Original Research Article

An inventory control model for deteriorating items with shortage under time varying holding cost for instantaneous replenishment

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Abstract

In this paper, an Economic Order Quantity (EOQ) model has been developed. An item deteriorates with three parameter Weibull distribution where shortages are allowed. The optimal cycle time, optimal holding cost and total optimal cost has been derived for the model. The optimal Holding cost has been derived by minimizing the total inventory cost. A numerical example and sensitivity analysis are presented to illustrate the proposed model.

Keywords: Inventory model, Deteriorating items, Time varying holding cost, Shortage, Three parameter Weibull distribution.

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INTRODUCTION

Deterioration plays an important role in many inventory systems. Deterioration is defined as Changee, damage, decay spoilage, obsolescence, evaporation, pilferage and loss of utility or loss of original value in a commodity from the original one. In such situation price discount can be another option to cover somewhat loss on these deteriorated items. Price discount is an attraction to the customers, therefore price discount are common practices by supplier it encourages the customer to purchase a lot size or defective items other than regular purchase.

P.N. Ghare and G.P. Schrader [1963] were the pioneer in deterioration they have considered constant deterioration rate in their inventory model. Covert and Philip [1973] extended Ghare and Schrader model to obtain an economic order quantity (EOQ) model for a variable rate of deterioration by assuming a two-parameter Weibull distribution. B.C. Giri, *et al* [1996] generalized an EOQ model for deteriorating items over a finite time horizon with demand and costs are varying with time. N.H. Shah and K.T. Shukla [2009] have developed deterministic inventory model for deterioration with shortages. S.V. Kawale and P.B. Bansode [2012] continued this work and developed EPQ model for deteriorating items with Weibull deterioration under time-varying holding cost. T. Chakrabarty, *et al* [1998] developed inventory model using three parameters Weibull distribution in deterioration. V.K. Mishra, *et al* [2013] developed deterministic inventory model for time-dependent demand and time-varying holding cost under partial backlogging. N. Ghasemi and B.A. Nadjafi [2013] used holding cost is an increasing function of period length to develop inventory model for deteriorated items. S.V. Kawale and P.B. Bansode [2013]developed inventory model for time varying holding cost and Weibull distribution for deterioration with fully backlogged.

Develop a model using three parameter Weibull distribution for deterioration with time-varying holding cost and shortage are allowed. Price discount is offered to

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deteriorated items. Numerical example is given to support developed model and analyzed the same.

NOTATIONS

A : Ordering cost per order.

h : holding cost per unit per unit time, $h = a + bt$ where a and b are positive constants.

$\theta(t)$: The deterioration rate at time t, where $\theta(t) = \alpha\beta(t-\gamma)^{\beta-1}$.

IM : The maximum inventory level during $(0, t_2)$.

IB : The maximum inventory level during shortage period.

$I(t)$: The inventory level at time t.

d : Demand rate.

T : The length of replenishment cycle.

s : Shortage cost per unit per unit time.

r : Price discount per unit cost.

TC : Total cost per time unit.

ASSUMPTIONS

1) The demand rate for the product is known and finite.
2) The lead time is zero.
3) The replenishment rate is instantaneous.
4) Shortages are allowed and fully back logged.
5) The deterioration rate is a three-parameter Weibull distribution during the time interval $[0, t_1]$.
6) No repairs or replacement of the deteriorated item takes place during a given cycle.
7) Finite time horizon period is considered.
8) A single item is considered over the prescribed period of time.
9) Holding cost is a linear function of time given by $h = a + bt$ $(a, b > 0)$.

MATHEMATICAL FORMULATIONS

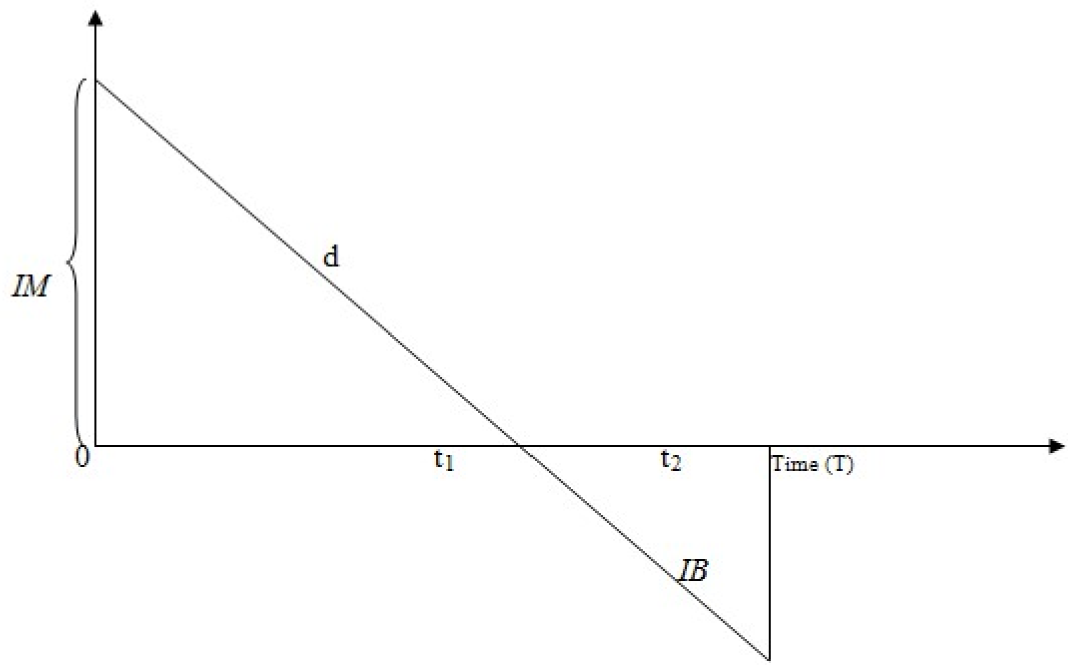

Figure 1: Representation of inventory model

The rate of Changee of inventory by the following differential equation.

$$\frac{dI(t)}{dt} + \theta(t)I(t) = -d \text{ for } (0 \le t \le t_1) \quad (1)$$

$$\frac{dI(t)}{dt} = -d \text{ for } (t_1 \le t \le t_2) \quad (2)$$

The conditions associated with these equations are: $I(0) = IM$, $I(t_1) = 0$

ANALYTICAL SOLUTION OF THE MODEL

Case A: Inventory level without shortage

During the period $\left[0,t_1\right]$, the inventory depletes due to the deterioration and demand. Hence the inventory level at any time during the period $\left[0,t_1\right]$ is described by the differential equation

$$\frac{dI(t)}{dt}+\alpha\beta(t-\gamma)^{\beta-1}I(t)=-d \text{ ,for } (0\leq t\leq t_1) \tag{3}$$

With the boundary condition are $I(0)=IM$ and $I(t_1)=0$

The solution of the linear differential equation (3) is,

$$I(t)=d\left[(t_1-t)+\frac{\alpha}{\beta+1}\left((t_1-\gamma)^{\beta+1}-(t-\gamma)^{\beta+1}\right)\right]e^{-\alpha(t-\gamma)^{\beta}}, \qquad \text{for } 0\leq t\leq t_1$$

Now taking the first two terms of the exponential series and neglecting the term containing α^2 of the above equation.

$$I(t)=d\left[(t_1-t)(1-\alpha(1-\gamma)^{\beta})+\frac{\alpha}{\beta+1}((t_1-\gamma)^{\beta+1}-(t-\gamma)^{\beta})\right], \qquad \text{for } 0\leq t\leq t_1 \tag{4}$$

Case B: Inventory level with shortages.

During the interval $\left[t_1,t_2\right]$ the inventory level depends on demand and fraction of demand is back logged.

Hence the inventory during $\left[t_1,t_2\right]$ can be represented by the differential equation

$$\frac{dI(t)}{dt}=-d \qquad \text{for } t_1\leq t\leq t_2 \tag{5}$$

With initial and boundary conditions $I(t_1)=0$ and $I(t_2)=IB$, respectively.

The solution of differential equation (5) is,

$$I(t)=-d\times t \qquad \text{for } t_1\leq t\leq t_2 \tag{6}$$

Therefore the total cost per replenishment cycle consists of the following components.

Ordering cost: Ordering cost per order is given by,

$$I_{OC}=A \tag{7}$$

Holding cost: The holding cost is given by,

$$I_{HC}=\int_0^{t_1}(a+bt)I(t)dt$$

$$I_{HC}=\int_0^{t_1}(a+bt)\times d\left[(t_1-t)(1-\alpha(1-\gamma)^{\beta})+\frac{\alpha}{\beta+1}((t_1-\gamma)^{\beta+1}-(t-\gamma)^{\beta+1})\right]dt$$

$$I_{HC}=d(1-\alpha(1-\gamma)^{\beta})\left(\frac{at_1^2}{2}+\frac{bt_1^3}{6}\right)+\frac{d\alpha}{\beta+1}(t_1-\gamma)^{\beta-1}(at_1+t_1^2)$$
$$-\frac{d\alpha}{(\beta+1)(\beta+2)}(t_1-\gamma)^{\beta+2}(a+bt_1)+\frac{bd\alpha(t_1-\gamma)^{\beta+3}}{(\beta+1)(\beta+2)(\beta+3)} \tag{8}$$
$$+\frac{d\alpha(\gamma^{\beta+3}-\gamma^{\beta+2})}{(\beta+1)(\beta+2)(\beta+3)}$$

Deterioration cost: The deterioration cost is given by,

$$I_D=\int_0^{t_1}\theta(t)I(t)dt$$

$$I_D = \int_0^{t_1} \alpha\beta(t-\gamma)^{\beta-1} I(t)dt$$

$$I_D = \frac{\alpha d}{\beta+1}\left((t_1-\gamma)^{\beta+1}+\gamma^{\beta+1}\right) \tag{9}$$

Shortage cost: Shortage cost is given by,

$$I_{SC} = s\int_{t_1}^{t_2} I(t) = s\int_{t_1}^{t_2} -d\times tdt$$

$$I_{SC} = sd\left(\frac{t_1^2-t_2^2}{2}\right) \tag{10}$$

Price discount: Price discount is given by:

$$I_{PD} = \int_0^{t_1} r\theta(t)dt = r\int_0^{t_1} \alpha\beta(t-\gamma)^{\beta-1}dt$$

$$I_{PD} = r\alpha((t_1-\gamma)^{\beta}+\gamma^{\beta}) \tag{11}$$

Therefore the average total cost per unit time is given by:

$$TC = \frac{1}{(t_1+t_2)}\left[I_{OC}+I_{HC}+I_D+I_{SC}+I_{PD}\right]$$

$$TC = \frac{1}{(t_1+t_2)}\left\{\begin{array}{l} A+d(1-\alpha(1-\gamma)^{\beta})\left(\frac{at_1^2}{2}+\frac{bt_1^3}{6}\right)+\frac{d\alpha}{\beta+1}(t_1-\gamma)^{\beta-1}(at_1+t_1^2) \\ -\frac{d\alpha}{(\beta+1)(\beta+2)}(t_1-\gamma)^{\beta+2}(a+bt_1)+\frac{bd\alpha(t_1-\gamma)^{\beta+3}}{(\beta+1)(\beta+2)(\beta+3)} \\ +\frac{d\alpha(\gamma^{\beta+3}-\gamma^{\beta+2})}{(\beta+1)(\beta+2)(\beta+3)}+\frac{\alpha d}{\beta+1}\left((t_1-\gamma)^{\beta+1}+\gamma^{\beta+1}\right)+sd\left(\frac{t_1^2-t_2^2}{2}\right) \\ +r\alpha((t_1-\gamma)^{\beta}+\gamma^{\beta}) \end{array}\right\} \tag{12}$$

To minimize the total cost (TC) per unit time, the optimal value of t_1 and $t_2$can be obtained by solving the following equation:

$$\frac{\partial TC}{\partial t_1} = 0$$

$$= \frac{1}{t_1+t_2}\left(\begin{array}{l} dst_1+d\left(at_1+\frac{bt_1}{2}\right)(1-\alpha(1-\gamma)^{\beta})+r\alpha\beta(t_1-\gamma)^{-1+\beta}+d\alpha(t_1-\gamma)^{\beta} \\ +d(at_1+bt_1^2)\alpha(t_1-\gamma)^{\beta}-\frac{d(a+bt_1)\alpha(t_1-\gamma)^{1+\beta}}{1+\beta}+\frac{d(a+2bt_1)\alpha(t_1-\gamma)^{1+\beta}}{1+\beta} \end{array}\right) -$$

$$\frac{1}{(t_1+t_2)^2}\left(\begin{array}{l} A+\frac{1}{2}ds(t_1^2-t_2^2)+d\left(\frac{at_1^2}{2}+\frac{bt_1^3}{6}\right)(1-\alpha(1-\gamma)^{\beta})+\frac{d(at_1+bt_1^2)\alpha(t_1-\gamma)^{1+\beta}}{1+\beta} \\ -\frac{d(a+bt_1)\alpha(t_1-\gamma)^{2+\beta}}{(1+\beta)(2+\beta)}+\frac{bd\alpha(t_1-\gamma)^{3+\beta}}{(1+\beta)(2+\beta)(3+\beta)}+r\alpha((t_1-\gamma)^{\beta}+\gamma^{\beta}) \\ +\frac{d\alpha((t_1-\gamma)^{1+\beta}+\gamma^{1+\beta})}{1+\beta}+\frac{d\alpha(-\gamma^{2+\beta}+\gamma^{3+\beta})}{(1+\beta)(2+\beta)(3+\beta)} \end{array}\right) \tag{13}$$

$$\frac{\partial TC}{\partial t_2}=0$$

$$\frac{dst_2}{(t_1+t_2)}-\frac{1}{(t_1+t_2)^2}\left(\begin{array}{l} A+\frac{1}{2}ds(t_1^2-t_2^2)+d\left(\frac{at_1^2}{2}+\frac{bt_1^3}{6}\right)(1-\alpha(1-\gamma)^{\beta}) \\ +\frac{d(at_1+bt_1^2)\alpha(t_1-\gamma)^{1+\beta}}{1+\beta}-\frac{d(a+bt_1)\alpha(t_1-\gamma)^{2+\beta}}{(1+\beta)(2+\beta)}+ \\ \frac{bd\alpha(t_1-\gamma)^{3+\beta}}{(1+\beta)(2+\beta)(3+\beta)}+r\alpha((t_1-\gamma)^{\beta}+\gamma^{\beta}) \\ +\frac{d\alpha((t_1-\gamma)^{1+\beta}+\gamma^{1+\beta})}{1+\beta}+\frac{d\alpha(-\gamma^{2+\beta}+\gamma^{3+\beta})}{(1+\beta)(2+\beta)(3+\beta)} \end{array}\right) \quad (14)$$

Providing that Equation satisfies the following condition

$$\left(\frac{\partial^2 TC}{\partial t_1^2}\right)\left(\frac{\partial^2 TC}{\partial t_2^2}\right)-\left(\frac{\partial^2 TC}{\partial t_1\partial t_2}\right)>0$$

By solving equation (13) and (14), the value of t_1 and t_2 can be obtained.

NUMERICAL EXAMPLE

Let us consider an inventory system with the following parametric in their proper units as given in assumptions [A, a,b, α, β, γ, r, d] = [300, 12,8, 0.07, 4, 0.3, 0.5, 10]. Using these values, we get, t_1^*= 1.50444. Putting the optimum values of t_1^* in equation (8) and equation (12) we get the optimum values of Holding Cost (HC) =125.876 and minimum Total average cost per unit time (TC) = 330.522 respectively.

SENSITIVITY ANALYSIS

Sensitivity Analysis depicts the extent to which the optimal solution of the model is affected by the Changes in its input parameter values. Here, we study the sensitivity for the cycle length T, Holding cost HC and total cost per time unit TC with respect to the Changes in the values of the parameters α, β and γ.

We will be using values of the numerical example given in the previous section for performing the senility analysis. The results are given in tabulated form in Table 1 to Table 3.

Table 1 Variation in deterioration rate "α"

Parameter A	Change in t_1	Change in HC	Change in TC
0.06	1.51944	127.023	329.678
0.07	1.50444	125.876	330.522
0.08	1.49098	124.833	331.301

In Table 1, shows that increasing the deterioration rate 'α' results in decrease of Total Cost (TC) of an inventory system and also decreases the Holding Cost (HC).

Table 2: Variation in deterioration rate "β"

Parameter Β	Change in t_1	Change in HC	Change in TC
4	1.50444	125.876	330.522
5	1.48407	123.861	331.225
6	1.46704	122.09ssss	331.785

By Table 2, It is observed that increase of deterioration rate 'β' results in decrease the Total Cost (TC) of an inventory system and also decrease the Holding Cost (HC).

Table 3.3: Variation in deterioration rate "γ"

Parameter γ	Change in t_1	Change in HC	Change in TC
0.1	1.44011	120.989	334.727
0.2	1.47264	123.523	332.556
0.3	1.50444	125.876	330.522

In Table 3, It is observe that increase in deterioration rate 'γ' results in increase Total Cost (TC) of an inventory system and also increase in Holding Cost (HC).

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