Some Probability Models for the Duration of Stay of Migrants at the Destination Place

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Research Article

Abstract: This present paper deals with a probability model for the number of return migration according to the duration of stay under certain assumptions. Parameters involved in this model have been estimated by the maximum likelihood (ML) method and method of moment (MM). Model has been applied to the real data set taken from a flooded area of Koshi river in Bihar.

Key Word: Return migration, Two parameter exponential distribution, Maximum likelihood (ML) method and Method of moment (MM).

Introduction

There is a natural tendency that a majority of the rural migrants return back to their own household whenever required condition arises. The return behavior of the migrants always operates them to give more emphasis on the improvements of social and economic condition of the household at origin rather than the place of destination. Migration from rural to urban and the return migration from urban to rural play a significant role in reducing the existing socio-economic and cultural disparities between the rural and urban areas (Yadava et al., 1996). However, the mechanism under which these processes take place requires a more complete knowledge of the nature and pattern of urban to rural migration. Rural urban migration is a result of diverse economic opportunities across space. In the past, it has played an important role in the urbanization process in different parts of the world and continues to be significant, even though migration rates have been slowed down in many countries (Lall et. al., 2006). Though the migrants and their households might benefited greatly, it is seen that this benefit occurs at the cost of decline in social welfare, increased population in urban destination areas and a greater regional concentration of wealth, income and human capital (Oberai et. al., 1984). The theoretical and empirical literature on migration has paid little attention to the fact that many migrants return to their home after having spent some years at the destination place. To understand the motives of return migrations, as well as the factors which explain variation in migration durations, it is important for designing optimal migration policies. Return migration from urban to rural areas of rural out migrants and its implications at both rural and urban places has become an important topic to be discussed among researchers, planners and administrators. In

urban areas of India, the increasing population concentration several problems, viz. problems has created of overcrowding, difficulties of waste disposals, housing shortages, inadequate educational facilities, poor water and electric supply, traffic congestion etc. In India about 31 per cent population lives in urban areas (Census, 2011), which is less than that of many developing countries, but in absolute numbers, it is more than the total population of many other nations. However, the rural-urban migration in India is mostly circular in nature and enhancing the development of rural area by the flow of money from urban to rural area through remittances. Due to stagnant rural economy, the rural population, mainly males, move out to urban areas in search of better economic opportunities, leaving their wives and children in villages who subsists in remittances. It was found that a household having at least one migrant has a better socio-economic status as compared to others (Singh and Yadava, 1981). The return migration to rural area is important for reducing the pressure of urban and town planners and, at the same time it enhances the living standard of household as well as the socio-economic development of rural area by bringing new ideas and cultures. Generally, a model explains only a particular behavior of the related observations which are introduced in it. In the field of social sciences, variables are highly interrelated and thus it is very difficult to introduce any realistic model. As such migration models are more problematic than the models used in physical sciences (Sivamurthy, 1982) and there are a set of specified models which explain only a certain aspect of migration. In recent past few attempts have been made at micro level to explain the pattern of rural to urban migration through some probabilistic and deterministic models (Sharma, 1984, 1987; Singh, 1986; Singh and Yadava, 1981; Yadava and Singh, 1983). But very few models have been proposed so far to describe the nature of return migration from urban to rural area (Kushwaha, 1992; Yadava et. al. 1989). The aim of this paper is to study the pattern of number of return migrants according to their duration of stay at the place of destination through probability model by modifying the existing one.

Model-I

When we observe the data of number of migrants according to their duration of stay at destination place, we see that it declines with the increase in duration of stay. In fact, the chance of return also increases with the increase in duration of stay (except where a migrant settles permanently at the place of destination). Thus, the number of migrants according to the duration of stay follow an exponential distribution (Yadava et. al., 1989).

$$f(x) = \begin{cases} \lambda \exp(-\lambda x), x \ge 0\\ 0, otherwise \end{cases}$$
(1)

Here λ is the risk of return migration. Then, the proportion of migrants with the duration of stay less than or equal to X is given by

$$F(x) = \begin{cases} 1 - \exp(-\lambda x), x \ge 0\\ 0, otherwise \end{cases}$$
(2)

But it has been seen that migrants stay some time at the place of destination and search an opportunity of job. If they are satisfied with the job then they continue to stay otherwise return back to the place of origin. Thus the exponential distribution is not an appropriate model. It need some modification and model-II has been proposed.

Model-II

Let *X* has a two parameter exponential distribution i.e. $X \sim Exp(\lambda, \theta)$ then the probability density function of *X* is

$$f(x) = \lambda \exp[-\lambda(x - \theta)]$$
, where $x \ge \theta$ and $\lambda > 0$ (3)
and the cumulative distribution function is

 $F(x) = 1 - \exp\left[-\lambda(x - \theta)\right]$

Estimation Procedure

Estimation of parameter of Model-I: This model contains only one parameter to be estimated from the observed distribution. The estimated value by the method of maximum likelihood is given as

 $\hat{\lambda} = \frac{1}{x}$ here \bar{x} is the mean duration of stay of migrants.

Estimation of parameters of Model-II

The Method of Maximum Likelihood (ML) Solution for the parameters of the 2-parameter Exponential Distribution:

If $X_{1,X_{2,X_{3,\dots}}}$ are iid Exp (λ, θ) then the likelihood

$$L(\theta,\lambda) = \lambda^n \exp\left[-\lambda \sum_{i=1}^n (x_i - \theta)\right] \text{ provided } x_{(1)} \ge \theta \tag{5}$$

and the log likelihood

$$\log L(\theta, \lambda) = l = n \log \lambda - \lambda \sum_{i=1}^{n} (x_i - \theta)$$
(6)

Thus Since $\hat{\theta} = x_{(1)}$, the first order statistics is 1. The m.l.e. of θ is given by

$$\hat{\theta} = 1$$

To find the solution of λ , we have to solve the following equation

$$\frac{\partial L}{\partial \lambda} = \frac{n}{\hat{\lambda}} + n\hat{\theta} - \sum_{i=n}^{n} x_i = 0$$
⁽⁷⁾

from the above equation we have

$$\hat{\lambda} = \frac{n}{(\sum_{i=1}^{n} x_i) - n\hat{\theta}}$$
(8)

by using the value of $\hat{\theta} = 1$ we can estimate the value of λ as

$$\hat{\lambda} = \frac{n}{(\sum_{i=1}^{n} x_i) - n}$$

The Method of Moments (MM) Solution for the parameters of the 2-parameter Exponential Distribution:

$$E(X) = (\theta + \frac{1}{\lambda})$$
 and $E(X^2) = (\theta^2 + \frac{2}{\lambda^2} + \frac{2\theta}{\lambda})$ (9)

therefore the Variance(X) =
$$\frac{1}{\lambda^2}$$

From equation (10) we have

 $\hat{\lambda} = \sqrt{\frac{1}{Variance(X)}}$ thus putting the value of $\hat{\lambda}$ in equation (9)

we can obtained the estimate of heta .

Application

(4)

The models discussed are applied to an observed set of data collected in the survey entitled "Migration and Related Characteristics-A Case Study of North-eastern Bihar" conducted during October 2008 to March 2009 and a resurvey of migrating household in February and March in 2012 after a gap of three years in flooded area of Koshi river. In the present study we have considered a person as migrate only if he has spent at least one year at destination place otherwise he was considered as a seasonal migrant. A person who returns his home daily is considered as a commuter. In this study we have not considered the seasonal migrants and commuters.

Discussions and Conclusion

The observed and expected distributions of return migrants according to duration of stay at destination place are given in Table 1. The results reveal that in case of exponential model, p-value is very small whereas in 2parameter exponential model, p-value indicates a strong agreement between observed and expected observations. Hence it can be concluded that 2-parameter exponential distribution describes the pattern of number of migrants according to their duration of stay quite well. Figure 1 portrays the observed frequencies and expected frequencies of the return migrants. It reflects that 2-parameter exponential model is more nearer to reality and thus the proposed model may be utilized for estimation and prediction purposes. The p-value in case of fitting by

(10)

moment estimate is slightly more than that of maximum likelihood estimate. The moment estimation procedure is easier than maximum likelihood estimation procedure in terms of calculation and derivation of the formulae. The mean of duration of stay for migrants has been found to be 5.113 years.

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	Observed frequency	Expected frequency				
Duration (in Years)		One non-star Frances the Distribution	Two parameter exponential distribution			
		One parameter Exponential Distribution	ML estimate	MM estimate		
≤2	24	43.06	28.71	29.29		
2-4	38	29.12	40.16	39.71		
4-6	23	19.69	24.70	24.51		
6 - 8	15	13.32	15.18	15.12		
8-10	11	9.01	9.34	9.33		
10 - 12	8	6.09	5.74	5.76		
12 - 14	7		9.17			
14 - 16	4	12.72		9.28		
16 - 18	2	12.72				
18 - 20	1					
Total	133	133.00	133.00	133.00		
The value of parameters		λ =0.19559	$\lambda = 0.24314$ and $\theta = 1$	λ =0.24135 and θ =0.96939		
χ^2		13.08	4.74	4.69		
<i>p</i> -value		0.023	0.449	0.455		



Figure 1: Observed and expected frequency of number of return migrants

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