed in these decades. The proportional pulse method was introduced by Matias and Guemez [7, 10].After that N.P.Chau[2] discussed in a similar manner but gave some restrictions on the initial conditions by which chaos can be controlled. We have taken the method of periodic proportional pulse and OGY method to control chaos[2,9,10,11].

In section 2 we have applied Chau's method [2] on one dimensional non linear chaotic map $x_{n+1} = \mu\, x_n(\,1 - x_n^2) - x_n^3$ with control parameter μ. We have observed that chaotic region begins for $\mu > 2.302283462700\ldots$.We consider the parameter value $\mu = \mu_0 = 2.35$ (say) which is beyond the accumulation point and shows a chaotic attractor [3, 5].

In section 3.2, the same technique is applied on two dimensional map

$$x_{n+1} = (1 + r)y_n - r{y_n}^3\,;\; y_{n+1} = (1 + r)x_n - r{x_n}^3$$

at the parameter value r=$r_0 = 1.5$ which is beyond accumulation point [4]. In section 4.1 OGY method is also applied on the above said two dimensional map for controlling chaos.

2. Control of chaos in one dimensional map [2, 5, 6, 9,10]

Matias and Guemez have proposed a specific method for controlling chaos by applying instantaneous pulses on the variables, x_i, of a chaotic dynamical system, once every p iterations, in the form

$x_i \rightarrow kx_i$ (i is a multiple of p) (1)

where k is a constant.

In 1997 N.P.Chau has discussed, in more details, a technique of controlling chaos known as periodic proportional pulses method for discrete dynamical systems which is as follows:

Let us consider a discrete one dimensional dynamical system of the form

$x_{n+1} = f(x_n)$ (2)

where $f\colon I \rightarrow I$, is an interval.

The fixed point of (2) is the solution x^* of the equation $x^* = f(x^*\,)$ and the point x^* is stable if $\left|\frac{df(x)}{dx}\right|_{x=x^*} < 1$.

Let us define a composite function

$F(x) = kf^p(x)$ (3)

where we kick the dynamics of the system by multiplying its value with a factor k, once every p iterations. A fixed point of F is any solution of the equation

$F(x_s) = kf^p(\,x_s) = x_s$ (4)

And this fixed point is stable if

$$\left|k\,\frac{df^p(x)}{dx}\right|_{x=x_s} < 1.$$

A stable fixed point of F can be viewed as a stable periodic point of period p of the original dynamics f, kicked by the control procedure. We suppose that the original map is chaotic and

wish to control it so as to obtain a stable periodic orbit of period p, by kicking at its orbit once every p iterations, following equation(1).

Now another new function $C^p(x) = \frac{x}{f^p(x)}\frac{df^p}{dx}$ is defined. So, for stability for F at x_s $|C^p(x_s)| < 1$. So, if a value of the variable x is picked in such a way that $|C^p(x)| < 1$ and find a suitable value of k using equation (4) such that x becomes a fixed point, and then automatically x becomes a stable fixed point of F. If the value of F is injected after p iterations of f, F gives a stable periodic point of period p. It is also important to note that, if the impulse k is too strong, it may kick the dynamics out of the basin of attraction, and in that case, the orbit may escape to infinity. In using pulse control, one must have this precaution in mind [2, 5, 6].

2.1. Periodic proportional pulses on the model

Here we consider a nonlinear cubic model of the form $x_{n+1} = \mu\, x_n(\,1 - x_n^2) - x_n^3$ with control parameter μ. In [3] we see that the model develops chaos through period-doubling route. The period-doubling cascade accrues at the accumulation point 2.302283462700… beyond which chaos occurs. The following (Fig.2.1.1) bifurcation graph evinces the chaotic behaviour after the accumulation point. Evidently for the parameter $\mu = 2.35$ the system is chaotic.

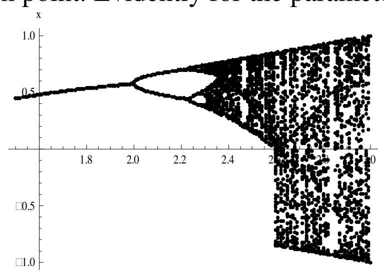

Figure 2.1.1: Bifurcation graph of the model abscissa represents the value of the control parameter and ordinate represents f(x).

Now for p=1 and $\mu = 2.35$, the curve $C^1(x)$ is drawn as in the figure 2.1.2

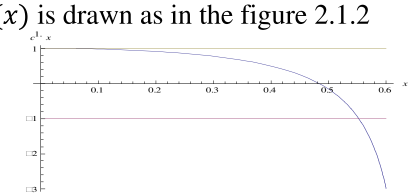

Figure 2.1.2: Control curve of $C^1(x)$ for the parameter $\mu = 2.35$

Here we consider the value of x such that the range is $-1 < C^p(x) < 1\,, p = 1$ (the figure help in choosing the value of x)and get the value of k satisfying equation(4).After solving the equation $C^1(x) = -1$, we get x=0.592238 . We can stabilize orbit of period one in the range (0, 0.592238). Now taking x=0.567721 in the above stated range the value of the kicking factor k=0.787234 for which x becomes a stable fixed point of F. Now if the value of k is applied at every iteration in our map the trajectory goes to the stable fixed point x.

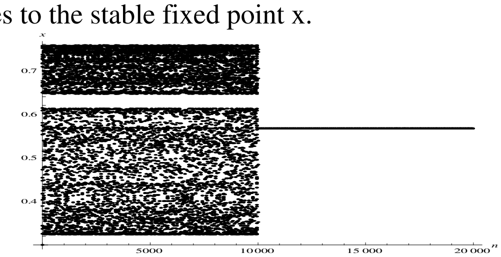

Figure 2.1.3(a): Abscissa represents the number of iterations, while the ordinate represents the value of x at every iteration

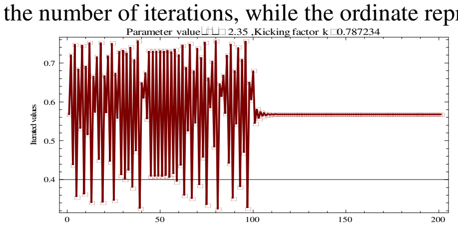

Figure 2.1.3 (b): Time series graph showing effect of controlling chaos of the map to periodic orbit of period one

In the figure 2.1.3(a) up to 10000 iterations it has been observed that chaos occurs, and after that the control is switched on, which shows that it converges to a point.

Also in the figure 2.1.3(b) we see that up to 100 iterations chaos occurs and after that the control is switched on, which shows that it converges to a point.

Again for p=2 and $\mu = 2.35$, the curve $C^2(x)$ is drawn as in the figure 2.1.4

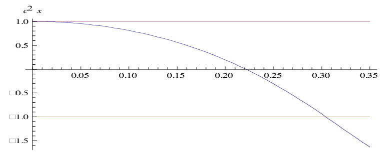

Figure 2.1.4: The curve $C^2(x)$, y=1 and y= -1. The figure evinces positive values of x for which $C^2(x)$lies between -1 and 1.

Now solving the equation $C^2(x) = -1$, we get x=0.304312. Hence the range of possible stable fixed points of F for orbit of period two may be taken as (0, 0.304312).). Now taking x=0.282131 in the above stated range the value of the kicking factor k=0.402466 for which x becomes a stable fixed point of F. Now if the value of k is applied at every (p=2) iteration in our map the trajectory goes to the stable fixed point x.

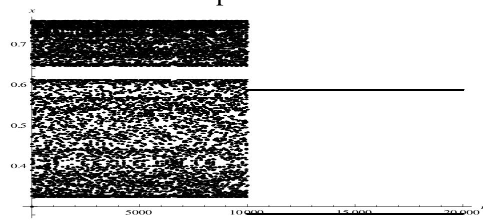

Figure 2.1.5 (a): Abscissa represents the number of iterations, while the ordinate represents the value of x at every (p=2) iteration.

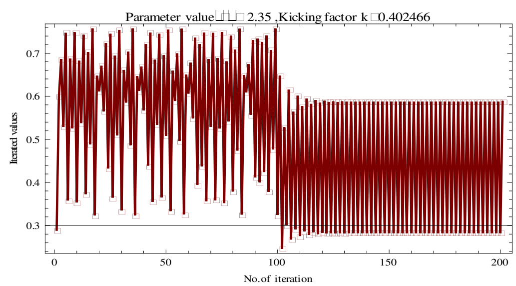

Figure 2.1.5 (b): Time series graph showing effect of controlling chaos of the map to periodic orbit of period two.

In the figure 2.1.5(a) up to 10000 iterations it has been observed that chaos occurs, and after that the chaos is suppressed to form the periodic points of period two.

Also in the figure 2.1.5(b) we see that up to 100 iterations chaos occurs and after that the chaos is suppressed to form the periodic points of period two.

Similarly at the parameter $\mu = 2.35$, the curve $C^3(x)$ is plotted in figure 2.1.6. Solving $C^3(x) = 1$ and $C^3(x) = -1$, we get one of the possible ranges of stable fixed point is (0.343998,0.516468).

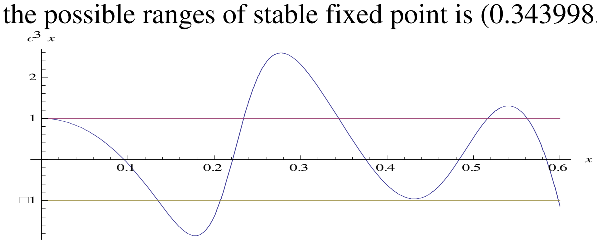

Figure 2.1.6: The curve$C^3(x)$, y=1 and y= -1. The figure evinces positive values of x for which $C^3(x)$lies between -1 and 1.

The value of x=0.457667 is taken in above stated range and hence the value of the corresponding kicking factor satisfying equation (4) is found as k= 0.692562, for which x becomes a stable fixed point of F. Now if the value of k is applied at every (p=3) iteration in our map the trajectory goes to the stable fixed point x. The diagram showing control of chaos is in fig 2.1.7(a) and 2.1.7(b)

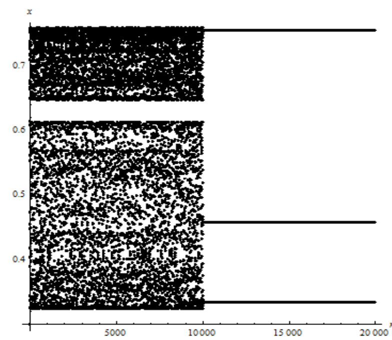

Figure 2.1.7 (a): Abscissa represents the number of iterations, while the ordinate represents the value of x at every (p=3) iteration

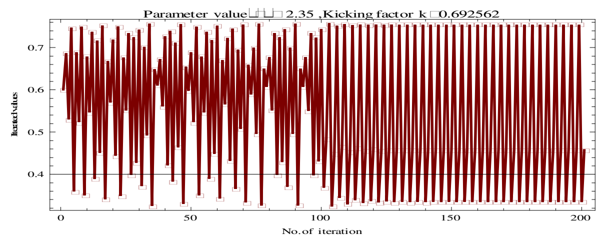

Figure 2.1.7 (b): Time series graph showing effect of controlling chaos of the map to periodic orbit of period three

Similarly at the parameter $\mu = 2.35$, the curve $C^4(x)$ is plotted in figure 2.1.8. Solving $C^4(x) = 1$ & $C^4(x) = -1$, we get one of the possible ranges of stable fixed point is (0.257622, 0.379555)

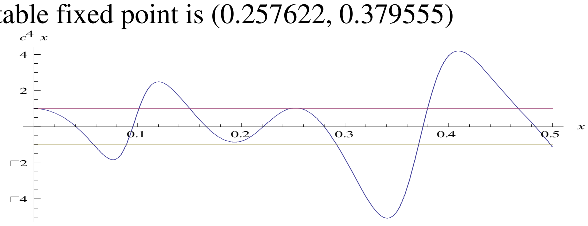

Figure 2.1.8: The curve $C^4(x)$, y=1 and y= -1. The figure evinces positive values of x for which $C^4(x)$ lies between -1 and 1.

The value of x=0.288681 is taken in above stated range and hence the value of the corresponding kicking factor satisfying equation (4) is found as k= 0.385016, for which x becomes a stable fixed point of F. Now if the value of k is applied at every (p=4) iteration in our map the trajectory goes to the stable fixed point x. The diagrams showing control of chaos are in fig 2.1.9(a) and 2.1.9(b).

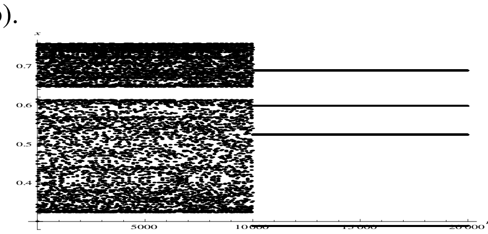

Figure 2.1.9 (a): Abscissa represents the number of iterations, while the ordinate represents the value of x at every (p=4) iteration

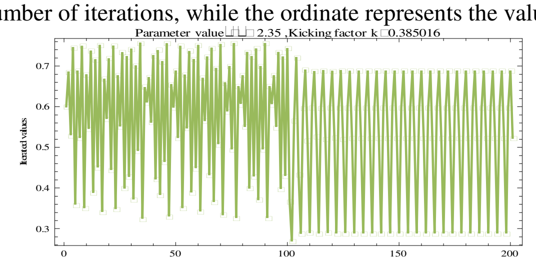

Figure 2.1.9 (b): Abscissa represents the number of iterations, while the ordinate represents the value of x at every (p=4) iteration

3. Controlling of Chaos in Two Dimensional Map

In this heading a two dimensional neural discrete map is considered. The map is

$$x_{n+1} = (1 + r)y_n - ry_n{}^3 \; ; \; y_{n+1} = (1 + r)x_n - rx_n{}^3 \tag{5}$$

where r is a control parameter. In [4], we have seen that the accumulation point of the system is 1.302283462700...from where chaotic region starts. We consider the parameter value $r = r_0 = 1.5$ (say) which is far behind the accumulation point and shows a chaotic attractor.

3.1: Control procedure for two dimensional map [2, 5]

Here we have considered the extended version of the above method [2, 5].The procedure is as follows

Let us consider a two dimensional discrete system

$x_{n+1} = f(x_n, y_n)$, $y_{n+1} = g(x_n, y_n)$

The model can be written as

$$X_{n+1} = F(X_n) \tag{6}$$

where X is a vector in R^2.

To control the dynamics, kick is applied to the orbit of the composite map F^p , once every p steps, by multiplying the x component of the dynamics by a factor k_1 and the y component by a factor k_2 .

Now the kicked map is defined a follows

$$H = K\,F^p \tag{7}$$

where K is a diagonal matrix whose diagonal elements are k_1 and k_2 and F^p represents composition of F, p times. Any fixed point of H let's say X is stable if

$$H = K\,F^p(X) = X \tag{8}$$

and Jacobian matrix has two eigenvalues whose modulus <1 (unity) (9)

Now we have to determine the values of k_1 and k_2 such that chaos is controlled.

3.2Proposed map and control procedure

The above extended version of the control procedure, is now applied in the two dimensional neural map (5) with r=1.5. From [4] it is clear that r=1.5 lies in the chaotic region. Now we consider p=1. The Jacobian matrix of (5) is

$$\begin{pmatrix} 0 & (r+1)-3ry^2 \\ (r+1)-3rx^2 & 0 \end{pmatrix}$$

So Jacobian of H will be

$$\begin{pmatrix} k_1 & 0 \\ 0 & k_2 \end{pmatrix}\begin{pmatrix} 0 & (r+1)-3ry^2 \\ (r+1)-3rx^2 & 0 \end{pmatrix}$$

$$= \begin{pmatrix} 0 & k_1((r+1)-3ry^2) \\ k_2((r+1)-3rx^2) & 0 \end{pmatrix}$$

The characteristic polynomial of the Jacobian matrix is given as follows:

$\lambda^2 - \lambda X + Y = 0$, where X= Sum of the diagonal elements= 0

and Y $= -\{k_1(r+1) - 3rx^2\}\,\{k_2(r+1) - 3ry^2\}$

The eigenvalues of the Jacobian matrix for any point (x, y) are given by

$\lambda = \frac{X \pm \sqrt{X^2 - 4Y}}{2}$ => $\lambda = \pm\sqrt{\{k_1((r+1) - 3rx^2)\}\{k_2((r+1) - 3ry^2)\}}$

The fixed point will be stable if $-1 < \lambda < 1$ (10)

We put $k_1 = \frac{x}{(1+r)y - ry^3}$ and $k_2 = \frac{y}{(1+r)x - rx^3}$ (11)

Now we pick those values of x, y which satisfy (10) and (11).For this purpose we make a suitable c-programming and draw the basin of attraction of period 1 i.e. for p=1, as shown in figure 3.2.1a

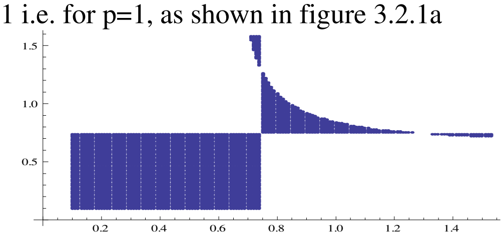

Figure 3.2.1a: Basin of attraction for period 1, where the points (x, y) satisfy equation (10) & (11)

Now any one point from the shaded portion is picked say (x=0.250000,y=0.500000) and determined kicking factor $k_1 = 0.235294, k_2 = 0.831169$ from(10) such that equation (9) is satisfied and eigenvalues $\lambda_1 = 0.772424$ and λ_2 ==-0.772424 are obtained .Applying control procedure with above kicking factor we have obtained following figure3.2.1b

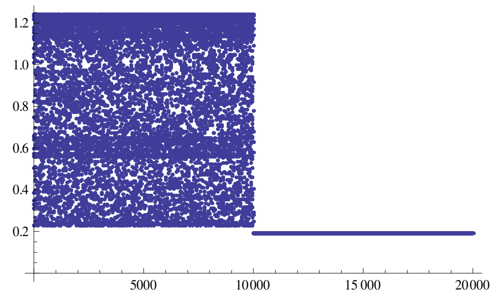

Figure 3.2.1b: Chaos is controlled by taking initial value of (x, y) from the shaded portion of the fig3.2.1a.

In figure 3.2.1b up to 10000 iterations are done at the parameter r=1.5 showing chaotic region and after that controlling parameters are switched on to get the periodic orbit of period one.

Now for the other values of p, we have

$$k_1 = \frac{x}{(1+r)y_{p-1} - r{y_{p-1}}^3} \quad \text{and} \quad k_2 = \frac{y}{(1+r)x_{p-1} - r{x_{p-1}}^3}$$

where x_{p-1} is the first component of f^{p-1} and y_{p-1} is the second component of f^{p-1}.

Also the Jacobian matrix is given as $\begin{pmatrix} k_1 & 0 \\ 0 & k_2 \end{pmatrix} \begin{pmatrix} \frac{\partial x_p}{\partial x} & \frac{\partial x_p}{\partial y} \\ \frac{\partial y_p}{\partial x} & \frac{\partial y_p}{\partial y} \end{pmatrix}$

Following above discussed procedure we show controlling of chaos for different periodic orbit i.e. using different values of p.

Now for p=2: Basin of attraction is as shown in figure 3.2.2a

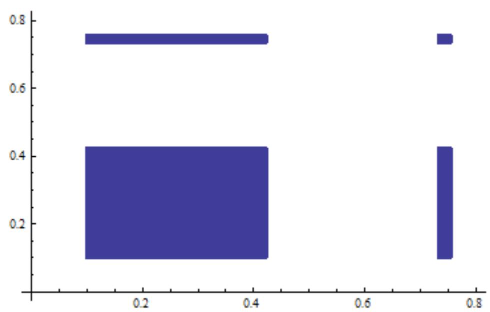

Figure 3.2.2a: Basin of attraction for period -2.Abscissa is the x co-ordinate and ordinate is the y co-ordinate of (x, y)

Now shaded portion of the above figure represents the point (x, y) which may be stable fixed point by taking suitable values k_1 and k_2.

Now picking any one value of co-ordinate of (x, y) =(x=0.390000, y=0.120000) from the shaded portion we have obtained $k_1 = 0.332845, \mathrm{k}_2 = 0.170440$ and corresponding eigenvalues are $\lambda_1 = 0.872433$ and $\square_2 = -0.624028$.

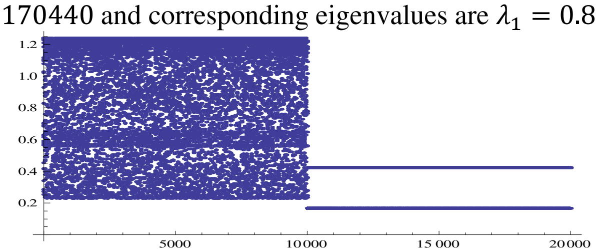

Figure 3.2.2b: Chaos is controlled by taking initial value of (x, y) from the shaded portion of the fig3.2.2a.

In figure 3.2.2b we see that up to 10000 iterations are done at the parameter r=1.5 showing chaotic region and after that controlling parameters are switched on to get the periodic orbit of period two.

For p=3.

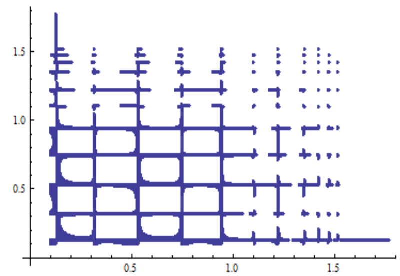

Figure 3.2.3a: Basin of attraction for period -3.Abscissa is the x co-ordinate and ordinate is the y co-ordinate of (x, y)

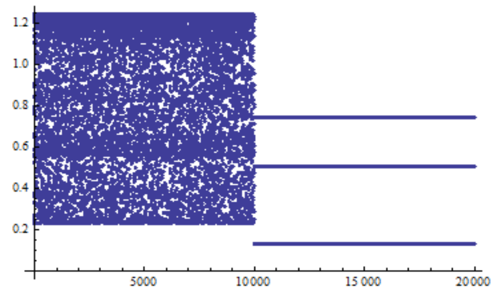

Figure 3.2.3b: Chaos is controlled by taking initial value of (x, y) from the shaded portion of the fig3.2.3a.

For p=4:

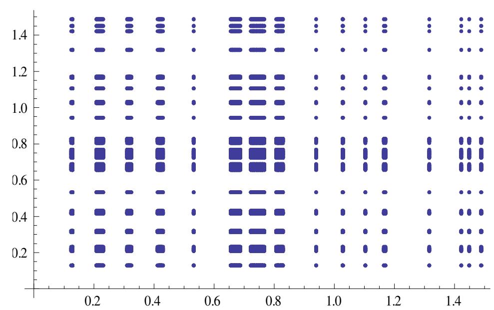

Figure 3.2.4a: Basin of attraction for period -4.Abscissa is the x co-ordinate and ordinate is the y co-ordinate of (x, y)

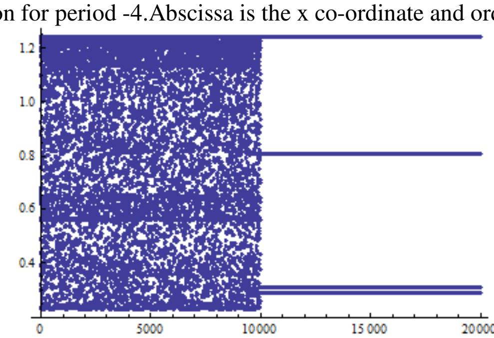

Fig 3.2.4b: Chaos is controlled by taking initial value of (x, y) from the shaded portion of the fig 3.2.4a.

For p=8

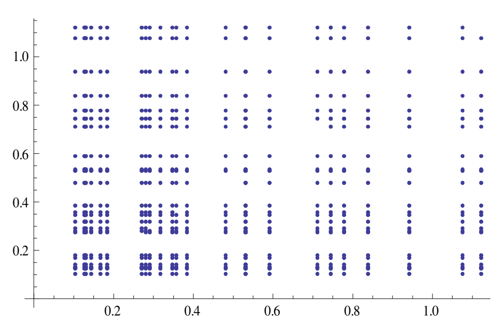

Figure 3.2.5a: Basin of attraction for period -8.Abscissa is the x co-ordinate and ordinate is the y co-ordinate of (x, y)

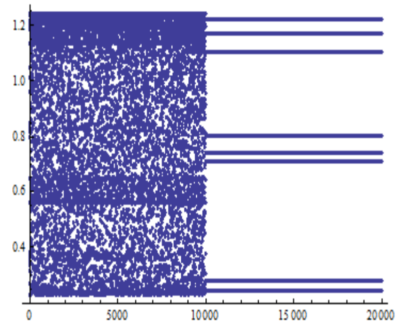

Figure 3.2.5b: Chaos is controlled by taking initial value of (x, y) from the shaded portion of the fig3.2.5a.

For p=16

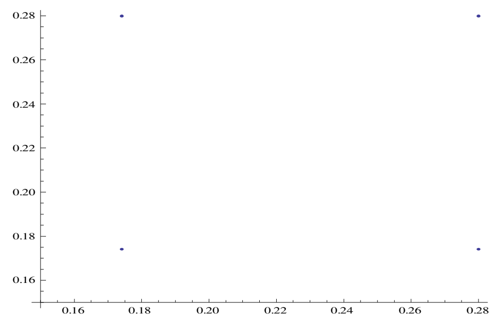

Figure 3.2.6a: Basin of attraction for period -16.Abscissa is the x co-ordinate and ordinate is the y co-ordinate of (x, y)

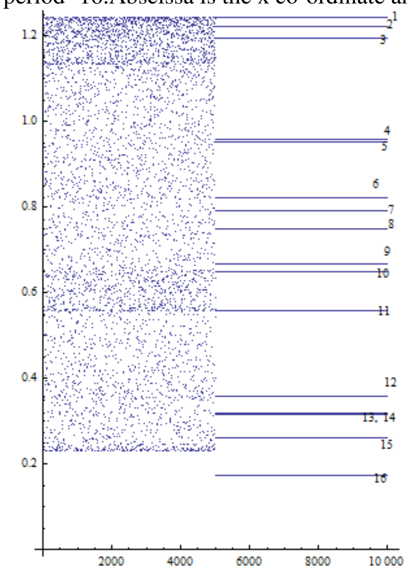

Figure 3.2.6b: Chaos is controlled by taking initial value of (x, y) from the shaded portion of the fig3.2.6a.

Here we want to say that although N.P.Chou said in[2] that his procedure is successful up to period 5 but in our two dimensional model we are able to control chaos up to period 16 by applying same procedure.

4. The OGY Method [9, 11]:

Consider the *n*-dimensional map

$$Z_{n+1} = h(Z_n, p),$$

where p is some accessible system parameter that can be changed in a small neighbourhood of its nominal value, say, p_0 . In case of continuous-time systems, such a map can be constructed using the Poincaré map.

It is well known that a chaotic attractor is densely filled with unstable periodic orbits and that any small region on the chaotic attractor will be visited by a chaotic orbit. The OGY method hinges on the existence of stable manifolds around

unstable periodic points. The basic idea is to make small time-dependent linear perturbations to the control parameter p in order to nudge the state towards the stable manifold of the desired fixed point.
Suppose that $Z_s(p)$ is an unstable fixed point of the system

$$c(x,y) = ((1+r)y - r\,y^3 \,, (1+r)x - rx^3) \tag{12}$$

The position of this fixed point moves smoothly as the parameter p is varied. For values of p close to p_0 in a small neighbourhood of $Z_s(p_0)$, the map can be approximated by a linear map given by

$$Z_{n+1} - Z_s(\,p_0) = J\,(\,Z_n - Z_s(\,p_0)\,) + C(p \;\, - p_0\,)$$

where J is the Jacobian and $C = \frac{\partial h}{\partial p}$. All the partial derivatives are evaluated at $Z_s(p_0)$ and p_0 .
Assume that in a small neighbourhood around the fixed point

$$P - p_0 = -K(\,Z_n - Z_s(\,p_0)\,) \tag{13}$$

where K is a constant vector of dimension n to be determined. Then in using equation (13) in (12) we have

$$Z_{n+1} - Z_s(\,p_0) = (J - CK)(Z_n - Z_s(\,p_0)) \tag{14}$$

The fixed point is then stable as long as the eigenvalues, or regulator poles, have modulus less than unity. The pole-placement technique from control theory can be applied to find the vector K [9, 11]

4.1Control of Chaos by OGY method in two dimensional map:

We know from [4] that the accumulation point for our model is r=1.302283462700… and the chaotic region begins when the control parameter cross accumulation point. Our aim is to control the chaos so we allow the control parameter a to vary around the nominal value $r_1 = 1.35$ for which the map has a chaotic attractor .For these values of parameter, the fixed points for period one are approximately detected at (-1.57527,1.57527), (-1.15212,-0 .64294), (-1, -1) , (- 0.64294 , 1.15212) , (0 ,0) ,(0.64294 , 1.15212) , (1 ,1) , (1.15212 , 1.15212) and (1.57527 , - 1.57527).Now if we discard the fixed points because of the fact that population cannot be negative , then the remaining fixed points are , (0 ,0) ,(0.64294 , 1.15212) , (1 ,1) , (1.15212 , 1.15212) and the eigenvalues in the form $(\lambda_1\,, \lambda_2\,)$ for these fixed points are (-2.35, 2.35), (1.43003i ,-1.43003i),(-1.7 ,1.7) ,(1.43003i ,-1.43003i).Clearly all of these fixed points are unstable .
Now let A (x_0, y_0) be an unstable point and we choose $Z_s(\,r_1) = A$. The fixed point is stable if the matrix $J - CK$ at A has eigenvalues with modulus less than one.

Here $J = \begin{pmatrix} 0 & (r+1) - 3ry^2 \\ (r+1) - 3rx^2 & 0 \end{pmatrix}$

$$C = \begin{pmatrix} \frac{\partial f}{\partial a} \\ \frac{\partial g}{\partial a} \end{pmatrix} = \begin{pmatrix} y - y^3 \\ x - x^3 \end{pmatrix}$$

And $K = (k_1 \quad k_2)$

And hence
$$J - CK = \begin{pmatrix} 0 & (r+1) - 3ry^2 \\ (r+1) - 3rx^2 & 0 \end{pmatrix} - \begin{pmatrix} y - y^3 \\ x - x^3 \end{pmatrix} (k_1 \quad k_2)$$
$$= \begin{pmatrix} 0 & (r+1) - 3ry^2 \\ (r+1) - 3rx^2 & 0 \end{pmatrix} - \begin{pmatrix} k_1(y - y^3) & k_2(\,y - y^3) \\ k_1(x - x^3\,) & k_2(x - x^3\,) \end{pmatrix}$$
$$= \begin{pmatrix} -k_1(y - y^3) & (r+1) - 3ry^2 - k_2(\,y - y^3) \\ (r+1) - 3rx^2 - k_1(x - x^3\,) & -k_2(x - x^3) \end{pmatrix}$$

Now at the fixed point A (0.64294, 1.15212),

$$J - CK = \begin{pmatrix} 0.377182k_1 & -3.02589 + 0.377182k_2 \\ 0.675844 - 0.377167k_1 & -0.377167k_2 \end{pmatrix}$$

And the characteristic polynomial is given by

$$\lambda^2 - (0.377182k_1 - 0.377167k_2)\lambda + (2.04503 - 1.14127k_1 - 0.254916k_2) = 0$$

Suppose that the eigenvalues (regulator poles) are given by λ_1 and λ_2 , then

$\lambda_1\,\lambda_2 = 2.04503 - 1.14127k_1 - 0.254916k_2$

and $\lambda_1 + \lambda_2 = 0.377182k_1 - 0.377167k_2$

$$\lambda_1 = \tfrac{1}{2}(0.377167k_1 - 0.377167k_2 - 0.377167\sqrt{-57.5025 + 32.0903k_1 + k_1{}^2 + 7.16759k_2 - 2k_1\,k_1 + k_2{}^2} = 1 - \epsilon \tag{15}$$

$$\lambda_2 = \tfrac{1}{2}(0.377167k_1 - 0.377167k_2 + 0.377167\sqrt{-57.5025 + 32.0903k_1 + k_1{}^2 + 7.16759k_2 - 2k_1\,k_1 + k_2{}^2} = 1 - \epsilon' \tag{16}$$

Solving (15) and (16) for k_1 and k_2 we have

$k_1 =$
$1.24976 \times 10^{-15} \left(-1.00557 \times 10^{46} - 1.69751 \times 10^{30}\, \epsilon - 1.69751 \times 10^{30}\, \epsilon' \mp 1.69751 \times 10^{30} \sqrt{3.50917 \times 10^{31} + 1.18476 \times 10^{16}\, \epsilon + \epsilon^2 + 1.18476 \times 10^{16}\, \epsilon' - 2.\, \epsilon\, \epsilon' + \epsilon'^2}\right.$
$k_2 =$
$5.10335 \times 10^{-23} \left(-2.46256 \times 10^{53} - 4.15704 \times 10^{37}\epsilon - 4.15704 \times 10^{37}\epsilon' \mp 4.15704 \times 10^{37} \sqrt{\left[3.50917 \times 10^{31} + 1.18476 \times 10^{16}\epsilon + \epsilon^2 + 1.18476 \times 10^{16}\epsilon' - 2.\, \epsilon\epsilon' + \epsilon'^2\right]}\right.$

We choose $\in, \in'$ in such a way that $|\lambda_1| < 1$ and $|\lambda_2| < 1$ and k_1, k_2 exists.
In particular, taking $\in = 1.7$, $\in' = 0.3$ we get the values of k_1 and k_2 as
$\mathrm{k}_1 = 1.8157024887897553$, $\mathrm{k}_2 = 1.8157024887897573$
Now applying these values and equation (14) the chaotic situation in the map (5 or12) is controlled as shown in the following figures.

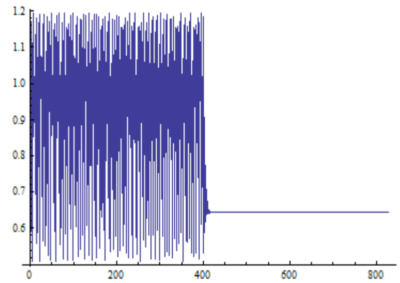

Figure 4.1: Time series plot of the map with a control where horizontal axis represents the number of iteration and vertical axis represents iterated values of the x-co-ordinate.

In figure 4.1 a time series plot is shown when the control is switched on after the 400th iterations and the control is left switched on until 810th iterate for the values of x-co-ordinate.

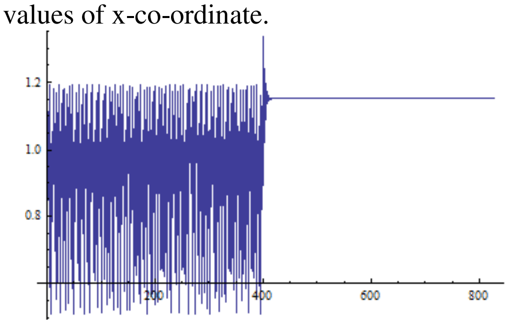

Figure 4.2: Time series plot of the map with a control where horizontal axis represents the number of iteration and vertical axis represents iterated values of the y-co-ordinate

In figure 4.2 a time series plot is shown when the control is switched on after the 400th iterate and the control is left switched on until 810th iterate for the values of y-co-ordinate.

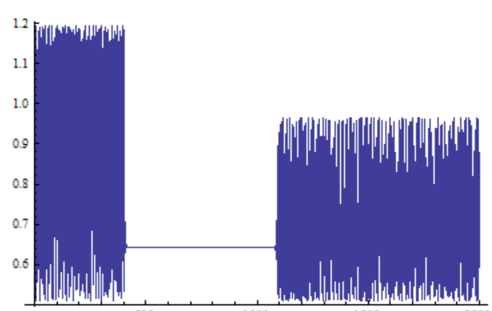

Figure 4.3: Time series plot of the map with control and without where horizontal axis represents the number of iteration and vertical axis represents iterated values of the x-co-ordinate.

In figure 4.3 a time series plot is shown when the control is switched on after the 400th iterate and then switched off after the 1100th iterate for the values of x-co-ordinate.

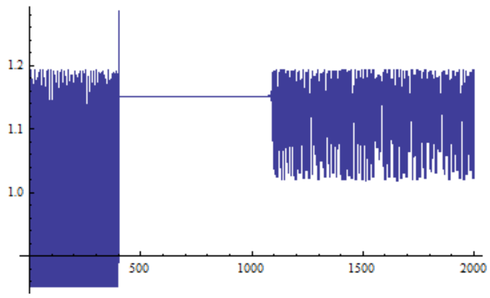

Figure 4.4: Time series plot of the map with control and without where horizontal axis represents the number of iteration and vertical axis represents iterated values of the y-co-ordinate.

In figure 4.3 a time series plot is shown when the control is switched on after the 400^{th} iterate and then switched off after the 1100^{th} iterate for the values of y-co-ordinate.

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